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VII. *On the Structure and Development of* Lepidosteus.

By F. M. BALFOUR, *LL.D., F.R.S., Fellow of Trinity College, Cambridge, and*
W. N. PARKER, *Assistant in the Biological Laboratory of the Normal School of Science, South Kensington.*

Received November 24,—Read December 8, 1881.

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INTRODUCTION.

THE following paper is the outcome of the very valuable gift of a series of embryos and larvæ of *Lepidosteus* by Professor ALEX. AGASSIZ, to whom we take this opportunity of expressing our most sincere thanks. The skull of these embryos and larvæ has been studied by Professor PARKER, and forms the subject of a memoir already presented to the Royal Society.

Considering that *Lepidosteus* is one of the most interesting of existing Ganoids, and that it is very closely related to species of Ganoids which flourished during the Triassic period, we naturally felt keenly anxious to make the most of the opportunity of working at its development offered to us by Professor AGASSIZ' gift. Professor AGASSIZ, moreover, most kindly furnished us with four examples of the adult Fish, which have enabled us to make this paper a study of the adult anatomy as well as of the development.

The first part of our paper is devoted to the segmentation, formation of the germinal layers, and general development of the embryo and larva. The next part consists of a series of sections on the organs, in which both their structure in the adult and their development are dealt with. This part is not, however, in any sense a monograph, and where already known, the anatomy is described with the greatest possible brevity. In this part of the paper considerable space is devoted to a comparison of the organs of *Lepidosteus* with those of other Fishes, and to a statement of the conclusions which follow from such comparison.

The last part of the paper deals with the systematic position of *Lepidosteus* and of the Ganoids generally.

GENERAL DEVELOPMENT.

The spawning of *Lepidosteus* takes place in the neighbourhood of New York about May 20th. AGASSIZ (No. 1)* gives an account of the process from Mr. S. W. GARMAN'S notes, which we venture to quote in full.

"Black Lake is well stocked with Bill-fish. When they appear, they are said to come in countless numbers. This is only for a few days in the spring, in the spawning season, between the 15th of May and the 8th of June. During the balance of the season they are seldom seen. They remain in the deeper parts of the lake, away from the shore, and, probably, are more or less nocturnal in habits. Out of season, an occasional one is caught on a hook baited with a Minnow. Commencing with the 20th of April, until the 14th of May we were unable to find the Fish, or to find persons who had seen them during this time. Then a fisherman reported having seen one rise to the surface. Later, others were seen. On the afternoon of the 18th, a few were found on the *points*, depositing the spawn. The temperature at the time was 68° to 69°

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* The numbers refer to the list of memoirs of the anatomy and development given at the end of this memoir (p. 433).

on the shoals, while out in the lake the mercury stood at 62° to 63°. The *points* on which the eggs were laid were of naked granite, which had been broken by the frost and heat into angular blocks of 3 to 8 inches in diameter. The blocks were tumbled upon each other like loose heaps of brickbats, and upon and between them the eggs were dropped. The *points* are the extremities of small capes that make out into the lake. The eggs were laid in water varying in depth from 2 to 14 inches. At the time of approaching the shoals, the Fish might be seen to rise quite often to the surface to take air. This they did by thrusting the bill out of the water as far as the corners of the mouth, which was then opened widely and closed with a snap. After taking the air, they seemed more able to remain at the surface. Out in the lake they are very timid, but once buried upon the shoals they become quite reckless as to what is going on about them. A few moments after being driven off, one or more of the males would return as if scouting. If frightened, he would retire for some time; then another scout would appear. If all promised well, the females, with the attendant males, would come back. Each female was accompanied by from one to four males. Most often, a male rested against each side, with their bills reaching up toward the back of her head. Closely crowded together, the little party would pass back and forth over the rocky bed they had selected, sometimes passing the same spot half-a-dozen times without dropping an egg, then suddenly would indulge in an orgasm; and, lashing and plashing the water in all directions with their convulsive movements, would scatter at the same instant the eggs and the sperm. This ended, another season of moving slowly back and forth was observed, to be in turn followed by another of excitement. The eggs were excessively sticky. To whatever they happened to touch, they stuck, and so tenaciously that it was next to impossible to release them without tearing away a portion of their envelopes. It is doubtful whether the eggs would hatch if removed. As far as could be seen at the time, upon or under the rocks to which the eggs were fastened there was an utter absence of anything that might serve as food for the young Fishes.

"Other Fishes, Bull-heads, &c., are said to follow the Bill-fish to eat the spawn. It may be so. It was not verified. Certainly the points under observations were unmolested. During the afternoon of the 18th of May a few eggs were scattered on several of the beds. On the 19th there were more. With the spear and the snare, several dozens of both sexes of the Fish were taken. Taking one out did not seem greatly to startle the others. They returned very soon. The males are much smaller than the average size of the females; and, judging from those taken, would seem to have as adults greater uniformity in size. The largest taken was a female, of 4 feet 1½ inch in length. Others of 2 feet 6 inches contained ripe ova. With the 19th of May all disappeared, and for a time—the weather being meanwhile cold and stormy—there were no signs of their continued existence to be met with. Nearly two weeks later, on the 31st of May, as stated by Mr. HENRY J. PERRY, they again came up, not in small detachments on scattered points as before, but in multitudes, on every shoal at all according with their ideas of spawning beds. They remained but two days. During the summer it happens now and then that one is seen to come up for his mouthful of air; beyond this there will be nothing to suggest the ravenous masses hidden by the darkness of the waters."

Egg membranes.—The ova of *Lepidosteus* are spherical bodies of about 3 millims. in diameter. They have a double investment consisting of (1) an outer covering formed of elongated, highly refractive bodies, somewhat pyriform at their outer ends (Plate 21, fig. 17, *f.e.*), which are probably metamorphosed follicular cells,* and (2) of an inner membrane, divided into two zones, viz.: an outer and thicker zone, which is radially striated, and constitutes the *zona radiata* (*z.r.*), and an inner and narrow homogeneous zone (*z.r′.*).

Segmentation.—We have observed several stages in the segmentation, which show that it is complete, but that it approaches the meroblastic type more nearly than in the case of any other known holoblastic ovum.

Our earliest stage showed a vertical furrow at the upper or animal pole, extending through about one-fifth of the circumference (Plate 21, fig. 1), and in a slightly later stage we found a second similar furrow at right angles to the first (Plate 21, fig. 2). We have not been fortunate enough to observe the next phases of the segmentation, but on the second day after impregnation (Plate 21, fig. 3), the animal pole is completely divided into small segments, which form a disc, homologous to the blastoderm of meroblastic ova; while the vegetative pole, which subsequently forms a large yolk-sac, is divided by a few vertical furrows, four of which nearly meet at the pole opposite the blastoderm (Plate 21, fig. 4). The majority of the vertical furrows extend only a short way from the edge of the small spheres, and are partially intercepted by imperfect equatorial furrows.

Development of the embryo.—We have not been able to work out the stages immediately following the segmentation, owing to want of material; and in the next stage satisfactorily observed, on the third day after impregnation, the body of the embryo is distinctly differentiated. The lower pole of the ovum is then formed of a mass in which no traces of the previous segments or segmentation furrows could any longer be detected.

Some of the dates of the specimens sent to us appear to have been transposed; so that our statements as to ages must only be taken as *approximately* correct.

Third day after impregnation.—In this stage the embryo is about 3·5 millims. in length, and has a somewhat dumb-bell shaped outline (Plate 21, fig. 5). It consists of (1) an outer area (*p.z.*) with some resemblance to the area pellucida of the Avian embryo, forming the parietal part of the body; and (2) a central portion consisting of the vertebral and medullary plates and the axial portions of the embryo. In hardened specimens the peripheral part forms a shallow depression surrounding the central part of the embryo.

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* We have examined the structure of the ovarian ova in order to throw light on the nature of these peculiar pyriform bodies. Unfortunately, the ovaries of our adult examples of *Lepidosteus* were so badly preserved, that we could not ascertain anything on this subject. The ripe ova in the ovary have an investment of pyriform bodies similar to those of the just laid ova.

With reference to the structure of the ovarian ova we may state that the germinal vesicles are provided with numerous nucleoli arranged in close proximity with the membrane of the vesicle.

The central part constitutes a somewhat prominent ridge, the axial part of it being the medullary plate. Along the anterior half of this part a dark line could be observed in all our specimens, which we at first imagined to be caused by a shallow groove. We have, however, failed to find in our sections a groove in this situation except in a single instance (Plate 22, fig. 20, *x*), and are inclined to attribute the appearance above-mentioned to the presence of somewhat irregular ridges of the outer layer of the epiblast, which have probably been artificially produced in the process of hardening.

The anterior end of the central part is slightly dilated to form the brain (*b.*); and there is present a pair of lateral swellings near the anterior end of the brain which we believe to be the commencing optic vesicles. We could not trace any other clear indications of the differentiation of the brain into distinct lobes.

At the hinder end of the central part of the embryo a very distinct dilatation may also be observed, which is probably homologous with the tail swelling of Teleostei. Its structure is more particularly dealt with in the description of our sections of this stage.

After the removal of the egg-membranes described above we find that there remains a delicate membrane closely attached to the epiblast. This membrane can be isolated in distinct portions, and appears to be too definite to be regarded as an artificial product.

We have been able to prepare several more or less complete series of sections of embryos of this stage (Plate 22, figs. 18–22). These sections present as a whole a most striking resemblance to those of Teleostean embryos at a corresponding stage of development.

Three germinal layers are already fully established. The epiblast (*ep.*) is formed of the same parts as in Teleostei, viz.:—of an outer epidermic and an inner nervous or mucous stratum. In the parietal region of the embryo these strata are each formed of a single row of cells only. The cells of both strata are somewhat flattened, but those of the epidermic stratum are decidedly the more flattened of the two.

Along the axial line there is placed, as we have stated above, the medullary plate. The epidermic stratum passes over this plate without undergoing any change of character, and the plate is *entirely constituted of the nervous stratum of the epidermis.*

The medullary plate has, roughly speaking, the form of a solid keel, projecting inwards towards the yolk. There is no trace, at this stage at any rate, of a medullary groove; and as we shall afterwards show, the central canal of the cerebro-spinal cord is formed in the middle of the solid keel. The shape of this keel varies according to the region of the body. In the head (Plate 22, fig. 18, *m.c.*), it is very prominent, and forming, as it does, the major part of the axial tissue of the body, impresses its own shape on the other parts of the head and gives rise to a marked ridge on the surface of the head directed towards the yolk. In the trunk (Plate 22, figs. 19, 20) the keel is much less prominent, but still projects sufficiently to give a convex form to the surface of the body turned towards the yolk.

In the head, and also near the hind end of the trunk, the nervous layer of the epiblast continuous with the keel on each side is considerably thicker than the lateral parts of the layer. The thickening of the nervous layer in the head gives rise to what has been called by GÖTTE* "the special sense plate," owing to its being subsequently concerned in the formation of parts of the organs of special sense. We cannot agree with GÖTTE in regarding it as part of the brain.

In the keel itself two parts may be distinguished, viz.: a superficial part, best marked in the region of the brain, formed of more or less irregularly arranged polygonal cells, and a deeper part of horizontally placed flatter cells. The upper part is mainly concerned in the formation of the cranial nerves, and of the dorsal roots of the spinal nerves.

The mesoblast (*ms.*) in the trunk consists of a pair of independent plates which are continued forwards into the head, and in the prechordal region of the latter, unite below the medullary keel.

The mesoblastic plates of the trunk are imperfectly divided into vertebral and lateral regions. Neither longitudinal sections nor surface views show at this stage any trace of a division of the mesoblast into somites. The mesoblast cells are polygonal, and no indication is as yet present of a division into splanchnic and somatic layers.

The notochord (*nc.*) is well established, so that its origin could not be made out. It is, however, much more sharply separated from the mesoblastic plates than from the hypoblast, though the ventral and inner corners of the mesoblastic plates which run in underneath it on either side, are often imperfectly separated from it. It is formed of polygonal cells, of which between 40 and 50 may as a rule be seen in a single section. No sheath is present around it. It has the usual extension in front.

The hypoblast (*hy.*) has the form of a membrane, composed of a single row of oval cells, bounding the embryo on the side adjoining the yolk.

In the region of the caudal swelling the relations of the germinal layers undergo some changes. This region may, from the analogy of other Vertebrates, be assumed to constitute the lip of the blastopore. We find accordingly that the layers become more or less fused. In the anterior part of the tail swelling, the boundary between the notochord and hypoblast becomes indistinct. A short way behind this point (Plate 22, fig. 21), the notochord unites with the medullary keel, and a neurenteric cord, homologous with the neurenteric canal of other Ichthyopsida, is thus established. In the same region the boundary between the lateral plates of mesoblast and the notochord, and further back (Plate 22, fig. 22), that between the mesoblast and the medullary keel, becomes obliterated.

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* "Üb. d. Entwick. d. Central-Nerven Systems d. Teleostier," Archiv für mikr. Anat., vol. xv., 1878.

Fifth day after impregnation.—Between the stage last described and the next stage of which we have specimens, a considerable progress has been made. The embryo (Plate 21, figs. 6 and 7) has grown markedly in length and embraces more than half the circumference of the ovum. Its general appearance is, however, much the same as in the earlier stage, but in the cephalic region the medullary plate is divided by constrictions into three distinct lobes, constituting the regions of the fore-brain, the mid-brain, and the hind-brain. The fore-brain (Plate 21, fig. 6, *f.b.*) is considerably the largest of the three lobes, and a pair of lateral projections forming the optic vesicles are decidedly more conspicuous than in the previous stage. The mid-brain (*m.b.*) is the smallest of the three lobes, while the hind-brain (*h.b.*) is decidedly longer, and passes insensibly into the spinal cord behind.

The medullary keel, though retaining to a great extent the shape it had in the last stage, is no longer completely solid. Throughout the whole region of the brain and in the anterior part of the trunk (Plate 22, figs. 23, 24, 25) a slit-like lumen has become formed. We are inclined to hold that this is due to the appearance of a space between the cells, and not, as supposed by OELLACHER for Teleostei, to an actual absorption of cells, though we must admit that our sections are hardly sufficiently well preserved to be conclusive in settling this point. Various stages in its growth may be observed in different regions of the cerebro-spinal cord. When first formed, it is a very imperfectly defined cavity, and a few cells may be seen passing right across from one side of it to the other. It gradually becomes more definite, and its wall then acquires a regular outline.

The optic vesicles are now to be seen in section (Plate 22, fig. 23, *op.*) as flattish outgrowths of the wall of the fore-brain, into which the lumen of the third ventricle is prolonged for a short distance.

The brain has become to some extent separate from the superjacent epiblast, but the exact mode in which this is effected is not clear to us. In some sections it appears that the separation takes place in such a way that the nervous keel is only covered above by the epidermic layer of the epiblast, and that the nervous layer, subsequently interposed between the two, grows in from the two sides. Such a section is represented in Plate 22, fig. 24. Other sections again favour the view that in the isolation of the nervous keel, a superficial layer of it remains attached to the nervous layer of the epidermis at the two sides, and so, from the first, forms a continuous layer between the nervous keel and the epidermic layer of the epiblast (Plate 22, fig. 25). In the absence of a better series of sections we do not feel able to determine this point. The posterior part of the nervous keel retains the characters of the previous stage.

At the sides of the hind-brain very distinct commencements of the auditory vesicles are apparent. They form shallow pits (Plate 22, fig. 24, *au.*) of the thickened part of the nervous layer adjoining the brain in this region. Each pit is covered over by the epidermic layer above, which has no share in its formation.

In many parts of the lateral regions of the body the nervous layer of the epidermis is more than one cell deep.

The mesoblastic plates are now divided in the anterior part of the trunk into a somatic and a splanchnic layer (Plate 22, fig. 25, *so.*, *sp.*), though no distinct cavity is as yet present between these two layers. Their vertebral extremities are somewhat wedge-shaped in section, the base of the wedge being placed at the sides of the medullary keel. The wedge-shaped portions are formed of a superficial layer of palisade-like cells and an inner kernel of polygonal cells. The superficial layer on the dorsal side is continuous with the somatic mesoblast, while the remainder pertains to the splanchnic layer.

The diameter of the notochord has diminished, and the cells have assumed a flattened form, the protoplasm being confined to an axial region. In consequence of this, the peripheral layer appears clear in transverse sections. A delicate cuticular sheath is formed around it. This sheath is probably the commencement of the permanent sheath of later stages, but at this stage it cannot be distinguished in structure from a delicate cuticle which surrounds the greater part of the medullary cord.

The hypoblast has undergone no changes of importance.

The layers at the posterior end of the embryo retain the characters of the last stage.

Sixth day after impregnation.—At this stage (Plate 21, fig. 8) the embryo is considerably more advanced than at the last stage. The trunk has decidedly increased in length, and the head forms a relatively smaller portion of the whole. The regions of the brain are more distinct. The optic vesicles (*op.*) have grown outwards so as to nearly reach the edges of the area which forms the parietal part of the body. The fore-brain projects slightly in front, and the mid-brain is seen as a distinct rounded prominence. Behind the latter is placed the hind-brain, which passes insensibly into the spinal cord. On either side of the mid- and hind-brain a small region is slightly marked off from the rest of the parietal part, and on this are seen two more or less transversely directed streaks, which, by comparison with the Sturgeon,* we are inclined to regard as the two first visceral clefts (*br.c.*). We have, however, failed to make them out in sections, and owing to the insufficiency of our material, we have not even studied them in surface views as completely as we could have wished.

The body is now laterally compressed, and more decidedly raised from the yolk than in the previous stages. In the lateral regions of the trunk the two segmental or archinephric ducts (*sg.*) are visible in surface views: the front end of each is placed at the level of the hinder border of the head, and is marked by a flexure inwards towards the middle line. The remainder of each duct is straight, and extends backwards for about half the length of the embryo. The tail has much the same appearance as in the last stage.

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* SALENSKY, "Recherches s. le Développement du Sterlet." Archives de Biol., vol. ii., 1881, pl. xvii., fig. 27.

The vertebral regions of the mesoblastic plates are now segmented for the greater part of the length of the trunk, and the somites of which they are composed (Plate 23, fig. 30, *pr.*) are very conspicuous in surface views.

Our sections of this stage are not so complete as could be desired: they show, however, several points of interest.

The central canal of the nervous system is large, with well-defined walls, and in hardened specimens is filled with a coagulum. It extends nearly to the region of the tail.

The optic vesicles, which are so conspicuous in surface views, appear in section (Plate 22, fig. 26, *op.*) as knob-like outgrowths of the fore-brain, and very closely resemble the figures given by OELLACHER of these vesicles in Teleostei.*

From the analogy of the previous stage, we are inclined to think that they have a lumen continuous with that of the fore-brain. In our only section through them, however, they are solid, but this is probably due to the section merely passing through them to one side.

The auditory pits (Plate 22, fig. 27, *au.*) are now well marked, and have the form of somewhat elongated grooves, the walls of which are formed of a single layer of columnar cells belonging to the nervous layer of the epidermis, and extending inwards so as nearly to touch the brain.

In an earlier stage it was pointed out that the dorsal part of the medullary keel was different in its structure from the remainder, and that it was destined to give rise to the nerves. The process of differentiation is now to a great extent completed, and may best be seen in the auditory region (Plate 22, fig. 27, VIII.). In this region there was present during the last stage a great rhomboidal mass of cells at the dorsal region of the brain (Plate 22, fig. 24, VIII.). In the present stage, this, which is the rudiment of the seventh and auditory nerves, is seen growing down on each side from the roof of the hind-brain, between the brain and the auditory involution, and abutting against the wall of the latter.

Rudiments of the spinal nerves are also seen at intervals as projections from the dorsal angles of the spinal cord (Plate 23, fig. 29, *sp.n.*). They extend only for a short distance outwards, gradually tapering off to a point, and situated between the epiblast and the dorsal angles of the mesoblastic somites.

The process of formation of the cranial nerves and dorsal roots of the spinal nerves is, it will be seen, essentially the same as that already known in the case of Elasmobranchii, Aves, &c. The nerves arise as outgrowths of a special crest of cells, the *neural crest* of MARSHALL, which is placed along the dorsal angle of the cord. The peculiar position of the dorsal roots of the spinal nerves is also very similar to what has been met with in the early stages of these structures by MARSHALL in Birds,* and by one of us in Elasmobranchs.†

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* "Beiträge zur Entwick. d. Knochenfische," Zeit. f. wiss. Zool., vol. xxiii., 1873, taf. iii., fig. ix., 2.

In the parietal region a cavity has now appeared in part of the trunk between the splanchnic and somatic layers of the mesoblast (Plate 23, fig. 29, *b.c.*), the somatic layer (*so.*) consisting of a single row of columnar cells on the dorsal side, while the remainder of each somite is formed of the splanchnic layer (*sp.*). In many of the sections the somatic layer is separated by a considerable interval from the epiblast.

We have been able to some extent to follow the development of the segmental duct. The imperfect preservation of our specimens has, as in other instances, rendered the study of the point somewhat difficult, but we believe that the figure representing the development of the duct some way behind its front end (Plate 23, fig. 29) is an accurate representation of what may be seen in a good many of our sections.

It appears from these sections that the duct (Plate 23, fig. 29, *sg.*) is developed as a hollow ridge-like outgrowth of the somatic layer of mesoblast, directed towards the epiblast, in which it causes a slight bulging. The cavity of the ridge freely communicates with the body-cavity. The anterior part of this ridge appears to be formed first. Very soon, in fact, in an older embryo belonging to this stage, the greater part of the groove becomes segmented off as a duct lying between the epiblast and somatic mesoblast (Plate 23, fig. 28, *sg.*), while the front end still remains, as we believe, in communication with the body cavity by an anterior pore.

This mode of development corresponds in every particular with that observed in Teleostei by ROSENBERG and OELLACHER.

The structure of the notochord (*nc.*) at this stage is very similar to that observed by one of us in Elasmobranchii.‡ The cord is formed of transversely arranged flattened cells, the outer parts of which are vacuolated, while the inner parts are granular, and contain the nuclei. This structure gives rise to the appearance in transverse sections of an axial darker area and a peripheral lighter portion.

The hypoblast retains for the most part its earlier constitution, but underneath the notochord, in the trunk, it is somewhat thickened, and the cells at the two sides spread in to some extent under the thickened portion (Plate 23, fig. 29, *s.nc.*). This thickening, as is shown in transverse sections at the stage when the segmental duct becomes separated from the somatic mesoblast (Plate 23, fig. 28, *s.nc.*), is the commencement of the subnotochordal rod.

The tail end of the embryo still retains its earlier characters.

Seventh day after impregnation.—Our series of specimens of this stage is very imperfect, and we are only able to call attention to the development of a certain number of organs.

Our sections clearly establish the fact that the optic vesicles are now hollow processes of the fore-brain. Their outer ends are dilated, and are in contact with the external skin. The formation of the optic cup has not, however, commenced. The nervous layer of the skin adjoining the outer wall of the optic cup is very slightly thickened, constituting the earliest rudiment of the lens.

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* Journal of Anat. and Physiol., vol. xi., p. 491, plates xx. and xxi.

† 'Elasmobranch Fishes,' p. 156, plates x. and xiii.

‡ Ibid., p. 136, plate xi., fig. 10.

In one of our embryos of this day the developing auditory vesicle still has the form of a pit, but in the other it is a closed vesicle, already constricted off from the nervous layer of the epidermis.

With reference to the development of the excretory duct we cannot add much to what we have already stated in describing the last stage.

The duct is considerably dilated anteriorly (Plate 23, fig. 31, *sg.*); but our sections throw no light on the nature of the abdominal pore. The posterior part of the duct has still the form of a hollow ridge united with somatic mesoblast (Plate 23, fig. 32, *sg.*).

During this stage, the embryo becomes to a small extent folded off from the yolk-sac both in front and behind, and in the course of this process the anterior and posterior extremities of the alimentary tract become definitely established.

We have not got as clear a view of the process of formation of these two sections of the alimentary tract as we could desire, but our observations appear to show that the process is in many respects similar to that which takes place in the formation of the anterior part of the alimentary tract in Elasmobranchii.* One of us has shown that in Elasmobranchs the ventral wall of the throat is formed *not* by a process of folding in of the hypoblastic sheet as in Birds, but by a growth of the ventral face of the hypoblastic sheet on each side of and at some little distance from the middle line. Each growth is directed inwards, and the two eventually meet and unite, thus forming a complete ventral wall for the gut. Exactly the same process would seem to take place in *Lepidosteus*, and after the lumen of the gut is in this way established, a process of mesoblast on each side also makes its appearance, forming a mesoblastic investment on the ventral side of the alimentary tract. Some time after the alimentary tract has been thus formed, the epiblast becomes folded in, in exactly the same manner as in the Chick, the embryo becoming thereby partially constricted off from the yolk (Plate 23, figs. 33, 34).

The form of the lumen of the alimentary tract differs somewhat in front and behind. In front, the hypoblastic sheet remains perfectly flat during the formation of the throat, and thus the lumen of the latter has merely the form of a slit. The lumen of the posterior end of the alimentary tract is, however, narrower and deeper (Plate 23, figs. 33, 34, *al.*). Both in front and behind, the lateral parts of the hypoblastic sheet become separated from the true alimentary tract as soon as the lumen of the latter is established.

It is quite possible that at the extreme posterior end of the embryo a modification of the above process may take place, for in this region the hypoblast appears to us to have the form of a solid cord.

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* F. M. BALFOUR, 'Monagraph on the Development of Elasmobranch Fishes,' p. 87, plate ix., fig. 2.

We could detect no true neurenteric canal, although a more or less complete fusion of the germinal layers at the tail end of the embryo may still be traced.

During this stage the protoplasm of the notochordal cells, which in the last stage formed a kind of axial rod in the centre of the notochord, begins to spread outwards toward the sheath of the notochord.

Eighth day after impregnation.—The external form of the embryo (Plate 21, fig. 9) shows a great advance upon the stage last figured. Both head and body are much more compressed laterally and raised from the yolk, and the head end is folded off for some distance. The optic vesicles are much less prominent externally. A commencing opercular fold is distinctly seen. Our figure of this stage is not, however, so satisfactory as we could wish.

A thickening of the nervous layer of the external epiblast which will form the lens (Plate 23, fig. 35, *l.*) is more marked than in the last stage, and presses against the slightly concave exterior wall of the optic vesicle (*op.*). The latter has now a large cavity, and its stalk is considerably narrowed.

The auditory vesicles (Plate 23, fig. 36, *au.*) are closed, appearing as hollow sacs one on each side of the brain, and are no longer attached to the epiblast.

The anterior opening of the segmental duct can be plainly seen close behind the head. The lumen of the duct is considerably larger.

The two vertebral portions of the mesoblast are now separated by a considerable space from the epiblast on one side and from the notochord on the other, and the cells composing them have become considerably elongated from side to side (Plate 23, fig. 37, *ms.*).

In some sections the aorta can be seen (Plate 23, fig. 37, *ao.*) lying close under the sub-notochordal rod, between it and the hypoblast, and on either side of it a slightly larger cardinal vein (*cd.v.*).

The protoplasm of the notochord has now again retreated towards the centre, showing a clear space all round. This is most marked in the region of the trunk (Plate 23, fig. 37). The sub-notochordal rod (*s.nc.*) lies close under it.

A completely closed fore-gut, lined by thickened hypoblast, extends about as far back as the auditory sacs (Plate 23, figs. 35 and 36, *al.*). In the trunk the hypoblast, which will form the walls of the alimentary tract, is separated from the notochord by a considerable interval.

Ninth day after impregnation: External characters.—Very considerable changes have taken place in the external characters of the embryo. It is about 8 millims. in length, and has assumed a completely piscine form. The tail especially has grown in length, and is greatly flattened from side to side: it is wholly detached from the yolk, and bends round towards the head, usually with its left side in contact with the yolk. It is provided with well-developed dorsal and ventral fin-folds, which meet each other round the end of the tail, the tail fin so formed being nearly symmetrical. The head is not nearly so much folded off from the yolk as the tail. At its front end is placed a disc with numerous papillæ, of which we shall say more hereafter. This disc is somewhat bifid, and is marked in the centre by a deep depression.

Dorsal to it, on the top of the head, are two widely separated nasal pits. On the surface of the yolk, in front of the head, is to be seen the heart, just as in Sturgeon embryos. Immediately below the suctorial disc is a slit-like space, forming the mouth. It is bounded below by the two mandibular arches, which meet ventrally in the median line. A shallow but well marked depression on each side of the head indicates the posterior boundary of the mandibular arch. Behind this is placed the very conspicuous hyoid arch with its rudimentary opercular flap; and in the depression, partly covered over by the latter, may be seen a ridge, the external indication of the first branchial arch.

Eleventh day after impregnation: External characters.—The embryo (Plate 21, fig. 10) is now about 10 millims. in length, and in several features exhibits an advance upon the embryo of the previous stage.

The tail fin is now obviously not quite symmetrical, and the dorsal fin-fold is continued for nearly the whole length of the trunk. The suctorial disc (Plate 21, fig. 11, *s.d.*) is much more prominent, and the papillæ (about 30 in number) covering it are more conspicuous from the surface. It is not obviously composed of two symmetrical halves. The opercular flap is larger, and the branchial arches behind it (two of which may be made out without dissection) are more prominent.

The anterior pair of limbs is now visible in the form of two *longitudinal* folds projecting in a vertical direction from the surface of the yolk-sac at the sides of the body.

The stages subsequent to hatching have been investigated with reference to the external features and to the habits by AGASSIZ, and we shall enrich our own account by copious quotations from his memoir.

He states that the first batch were hatched on the eighth* day after being laid. " The young Fish possessed a gigantic yolk-bag, and the posterior part of the body presented nothing specially different from the general appearance of a Teleostean embryo, with the exception of the great size of the chorda. The anterior part, however, was most remarkable; and at first, on seeing the head of this young *Lepidosteus*, with its huge mouth-cavity extending nearly to the gill-opening, and surmounted by a hoof-shaped depression edged with a row of protuberances acting as suckers, I could not help comparing this remarkable structure, so utterly unlike anything in Fishes or Ganoids, to the Cyclostomes, with which it has a striking analogy. This organ is also used by *Lepidosteus* as a sucker, and the moment the young Fish is hatched he attaches himself to the sides of the disc, and there remains hanging immovable; so firmly attached, indeed, that it requires considerable commotion in the water to make him loose his hold. Aerating the water by pouring it from a height did not always produce sufficient disturbance to loosen the young Fishes. The eye, in this stage, is rather less advanced than in corresponding stages in bony Fishes; the brain is also comparatively smaller, the otolith ellipsoidal, placed obliquely in the rear above the gill-opening. . . . Usually the gill-cover is pressed closely against the sides of the body, but in breathing an opening is seen through which water is constantly passing, a strong current being made by the rapid movement of the pectorals, against the base of which the extremity of the gill-cover is closely pressed. The large yolk-bag is opaque, of a bluish-gray colour. The body of the young *Lepidosteus* is quite colourless and transparent. The embryonic fin is narrow, the dorsal part commencing above the posterior end of the yolk-bag; the tail is slightly rounded, the anal opening nearer the extremity of the tail than the bag. The intestine is narrow, and the embryonic fin extending from the vent to the yolk-bag is quite narrow. In a somewhat more advanced stage,—hatched a few hours earlier,—the upper edge of the yolk-bag is covered with black pigment cells, and minute black pigment cells appear on the surface of the alimentary canal. There are no traces of embryonic fin-rays either in this stage or the one preceding; the structure of the embryonic fin is as in bony Fishes—previous to the appearance of these embryonic fin-rays—finely granular. Seen in profile, the yolk-bag is ovoid; as seen from above, it is flattened, rectangular in front, with rounded corners, tapering to a rounded point towards the posterior extremity, with re-entering sides."

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* This statement of AGASSIZ does not correspond with the dates on the specimens sent to us—a fact no doubt due to the hatching not taking place at the same time for all the larvæ.

We have figured an embryo of 11 millims. in length, shortly after hatching (Plate 21, fig. 12), the most important characters of which are as follows:—The yolk-sac, which has now become much reduced, forms an appendage attached to the ventral surface of the body, and has a very elongated form as compared with its shape just before hatching. The mouth, as also noticed by AGASSIZ, has a very open form. It is (Plate 21, fig. 13, *m.*) more or less rhomboidal, and is bounded behind by the mandibular arch (*mn.*) and laterally by the superior maxillary processes (*s.mx.*). In front of the mouth is placed the suctorial disc (*s.d.*), the central papillæ of which are arranged in groups. The opercular fold (*h.op.*) is very large, covering the arches behind. A well-marked groove is present between the mandibular and opercular arches, but so far as we can make out it is not a remnant of the hyomandibular cleft.

The pectoral fins (Plate 21, fig. 12, *pc.f.*) are very prominent longitudinal ridges, which, owing to their being placed on the surface of the yolk-sac, project in a nearly vertical direction: a feature which is also found in many Teleostean embryos with large yolk-sacs.

No traces of the pelvic fins have yet become developed.

The positions of the permanent dorsal, anal, and caudal fins, as pointed out by AGASSIZ, are now indicated by a deposit of pigment in the embryonic fin.

In an embryo on the sixth day after hatching, of about 15 millims. in length, of which we have also given a figure (Plate 21, fig. 14), the following fresh features deserve special notice.

In the region of the head there is a considerable elongation of the pre-oral part, forming a short snout, at the end of which is placed the suctorial disc. At the sides of the snout are placed the nasal pits, which have become somewhat elongated anteriorly.

The mouth has lost its open rhomboidal shape, and has become greatly narrowed in an antero-posterior direction, so that its opening is reduced to a slit. The mandibles and maxillary processes are nearly parallel, though both of them are very much shorter than in the adult. The operculum is now a very large flap, and has extended so far backwards as to cover the insertion of the pectoral fin. The two opercular folds nearly meet ventrally.

The yolk-sac is still more reduced in size, one important consequence of which is that the pectoral fins (*pc.f.*) appear to spring out more or less horizontally from the sides of the body, and at the same time their primitive line of attachment to the body becomes transformed from a longitudinal to a more or less transverse one.

The first traces of the pelvic fins are now visible as slight longitudinal projections near the hinder end of the yolk-sac (*pl.f.*).

The pigmentation marking the regions of the permanent fins has become more pronounced, and it is to be specially noted that the ventral part of the caudal fin (the permanent caudal) is considerably more prominent than the dorsal fin opposite to it.

The next changes, as AGASSIZ points out, "are mainly in the lengthening of the snout; the increase in length both of the lower and upper jaw; the concentration of the sucker of the sucking disc; and the adoption of the general colouring of somewhat older Fish. The lobe of the pectoral has become specially prominent, and the outline of the fins is now indicated by a fine milky granulation. Seen from above, the gill-cover is seen to leave a large circular opening leading to the gill-arches, into which a current of water is constantly passing, by the lateral expansion and contraction of the gill-cover; the outer extremity of the gill-cover covers the base of the pectorals. In a somewhat older stage the snout has become more elongated, the sucker more concentrated, and the disproportionate size of the terminal sucking-disc is reduced; the head, when seen from above, becoming slightly elongated and pointed."

In a larva of about 18 days old and 21 millims. in length, of which we have not given a figure, the snout has grown greatly in length, carrying with it the nasal organs, the openings of which now appear to be divided into two parts. The suctorial disc is still a prominent structure at the end of the snout. The lower jaw has elongated correspondingly with the upper, so that the gape is very considerable, though still very much less than in the adult.

The opercular flaps overlap ventrally, the left being superficial. They still cover the bases of the pectoral fins. The latter are described by AGASSIZ as being "kept in constant rapid motion, so that the fleshy edge is invisible, and the vibration seems almost involuntary, producing a constant current round the opening leading into the cavity of the gills."

The pelvic fins are somewhat more prominent.

The yolk-sac, as pointed out by AGASSIZ, has now disappeared as an external appendage.

After the stage last described the young Fish rapidly approaches the adult form. To show the changes effected we have figured the head of a larva of about a month old and 23 millims. in length (Plate 21, fig. 15). The suctorial disc, though much reduced, is still prominent at the end of the snout. Eventually, as shown by AGASSIZ, it forms the fleshy globular termination of the upper jaw.

The most notable feature in which the larva now differs in its external form from the adult is in the presence of an externally heterocercal tail, caused by the persistence of the primitive caudal fin as an elongated filament projecting beyond the permanent caudal (Plate 28, fig. 68).

Delicate dermal fin-rays are now conspicuous in the peripheral parts of all the permanent fins. These rays closely resemble the horny fin-rays in the fins of embryo Elasmobranchs in their development and structure. They appear gradually to enlarge to form the permanent rays, and we have followed out some of the stages of their growth, which is in many respects interesting. Our observations are not, however, complete enough to publish, and we can only say here that their early development and structure proves their homology with the horny fibres or rays in fins of Elasmobranchii. The skin is still, however, entirely naked, and without a trace of its future armour of enamelled scales.

The tail of a much older larva, 11 centims. in length, in which the scales have begun to be formed, is shown in Plate 21, fig. 16.

We complete this section of our memoir by quoting the following passages from AGASSIZ as to the habits of the young fish at the stages last described :—

"In the stages intervening between plate iii, fig. 19, and plate iii, fig. 30, the young *Lepidosteus* frequently swim about, and become readily separated from their point of attachment. In the stage of plate iii, fig. 30, they remain often perfectly quiet close to the surface of the water; but, when disturbed, move very rapidly about through the water. . . . The young already have also the peculiar habit of the adult of coming to the surface to swallow air. When they go through the process under water of discharging air again they open their jaws wide, and spread their gill-covers, and swallow as if they were choking, making violent efforts, until a minute bubble of air has become liberated, when they remain quiet again. The resemblance to a Sturgeon in the general appearance of this stage of the young *Lepidosteus* is quite marked."

BRAIN.

I. *Anatomy.*

The brain of *Lepidosteus* has been figured by BUSCH (whose figure has been copied by MIKLUCHO-MACLAY, and apparently by HUXLEY), by OWEN, and by WILDER

(No. 15). The figure of the latter author, representing a longitudinal section through the brain, is the most satisfactory, the other figures being in many respects inaccurate; but even WILDER's figure and description, though taken from the fresh object, appears to us in some respects inadequate. He offers, moreover, fresh interpretations of certain parts of the brain which we shall discuss in the sequel.

We have examined two brains which, though extremely soft, were, nevertheless, sufficiently well preserved to enable us to study the external form. We have, moreover, made a complete series of transverse sections through one of the brains, and our sections, though utterly valueless from a histological point of view, have thrown some light on the topographical anatomy of the brain.

Plate 25, figs. 47 A, B, and C, represent three views of the brain, viz.: from the side, from above, and from below. We will follow in our description the usual division of the brain into fore-brain, mid-brain, and hind-brain.

The fore-brain consists of an anterior portion forming the cerebrum, and a posterior portion constituting the thalamencephalon.

The cerebrum at first sight appears to be composed of (*a*) a pair of posterior and somewhat dorsal lobes, forming what have usually been regarded as the true cerebral hemispheres, but called by WILDER the prothalami, and (*b*) a pair of anterior and ventral lobes, usually regarded as the olfactory lobes, from which the olfactory nerves spring. Mainly from a comparison with our embryonic brains described in the sequel, we are inclined to think that the usual interpretations are not wholly correct, but that the true olfactory lobes are to be sought for in small enlargements (Plate 25, figs. 47 A, B, and C, *olf.*) at the front end of the brain * from which the olfactory nerves spring. The cerebrum proper would then consist of a pair of anterior and ventral lobes (*ce.*), and of a pair of posterior lobes (*ce'.*), both pairs uniting to form a basal portion behind.

The two pairs of lobes probably correspond with the two parts of the cerebrum of the Frog, the anterior of which, like that of *Lepidosteus*, was held to be the olfactory lobe, till GÖTTE's researches showed that this view was not tenable.

The anterior lobes of the cerebrum have a conical form, tapering anteriorly, and are completely separated from each other. The posterior lobes, as is best shown in side views, have a semicircular form. Viewed from above they appear as rounded prominences, and their dorsal surface is marked by two conspicuous furrows (Plate 25, fig. 47 B, *ce'.*), which have been noticed by WILDER, and are similar to those present in many Teleostei. Their front ends overhang the base of the anterior cerebral lobes. The basal portion of the cerebrum is an undivided lobe, the anterior wall of which forms the lamina terminalis.

What we have above described as the posterior cerebral lobes have been described by WILDER as constituting the everted dorsal border of the basal portion of the cerebrum.

The portion of the cerebro-spinal canal within the cerebrum presents certain primitive characters, which are in some respects dissimilar to those of higher types, and have led WILDER to hold the posterior cerebral lobes, together with what we have called the basal portion of the cerebrum, to be structures peculiar to Fishes, for which he has proposed the name "prothalami."

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* The homologies of the olfactory lobes throughout the group of Fishes require further investigation.

In the basal portion of the cerebrum there is an unpaired slit-shaped ventricle, the outer walls of which are very thick. It is provided with a floor formed of nervous matter, in part of which, judging from WILDER's description, a well-marked commissure is placed. We have found in the larva a large commissure in this situation (Plate 24, figs. 44 and 45, *a.c.*); and it may be regarded as the homologue of the anterior commissure of higher types. This part of the ventricle is stated by WILDER to be without a roof. This appears to us highly improbable. We could not, however, determine the nature of the roof from our badly preserved specimens, but if present, there is no doubt that it is extremely thin, as indeed it is in the larva (Plate 24, fig. 46 B). In a dorsal direction the unpaired ventricle extends so as to separate the two posterior cerebral lobes. Anteriorly the ventricle is prolonged into two horns, which penetrate for a short distance, as *the lateral ventricles*, into the base of the anterior cerebral lobes. The front part of each anterior cerebral lobe, as well as of the whole of the posterior lobes, appears solid in our sections; but WILDER describes the anterior horns of the ventricle as being prolonged for the whole length of the anterior lobes.

In the embryos of all Vertebrates the cerebrum is not at first divided into two lobes, so that the fact of the posterior part of the cerebrum in *Lepidosteus* and probably other Ganoids remaining permanently in the undivided condition does not appear to us a sufficient ground for giving to the lobes of this part of the cerebrum the special name of prothalami, as proposed by WILDER, or for regarding them as a section of the brain peculiar to Fishes.

The thalamencephalon (*th.*) contains the usual parts, but is in some respects peculiar. Its lateral walls, forming the optic thalami, are thick, and are not sharply separated in front from the basal part of the cerebrum; between them is placed the third ventricle. The thalami are of considerable extent, though partially covered by the optic lobes and the posterior lobes of the cerebrum. They are not, however, relatively so large as in other Ganoid forms, more especially the Chondrostei and *Polypterus*.

On the roof of the thalamencephalon is placed a large thin-walled vesicle (Plate 25, figs. 47 A and B, *v.th.*), which undoubtedly forms the most characteristic structure connected with this part of the brain. Owing to the wretched state of preservation of the specimens, we have found it impossible to determine the exact relations of this body to the remainder of the thalamencephalon; but it appears to be attached to the roof of the thalamencephalon by a narrow stalk only. It extends forwards so as to overlap part of the cerebrum in front, and is closely invested by a highly vascular layer of the pia mater.

No mention is made by WILDER of this body; nor is it represented in his figures or in those of the other anatomists who have given drawings of the brain of *Lepidosteus*. It might at first be interpreted as a highly-developed pineal gland, but a comparison with the brain of the larva (vide p. 378) shows that this is not the case, but that the body in question is represented in the larva by a special outgrowth of the roof of the thalamencephalon. The vesicle of the roof of the thalamencephalon is therefore to be regarded as a peculiar development of the tela choroidea of the third ventricle.

How far this vesicle has a homologue in the brains of other Ganoids is not certain, since negative evidence on this subject is all but valueless. It is possible that a vesicular sac covering over the third ventricle of the Sturgeon described by STANNIUS,* and stated by him to be wholly formed of the membranes of the brain, is really the homologue of our vesicle.

WIEDERSHEIM† has recently described in *Protopterus* a body which is undoubtedly homologous with our vesicle, which he describes in the following way :—

"Dorsalwärts ist das Zwischenhirn durch ein tiefes, von Hirnschlitz eingenommenes Thal von Vorderhirn abgesetzt; dasselbe ist jedoch durch eine häutige, mit der Pia mater zusammenhängende Kuppel oder Kapsel überbrückt."

This "Kuppel" has precisely the same relations and a very similar appearance to our vesicle. The true pineal gland is placed behind it. It appears to us possible that the body found by HUXLEY‡ in *Ceratodus*, which he holds to be the pineal gland, is in reality this vesicle. It is moreover possible that what has usually been regarded as the pineal gland in *Petromyzon* may in reality be the homologue of the vesicle we have found in *Lepidosteus*.

We have no observations on the pineal gland of the adult, but must refer the reader for the structure and relations of this body to the embryological section.

The infundibulum (Plate 25, fig. 47 A, *in.*) is very elongated. Immediately in front of it is placed the optic chiasma (Plate 25, figs. 47 A and C, *op.ch.*) from which the optic fibres can be traced passing along the sides of the optic thalami and to the optic lobes, very much as in MÜLLER's figure of the brain of *Polypterus*.

On the sides of the infundibulum are placed two prominent bodies, the lobi inferiores (*l.in.*), each of which contains a cavity continuous with the prolongation of the third ventricle into the infundibulum. The apex of the infundibulum is enlarged, and to it is attached a pituitary body (*pt.*).

The mid-brain is of considerable size, and consists of a basal portion connecting the optic thalami with the medulla, and a pair of large optic lobes (*op.l.*). The iter a tertio ad quartum ventriculum, which forms the ventricle of this part of the brain, is prolonged into each optic lobe, and the floor of each prolongation is taken up by a dome-shaped projection, the homologue of the torus semicircularis of Teleostei.

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[*] "Üb. d. Gehirns des Stors," MÜLLER's Archiv., 1843, and Lehrbuch d. vergl. Anat. d. Wirbelthiere. CATTIE (Archives de Biologie, vol. iii., 1882, has recently described in *Acipenser sturio* a vesicle on the roof of the thalamencephalon, whose cavity is continuous with the third ventricle. This vesicle is clearly homologous with that in *Lepidosteus*. (June 28, 1882.)

[†] R. WIEDERSHEIM, 'Morphol. Studien,' 1880, p. 71.

[‡] "On *Ceratodus Forsterei*," &c., Proc. Zool. Soc., 1876.

The hind-brain consists of the usual parts, the medulla oblongata and the cerebellum. The medulla presents no peculiar features. The sides of the fourth ventricle are thickened and everted, and marked with peculiar folds (Plate 25, figs. 47 A and B, *m.o.*).

The cerebellum is much larger than in the majority of Ganoids, and resembles in all essential features the cerebellum of Teleostei. In side views it has a somewhat S-shaped form, from the presence of a peculiar lateral sulcus (Plate 25, fig. 47 A, *cb.*). As shown by WILDER, its wall actually has in longitudinal section this form of curvature, owing to its anterior part projecting forwards into the cavity of the iter.* This forward projection is not, however, so conspicuous as in most Teleostei. The cerebellum contains a large unpaired prolongation of the fourth ventricle.

II. *Development.*

The early development of the brain has already been described; and, although we do not propose to give any detailed account of the later stages of its growth, we have thought it worth while calling attention to certain developmental features which may probably be regarded as to some extent characteristic of the Ganoids. With this view we have figured (Plate 24, figs. 44, 45) longitudinal sections of the brain at two stages, viz.: of larvæ of 15 and 26 millims., and transverse sections (Plate 24, figs. 46 A–G) of the brain of a larva at about the latter stage (25 millims.).

The original embryonic fore-brain is divided in both embryos into a cerebrum (*ce.*) in front and a thalamencephalon (*th.*) behind. In the younger embryo the cerebrum is a single lobe, as it is in the brains of all Vertebrate embryos; but in the older larva it is anteriorly (Plate 24, fig. 46 A) completely divided into two hemispheres. The roof of the undivided posterior part of the cerebrum is extremely thin (Plate 24, fig. 46 B). Near the posterior border of the base of the cerebrum there is a great development of nervous fibres, which may probably be regarded as in part equivalent to the anterior commissure (Plate 24, figs. 44, 45, *a.c.*).

Even in the oldest of the two brains the olfactory lobes are very slightly developed, constituting, however, small lateral and ventral prominences of the front end of the hemispheres. From each of them there springs a long olfactory nerve, extending for the whole length of the rostrum to the olfactory sac.

The thalamencephalon presents a very curious structure, and is relatively a more important part of the brain than in the embryo of any other form which we know of. Its roof, instead of being, as usual, compressed antero-posteriorly,† so as to be almost concealed between the cerebral hemispheres and the optic lobes (mid-brain), projects on the surface for a length quite equal to that of the cerebral hemispheres (Plate 24, figs. 44 and 45, *th.*). In the median line the roof of the thalamencephalon is thin and folded; at its posterior border is placed the opening of the small pineal gland. This body is a papilliform process of the nervous matter of the roof of this part of the brain, and instead of being directed forwards, as in most Vertebrate types, tends somewhat backwards, and rests on the mid-brain behind (Plate 24, figs. 44, 45, and 46 C and D, *pn.*). The roof of the thalamencephalon immediately in front of the pineal gland forms a sort of vesicle, the sides of which extend laterally as a pair of lobes, shown in transverse sections in Plate 24, figs. 46 C and D, as *th.l.* This vesicle becomes, we cannot doubt, the vesicle on the roof of the thalamencephalon which we have described in the adult brain. Immediately in front of the pineal gland the roof of the thalamencephalon contains a transverse commissure (Plate 24, fig. 46 C, *z.*), which is the homologue of a similarly situated commissure present in the Elasmobranch brain,* while behind the pineal gland is placed the posterior commissure. The sides of the thalamencephalon are greatly thickened, forming the optic thalami (Plate 24, figs. 46 C and D, *op.th.*), which are continuous in front with the thickened outer walls of the hemispheres. Below, the thalamencephalon is produced into a very elongated infundibulum (Plate 24, figs. 44, 45, 46 E, *in.*), the apex of which is trilobed as in Elasmobranchii and Teleostei. The sides of the infundibulum exhibit two lobes, the lobi inferiores (Plate 24, fig. 46 D, *l.in.*), which are continued posteriorly into the crura cerebri.

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* In WILDER's figure the walls of the cerebellum are represented as much too thin.

† Vide F. M. BALFOUR, 'Comparative Embryology,' vol. ii., figs. 243 and 250.

The pituitary body† (Plate 24, figs. 44, 45, 46 E, *pt.*) is small, not divided into lobes, and provided with a very minute lumen.

In front of the infundibulum is the optic chiasma (Plate 24, fig. 46 D, *op.ch.*), which is developed very early. It is, as stated by MÜLLER, a true chiasma.

The mid-brain (Plate 24, figs. 44 and 45, *m.b.*) is large, and consists in both stages of (1) a thickened floor forming the crura cerebri, the central canal of which constitutes the iter a tertio ad quartum ventriculum; and (2) the optic lobes (Plate 24, figs. 46 E, F, G, *op.l.*) above, each of which is provided with a cavity continuous with the median iter. The optic lobes are separated dorsally and in front by a well-marked median longitudinal groove. Posteriorly they largely overlap the cerebellum. In the anterior part of the optic lobes, at the point where the iter joins the third ventricle, there may be seen slight projections of the floor into the lumen of the optic lobes (Plate 24, fig. 46 E). These masses probably become in the adult the more conspicuous prominences of the floor of the ventricles of the optic lobes, which we regard as homologous with the tori semicirculares of the brain of the Teleostei.

The hind-brain is formed of the usual divisions, viz.: cerebellum and medulla oblongata (Plate 24, figs. 44 and 45, *cb.*, *md.*). The former constitutes a bilobed projection of the roof of the hind-brain. Only a small portion of it is during these stages left uncovered by the optic lobes, but the major part extends forwards for a considerable distance under the optic lobes, as shown in the transverse sections (Plate 24, figs. 46 F and G, *cb.*); and its two lobes, each with a prolongation of its cavity, are continued forwards beyond the opening of the iter into the fourth ventricle.

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* Vide F. M. BALFOUR, 'Comparative Embryology,' vol. ii., pp. 355–6, where it is suggested that this commissure is the homologue of the grey commissure of higher types.

† We have not been able to work out the early development of the pituitary body as satisfactorily as we could have wished. In Plate 24, fig. 40, there is shown an invagination of the oral epithelium to form it; in Plate 24, figs. 41 and 42, it is represented in transverse section in two consecutive sections. Anteriorly it is still connected with the oral epithelium (fig. 41), while posteriorly it is free. It is possible that an earlier stage of it is shown in Plate 23, fig. 35. Were it not for the evidence in other types of its being derived from the epiblast we should be inclined to regard it as hypoblastic in origin.

It is probable that the anterior horns of the cerebellum are equivalent to the prolongations of the cerebellum into the central cavity of the optic lobes of Teleostei, which are continuous with the so-called fornix of GÖTTSCHE.

III. *Comparison of the larval and adult brain of* Lepidosteus, *together with some observations on the systematic value of the characters of the Ganoid brain.*

The brain of the older of the two larvæ, which we have described, sufficiently resembles in most of its features that of the adult to render material assistance in the interpretation of certain of the parts of the latter. It will be remembered that in the adult brain the parts usually held to be olfactory lobes were described as the anterior cerebral lobes. The grounds for this will be apparent by a comparison of the cerebrum of the larva and adult. In the larva the cerebrum is formed of (1) an unpaired basal portion with a thin roof, and (2) of a pair of anterior lobes, with small olfactory bulbs at their free extremities.

The basal portion in the larva clearly corresponds in the adult with the basal portion, together with the two posterior cerebral lobes, which are merely special outgrowths of the dorsal edge of the primitive basal portion. The pair of anterior lobes have exactly the same relations in the larva as in the adult, except that in the former the ventricles are prolonged for their whole length instead of being confined to their proximal portions. If, therefore, our identifications of the larval parts of the brain are correct, there can hardly be a question as to our identifications of the parts in the adult. As concerns these identifications, the comparison of the brain of our two larvæ appears conclusive in favour of regarding the anterior lobes as parts of the cerebrum, as distinguished from the olfactory lobes, in that they are clearly derived from the undivided anterior portion of the cerebrum of the younger larva.

The comparison of the larval brain with that of the adult again appears to us to leave no doubt that the vesicle attached to the roof of the thalamencephalon in the adult is the same structure as the bilobed outgrowth of this roof in the larva; and since there is in addition a well-developed pineal gland in the larva with the usual relations, there can be no ground for identifying the vesicle in the adult with the pineal gland.

MÜLLER, in his often quoted memoir (No. 13), states that the brains of Ganoids are peculiar and distinct from those both of Teleostei and Elasmobranchii; but in addition to pointing out that the optic nerves form a chiasma he does not particularly mention the features, to which he alludes in general terms. More recently WILDER (No. 15) has returned to this subject; and though, as we have already had occasion to point out, we cannot accept all his identifications of the parts of the Ganoid brain, yet he has called attention to certain characteristic features of the cerebrum which have an undoubted systematic value.

The distinctive characters of the Ganoid brain are, in our opinion, (1) the great elongation of the region of the thalamencephalon ; and (2) the unpaired condition of the posterior part of the cerebrum, and the presence of so thin a roof to the ventricle of this part as to cause it to appear open above.

The immense length of the region of the thalamencephalon is a feature in the Ganoid brain which must at once strike any one who examines figures of the brains of Chondrostrei, *Polypterus*, or *Amia*. It is less striking in the adult *Lepidosteus*, though here also we have shown that the thalamencephalon is really very greatly developed; but in the larva of *Lepidosteus* this feature is still better marked, so that the brain of the larva may be described as being more characteristically Ganoid than that of the adult.

The presence of a largely developed thalamencephalon at once distinguishes a Ganoid brain from that of a Teleostean Fish, in which the optic thalami are very much reduced; but *Lepidosteus* shows its Teleostean affinities by a commencing reduction of this part of the brain.

The large size of the thalamencephalon is also characteristic of the Ganoid brain in comparison with the brain of the Dipnoi ; but is not however so very much more marked in the Ganoids than it is in some Elasmobranchii.

On the whole, we may consider the retention of a large thalamencephalon as a primitive character.

The second feature which we have given as characteristic of the Ganoid brain is essentially that which has been insisted upon by WILDER, though somewhat differently expressed by him.

The simplest condition of the cerebrum is that found in the larva of *Lepidosteus*, where there is an anterior pair of lobes, and an undivided posterior portion with a simple prolongation of the third ventricle, and a very thin roof. The dorsal edges of the posterior portion, adjoining the thin roof, usually become somewhat everted (cf. WILDER), and in *Lepidosteus* these edges have in the adult a very great development, and form (vide Plate 25, fig. 47 A–C, *ce'*.) two prominent lobes, which we have spoken of as the posterior cerebral lobes.

These characters of the cerebrum are perhaps even more distinctive than those of the thalamencephalon.

In Teleostei the cerebrum appears to be completely divided into two hemispheres, which are, however, all but solid, the lateral ventricles being only prolonged into their bases. In Dipnoi again there is either (*Protopterus*, WIEDERSHEIM*) a completely separated pair of oval hemispheres, not unlike those of the lower Amphibia, or the oval hemispheres are not completely separated from each other (*Ceratodus*, HUXLEY,†

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* 'Morphol. Studien,' iii. Jena, 1880.

† "On *Ceratodus Forsteri*," Proc. Zool. Soc., 1876.

Lepidosiren, HYRTL*); in either case the hemispheres are traversed for the whole length by lateral ventricles which are either completely or nearly completely separated from each other.

In Elasmobranchii the cerebrum is an unpaired though bilobed body, but traversed by two completely separated lateral ventricles, and without a trace of the peculiar membranous roof found in Ganoids.

Not less interesting than the distinguishing characters of the Ganoid brain are those cerebral characters which indicate affinities between *Lepidosteus* and other groups. The most striking of these are, as might have been anticipated, in the direction of the Teleostei.

Although the foremost division of the brain is very dissimilar in the two groups, yet the hind-brain in many Ganoids and the mid-brain also in *Lepidosteus* approaches closely to the Teleostean type. The most essential feature of the cerebellum in Teleostei is its prolongation forwards into the ventricles of the optic vesicles as the valvula cerebelli. We have already seen that there is a homologous part of the cerebellum in *Lepidosteus;* STANNIUS also describes this part in the Sturgeon, but no such part is represented in MÜLLER'S figure of the brain of *Polypterus*, or described by him in the text.

The cerebellum is in most Ganoids relatively smaller, and this is even the case with *Amia;* but the cerebellum of *Lepidosteus* is hardly less bulky than that of most Teleostei.

The presence of tori semicirculares on the floor of the mid-brain of *Lepidosteus* again undoubtedly indicates its affinities with the Teleostei, and such processes are stated by STANNIUS to be absent in the Sturgeon, and have not, so far as we are aware, been described in other Ganoids. Lastly we may point to the presence of well-developed lobi inferiores in the brain of *Lepidosteus* as an undoubted Teleostean character.

On the whole, the brain of *Lepidosteus*, though preserving its true Ganoid characters, approaches more closely to the brain of the Teleostei than that of any other Ganoid, including even *Amia.*

It is not easy to point elsewhere to such marked resemblances of the Ganoid brain, as to the brain of the Teleostei.

The division of the cerebrum into anterior and posterior lobes, which is found in *Lepidosteus*, probably reappears again, as already indicated, in the higher Amphibia. The presence of the peculiar vesicle attached to the roof of the thalamencephalon has its parallel in the brain of *Protopterus*, and as pointing in the same direction a general similarity in the appearance of the brain of *Polypterus* to that of the Dipnoi may be mentioned.

There appears to us to be in no points a close resemblance between the brain of Ganoids and that of Elasmobranchii.

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* '*Lepidosiren paradoxa.*' Prag., 1845.

SENSE ORGANS.

Olfactory organ.

Development.—The nasal sacs first arise during the late embryonic period in the form of a pair of thickened patches of the nervous layer of the epiblast on the dorsal surface of the front end of the head (Plate 24, fig. 39, *ol.*). The patches very soon become partially invaginated; and a small cavity is developed between them and the epidermic layer of the epiblast (Plate 24, figs. 42 and 43, *ol.*). Subsequently, the roof of this space, formed by the epidermic layer of the epiblast, is either broken through or absorbed; and thus open pits, *lined entirely by the nervous layer of the epidermis,* are formed.

We are not acquainted with any description of an exactly similar mode of origin of the olfactory pits, though the process is almost identical with that of the other sense organs.

We have not worked out in detail the mode of formation of the double openings of the olfactory pits, but there can be but little doubt that it is caused by the division of the single opening into two.

The olfactory nerve is formed very early (Plate 24, fig. 39, I), and, as MARSHALL has found in Aves and Elasmobranchii, it arises at a stage prior to the first differentiation of an olfactory bulb as a special lobe of the brain.

The Eye.

Anatomy.—We have not made a careful histological examination of the eye of *Lepidosteus,* which in our specimens was not sufficiently well preserved for such a purpose; but we have found a vascular membrane enveloping the vitreous humour on its retinal aspect, which, so far as we know, is unlike anything which has so far been met with in the eye of any other adult Vertebrate.

The membrane itself is placed immediately outside the hyaloid membrane, *i.e.,* on side of the hyaloid membrane bounding the vitreous humour. It is easily removed from the retina, to which it is only adherent at the entrance of the optic nerve. In both the eyes we examined it also adhered, at one point, to the capsule of the lens, but we could not make out whether this adhesion was natural, or artificially produced by the coagulation of a thin layer of albuminous matter. In one instance, at any rate, the adhesion appeared firmer than could easily be produced artificially.

The arrangement of the vessels in the membrane is shown diagrammatically in Plate 25, fig. 49, while the characteristic form of the capillary plexus is represented in Plate 25, fig. 50.

The arterial supply appears to be derived from a vessel perforating the retina close to the optic nerve, and obviously homologous with the artery of the processus falciformis and pecten of Teleostei and Birds, and with the arteria centralis retinæ of Mammals. From this vessel branches diverge and pursue a course towards the periphery. They give off numerous branches, the blood from which enters a capillary plexus (Plate 25, figs. 49 and 50) and is collected again by veins, which pass outwards and finally bend over and fall into (Plate 25, fig. 49) a circular vein (*cr.v.*) placed at the outer edge of the retina along the insertion of the iris (*ir*). The terminal branches of some of the main arteries appear also to fall directly into this vein.

The membrane supporting the vessels just described is composed of a transparent matrix, in which numerous cells are embedded (Plate 25, fig. 50).

Development.—In the account of the first stages of development of *Lepidosteus*, the mode of formation of the optic cup, the lens, &c., have been described (vide Plates 22 and 23, figs. 23, 26, 35). With reference to the later stages in the development of the eye, the only subject with which we propose to deal is the growth of the mesoblastic processes which enter the cavity of the vitreous humour through the choroid slit.

Lepidosteus is very remarkable for the great number of mesoblast cells which thus enter the cavity of the vitreous humour, and for the fact that these cells are *at first unaccompanied by any vascular structures* (Plate 24, fig. 43, *v.h.*). The mesoblast cells are scattered through the vitreous humour, and there can be no doubt that during early larval life, at a period however when the larva is certainly able to see, every histologist would consider the vitreous humour to be a tissue formed of scattered cells, with a large amount of intercellular substance; and the fact that it is so appears to us to demonstrate that KESSLER'S view of the vitreous humour being a mere transudation is not tenable.

In the larva five or six days after hatching, and about 15 millims. in length, the choroid slit is open for its whole length. The edges of the slit near the lens are folded, so as to form a ridge projecting into the cavity of the vitreous humour, while nearer the insertion of the optic nerve they cease to exhibit any such structure. The mesoblast, though it projects between the lips of the ridge near the lens, only extends through the choroid slit into the cavity of the vitreous humour in the neighbourhood of the optic nerve. Here it forms a lamina with a thickened edge, from which scattered cells in the cavity of the vitreous humour seem to radiate.

At a slightly later stage than that just described, blood-vessels become developed within the cavity of the vitreous humour, and form the vascular membrane already described in the adult, placed close to the layer of nerve-fibres of the retina, but separated from this layer by the hyaloid membrane (Plate 25, fig. 48, *v.sh.*). The artery bringing the blood to the above vascular membrane is bound up in the same sheath as the optic nerve, and passes through the choroid slit very close to the optic nerve. Its entrance into the cavity of the vitreous humour is shown in Plate 25, fig. 48 (**vs.**); its relation to the optic nerve in Plate 24, fig. 46, C and D (*vs.*).

The above sheath has, so far as we know, its nearest analogue in the eye of *Alytes*, where, however, it is only found in the larva.

The reader who will take the trouble to refer to the account of the imperfectly-developed processus falciformis of the Elasmobranch eye in the treatise 'On Comparative

Embryology,' by one of us,* will not fail to recognise that the folds of the retina at the sides of the choroid slit, and the mesoblastic process passing through this slit, are strikingly similar in *Lepidosteus* and Elasmobranchii; and that, if we are justified in holding them to be an imperfectly-developed processus falciformis in the one case, we are equally so in the other.

JOHANNES MÜLLER mentions the absence of a processus falciformis as one of the features distinguishing Ganoids and Teleostei. So far as the systematic separation of the two groups is concerned, he is probably perfectly justified in this course; but it is interesting to notice that both in Ganoids and Elasmobranchii we have traces of a structure which undergoes a very special development in the Teleostei, and that the processus falciformis of Teleostei is therefore to be regarded, not as an organ peculiar to them, but as the peculiar modification within the group of a primitive Vertebrate organ.

SUCTORIAL DISC.

One of the most remarkable organs of the larval *Lepidosteus* is the suctorial disc, placed at the front end of the head, to which we have made numerous allusions in the first section of this memoir.

The external features of the disc have been fully dealt with by AGASSIZ, and he also explained its function by observations on the habits of the larva. We have already quoted (p. 371) a passage from AGASSIZ' memoir showing how the young Fishes use the disc to attach themselves firmly to any convenient object. The discs appear in fact to be highly efficient organs of attachment, in that the young Fish can remain suspended by them to the sides of the jar, even after the water has been lowered below the level at which they are attached.

The disc is formed two or three days before hatching, and from AGASSIZ' statements, it appears to come into use immediately the young Fish is liberated from the egg membranes.

We have examined the histological structure of the disc at various ages of its growth, and may refer the reader to Plate 21, figs. 11 and 13, and Plate 24, figs. 40 and 44. The result of our examination has been to show that the disc is provided with a series of papillæ often exhibiting a bilateral arrangement. The papillæ are mainly constituted of highly modified cells of the mucous layer of the epidermis. These cells have the form of elongated columns, the nucleus being placed at the base, and the main mass of the cells being filled with a protoplasmic reticulum. They may probably be regarded as modified mucous cells. In the mesoblast adjoining the suctorial disc there are numerous sinus-like vascular channels.

It does not appear probable that the disc has a true sucking action. It is unprovided with muscular elements, and there appears to be no mechanism by which it could act as a sucking organ. We must suppose, therefore, that its adhesive power depends upon the capacity of the cells composing its papillæ to pour out a sticky secretion.

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* Vol. ii., p. 414.

MUSCULAR SYSTEM.

There is a peculiarity in the muscular system of *Lepidosteus,* which so far as we know has not been previously noticed. It is that the lateral muscles of each side are not divided, either in the region of the trunk or of the tail, into a dorso-lateral and ventro-lateral division.

This peculiarity is equally characteristic of the older larvæ as of the adult, and is shown in Plate 28, figs. 67, 72, and 73, and Plate 29, figs. 74–76. In the Cyclostomata the lateral muscles are not divided into dorsal and ventral sections; but except in this group such a division has been hitherto considered as invariable amongst Fishes.

This character must, without doubt, be held to be the indication of a very primitive arrangement of the muscular system. In the embryos of all Fishes with the usual type of the lateral muscles, the undivided condition of the muscles precedes the divided condition; and in primitive forms such as the Cyclostomata and Amphioxus the embryonic condition is retained, as it is in *Lepidosteus.*

SKELETON.

Part I.—*Vertebral column and ribs of the adult.*

A typical vertebra from the trunk of *Lepidosteus* has the following characters (Plate 29, figs. 80 and 81).

The centrum is slightly narrrower in the middle than at its two extremities. It articulates with adjacent vertebræ by a convex face in front and a concave face behind, being thus, according to OWEN'S nomenclature, opisthocœlous. It presents on its under surface a well-marked longitudinal ridge, which in many vertebræ is only united at its two extremities with the main body of the vertebra.

From the lateral borders of the centrum there project, at a point slightly nearer the front than the hind end, a pair of prominent hæmal processes (*h.a.*), to the ends of which are articulated the ribs. These processes have a nearly horizontal direction in the greater part of the trunk, though bent downwards in the tail.

The neural arches (*n.a.*) have a somewhat complicated form. They are mainly composed of two vertical plates, the breadth of the basal parts of which is nearly as great as the length of the vertebræ, so that comparatively narrow spaces are left between the neural arches of successive vertebræ for the passage of the spinal nerves. Some little way from its dorsal extremity each neural arch sends a horizontal process inwards, which meets its fellow and so forms a roof for the spinal canal. These processes appear to be confined to the posterior parts of the vertebræ, so that at the front ends of the vertebræ, and in the spaces between them, the neural canal is without an osseous roof. Above the level of this osseous roof there is a narrow passage, bounded laterally by the dorsal extremities of the neural plates. This passage is mainly filled up by a series of cartilaginous elements (Plate 29, figs. 80 and 81, *i.c.*) (probably fibro-cartilage), which rest upon the roof of the neural canal. Each element is situated *intervertebrally*, its anterior end being wedged in between the two dorsal processes of the neural arch of the vertebra in front, and its posterior end extending for some distance over the vertebra behind. The successive elements are connected by fibrous tissue, and are continuous dorsally with a fibrous band, known as the ligamentum longitudinale superius (Plate 29, figs. 80 and 81, *l.l.*), characteristic of Fishes generally, and running continuously for the whole length of the vertebral column. Each of the cartilaginous elements is, as will be afterwards shown, developed as two independent pieces of cartilage, and might be compared with the dorsal element which usually forms the keystone of the neural arch in Elasmobranchs, were not the latter vertebral instead of intervertebral in position. More or less similar elements are described by GÖTTE in the neural arches of many Teleostei, which also, however, appear to be vertebrally placed, and he has compared them and the corresponding elements in the Sturgeon with the Elasmobranch cartilages forming the keystone of the neural arch. GÖTTE does not, however, appear to have distinguished between the cartilaginous elements, and the osseus elements forming the roof of the spinal canal, which are true membrane bones; it is probable that the two are not so clearly separated in other types as in *Lepidosteus.*

The posterior ends of the neural plates of the neural arches are continued into the dorsal processes directed obliquely upwards and backwards, which have been somewhat unfortunately described by STANNIUS as rib-like projections of the neural arch. The dorsal processes of the two sides do not meet, but between them is placed a median free spinous element, also directed obliquely upwards and backwards, which forms a kind of roof for the groove in which the cartilaginous elements and the ligamentum longitudinale are placed.

The vertebræ are wholly formed of a very cellular osseous tissue, in which a distinction between the bases of the neural and hæmal processes and the remainder of the vertebra is not recognisable. The bodies of the vertebræ are, moreover, directly continuous with the neural and hæmal arches.

The ribs in the region of the trunk are articulated to the ends of the long hæmal processes. They envelop the body cavity, their proximal parts being placed immediately outside the peritoneal membrane, along the bases of the intermuscular septa. Their distal ends do not, however, remain close to the peritoneal membrane, *but pass outwards along the intermuscular septa till their free ends come into very close proximity with the skin.* This peculiarity, which holds good in the adult for all the free ribs, is shown in one of the anterior ribs of an advanced larva in Plate 28, fig. 72 (*rb.*). We are not aware that this has been previously noticed, but it appears to us to be a point not without interest in all questions which concern the homology of rib-like structures occupying different positions in relation to the muscles. Its bearings are fully dealt with in the section of this paper devoted to the consideration of the homologies of the ribs in Fishes.

As regards the behaviour of the ribs in the transitional region between the trunk and the tail, we cannot do better than translate the description given by **GEGENBAUR** of this region (No. 6, p. 411):—"Up to the 34th vertebra the ribs borne by the laterally and posteriorly directed processes present nothing remarkable, though they have gradually become shorter. The ribs of the 35th vertebra exhibit a slight curvature outwards of their free ends, a peculiarity still more marked in the 36th. The last named pair of ribs converge somewhat in their descent backwards so that both ribs decidedly approach before bending outwards. The 37th vertebra is no longer provided with freely terminating ribs, but on the contrary, the same pair of processes which in front was provided with ribs, bears a short forked process as the hæmal arch. *The two, up to this point separated ribs, have here formed a hæmal arch by the fusion of their lower ends, which arch is movable just like the ribs, and, like them, is attached to the vertebral column.*"

In the region of the tail fin the hæmal arches supporting the caudal fin rays are very much enlarged.

Part II.—*Development of the vertebral column and ribs.*

The first development and early histological changes of the notochord have already been given, and we may take up the history of the vertebral column at a period when the notochord forms a large circular rod, whose cells are already highly vacuolated, while the septa between the vacuoles form a delicate wide-meshed reticulum. Surrounding the notochord is the usual cuticular sheath, which is still thin.

The first indications of the future vertebral column are to be found in the formation of a distinct mesoblastic investment of the notochord. On the dorsal aspect of the notochord, the mesoblast forms two ridges, one on each side, which are prolonged upwards so as to meet above the neural canal, for which they form a kind of sheath. On the ventral side of the notochord there are also two ridges, which are, however, except on the tail, much less prominent than the dorsal ridges.

The changes which next ensue are practically identical with those which take place in Teleostei. Around the cuticular sheath of the notochord there is formed an elastic membrane—the membrana elastica externa. At the same time the basal parts of the dorsal, or as we may perhaps more conveniently call them, the neural ridges of the notochord become enlarged at each intermuscular septum, and the tissue of these enlargements soon becomes converted into cartilage, thus forming a series of independent paired neural processes riding on the membrana elastica externa surrounding the notochord, and extending about two-thirds of the way up the sides of the medullary cord. They are shown in transverse section in Plate 28, fig. 67 (*n.a.*), and in a side view in fig. 68 (*n.a.*).

Simultaneously with the neural arches, the hæmal arches also become established, and arise by the formation of similar enlargements of the ventral or hæmal ridges. In the trunk they are very small, but in the region of the tail their condition is very different. At the front end of the anal fin the paired hæmal arches suddenly enlarge and extend ventralwards (Plate 28, fig. 67, *h.a.*).

Each succeeding pair of arches becomes larger than the one in front, and the two elements of each arch first nearly meet below the caudal vein (Plate 28, fig. 67) and finally actually do so, forming in this way a completely closed hæmal canal. At the point where they first meet the permanent caudal fin commences, and here (Plate 28, fig. 68) we find that not only do the hæmal arches meet and coalesce below the caudal vein, but they are actually produced into long spines supporting the fin rays of the caudal fin, which thus differs from the other fins in being supported by parts of the true vertebral column and not by independently formed elements of the skeleton.

Each of the large caudal hæmal arches, including the spine, forms a continous whole, and arises at an earlier period of larval life than any other part of the vertebral column. We noticed the first indications of the neural arches in the larva of about a week old, while they are converted into fully formed cartilage in the larva of three weeks.

The neural and hæmal arches, resting on the membrana elastica externa, do not at this early stage in the least constrict the notochord. They grow gradually more definite, till the larva is five or six weeks old and about 26 millims. in length, but otherwise for a long time undergo no important changes. During the same period, however, the true sheath of the notochord greatly increases in thickness, and the membrana elastica externa becomes more definite. So far it would be impossible to distinguish the development of the vertebral column of *Lepidosteus* from that of a Teleostean Fish.

Of the stages immediately following we have unfortunately had no examples, but we have been fortunate enough to obtain some young specimens of *Lepidosteus*,* which have enabled us to work out with tolerable completeness the remainder of the developmental history of the vertebral column. In the next oldest larva, of about 5·5 centims., the changes which have taken place are already sufficient to differentiate the vertebral column of *Lepidosteus* from that of a Teleostean, and to show how certain of the characteristic features of the adult take their origin.

In the notochord the most important and striking change consists in the appearance of a series of very well marked vertebral constrictions *opposite the insertions of the neural and hæmal arches.* The first constrictions of the notochord are thus, as in other Fishes, vertebral; and although, owing to the growth of the intervertebral cartilage, the vertebral constrictions are subsequently replaced by intervertebral constrictions, yet at the same time the primitive occurrence of vertebral constrictions demonstrates that the vertebral column of *Lepidosteus* is a modification of a type of vertebral column with biconcave vertebræ.

The structure of the gelatinous body of the notochord has undergone no important change. The sheath, however, exhibits certain features which deserve careful description. In the first place the attention of the observer is at once struck by the fact that, in the vertebral regions, the sheath is much thicker (·014 millim.) than in the intervertebral (·005 millim.), and a careful examination of the sheath in longitudinal sections shows that the thickening is due to the special differentiation of a superficial part (Plate 28, fig. 69, *sh.*) of the sheath in each vertebral region. This part is somewhat granular as compared to the remainder, especially in longitudinal sections. It forms a cylinder (the wall of which is about ·01 millim. thick) in each vertebral region, immediately within the membrana elastica externa. Between it and the gelatinous tissue of the notochord within there is a very thin unmodified portion of the sheath, which is continuous with the thinner intervertebral parts of the sheath. This part of the sheath is faintly, but at the same time distinctly, concentrically striated—a probable indication of concentric fibres. The inner unmodified layer of the sheath has the appearance in transverse sections through the vertebral regions of an inner membrane, and may perhaps be KÖLLIKER's "membrana elastica interna."

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* These specimens were given to us by Professor W. K. PARKER, who received them from Professor BURT G. WILDER.

We are not aware that any similar modification of the sheath has been described in other forms.

The whole sheath is still invested by a very distinct membrana elastica externa (*m.el*).

The changes which have taken place in the parts which form the permanent vertebræ will be best understood from Plate 28, figs. 69–71. From the transverse section (fig. 70) it will be seen that there are still neural and hæmal arches resting upon the membrana elastica externa; but longitudinal sections (fig. 69) show that laterally these arches join a cartilaginous tube, embracing the intervertebral regions of the notochord, and continuous from one vertebra to the next.

It will be convenient to treat separately the neural arches, the hæmal arches with their appendages, and the intervertebral cartilaginous rings.

The neural arches, except in the fact of embracing a relatively smaller part of the neural tube than in the earlier stage, do not at first sight appear to have undergone any changes. Viewed from the side, however, in dissected specimens, they are seen to be prolonged upwards so as to unite above with bars of cartilage directed obliquely backwards. An explanation of this appearance is easily found in the sections. The cartilaginous neural arches are invested by a delicate layer of homogeneous bone, developed in the perichondrium, and this bone is prolonged beyond the cartilage and joins a similar osseous investment of the dorsal bars above mentioned. The whole of these parts may, it appears to us, be certainly reckoned as parts of the neural arches, so that at this stage each neural arch consists of: (1) a pair of basal portions resting on the notochord consisting of cartilage invested by bone, (2) of a pair of dorsal cartilaginous bars invested in bone (*n.a'.*), and (3) of osseous bars connecting (1) and (2).

Though, in the absence of the immediately preceding stages, it is not perfectly certain that the dorsal pieces of cartilage are developed independently of the ventral, there appears to us every probability that this is so; and thus the cartilage of each neural arch is developed discontinuously, while the permanent bony neural arch, which commences as a deposit of bone partly in the perichondrium and partly in the intervening membrane, forms a continuous structure.

Analogous occurrences have been described by GÖTTE in Teleostei.

The dorsal portion of each neural arch becomes what we have called the dorsal process of the adult arch.

Between the dorsal processes of the two sides there is placed a median rod of cartilage (Plate 28, fig. 70, *i.s.*), which in its development is wholly independent of the true neural arches, and which constitutes the median spinous element of the adult. In tracing these backwards it becomes obvious that they are homologous with the interspinous elements supporting the dorsal fin, in that they are replaced by these interspinous elements in the region of the dorsal fin, and that the interspinous bones occupy the same position as the median spinous processes. This homology was first pointed out by GÖTTE in the case of the Teleostei.

Immediately beneath this rod is placed the longitudinal ligament (Plate 28, fig. 70, *l.l.*), but there is as yet no trace of a junction between the neural arches of the two sides in the space between the longitudinal ligament and the spinal cord.

The basal parts of the neural arches of the two sides are united dorsally by a thin cartilaginous layer resting on the sheath of the notochord, but they are not united ventrally with the hæmal arches.

The hæmal processes in the trunk are much more prominent than in the preceding stage, and their bases are united ventrally by a tolerably thick layer of cartilage. In the trunk they are continuous with the so-called ribs of the adult (Plate 28, fig. 70); but in order to study the nature of these ribs it is necessary to trace the modifications undergone by the hæmal arches in passing from the tail to the trunk.

It will be remembered that at an earlier stage the hæmal arches in the region of the tail-fin were fully formed, and that through the anterior part of the caudal region the hæmal processes were far advanced in development, and just in front of the caudal fin had actually met below the caudal vein.

The mode of development of the hæmal arches in the tail as *unjointed* cartilaginous bars investing the caudal arteries and veins is so similar to that of the caudal hæmal arches of Elasmobranchii, that it appears to us impossible to doubt their identity in the two groups.*

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* GEGENBAUR (No. 6) takes a different view on this subject, as is clear from the following passage in this memoir (pp. 369-370) :—" Each vertebra of *Lepidosteus* thus consists of a section of the notochord, and of the cartilaginous tissue surrounding its sheath, which gives origin to the upper arches for the whole length of the vertebral column, and in the caudal region to that of the lower arches also. *The latter do not however complete the enclosure of a lower canal, but this is effected by special independent elements*, which are to be interpreted as homologues of the ribs." (The italics are ours.) While we fully accept the homology between the ribs and the lower elements of the hæmal arches of the tail, the view expressed in the italicised section, to the effect that the lower parts of the caudal arches are not true hæmal arches but are independently formed elements, is entirely opposed to our observations, and has we

The changes which have taken place by this stage with reference to the hæmal arches of the tail are not very considerable.

In the case of a few more vertebræ the hæmal processes have united into an arch, and the spinous processes of the arches in the region of the caudal fin have grown considerably in length. A more important change is perhaps the commencement of a segmentation of the distal parts of the hæmal arches from the proximal. This process has not, however, as yet resulted in a complete separation of the two, such as we find in the adult.

If the hæmal processes are traced forwards (Plate 29, figs. 75 and 76) from the anterior segment where they meet ventrally, it will be found that each hæmal process consists of a basal portion, adjoining the notochord, and a peripheral portion. These two parts are completely continuous, but the line of a future separation is indicated by the structure of the cartilage, though not shown in our figures. As the true body cavity of the trunk replaces the obliterated body cavity of the caudal region, no break of continuity will be found in the structure of the hæmal processes (Plates 28 and 29, figs. 73 and 74), but while the basal portions grow somewhat larger, the peripheral portions gradually elongate and take the form of delicate rods of cartilage extending ventralwards, on each side of the body cavity, immediately outside the peritoneal membrane, and along the lines of insertion of the intermuscular septa. These rods obviously become the ribs of the adult.

As one travels forwards the ribs become continually longer and more important, and though they are at this stage united with the hæmal processes in every part of the trunk, yet they are much more completely separated from these processes in front than behind (Plate 28, fig. 72).

In front (Plate 28, fig. 72), each rib (*rb.*), after continuing its ventral course for some distance, immediately outside the peritoneal membrane, turns outwards, and passes along one of the intermuscular septa till it reaches the epidermis. This feature in the position of the ribs is, as has been already pointed out in the anatomical part of this section, characteristic of all the ribs of the adult.

It is unfortunate that we have had no specimens showing the ribs at an earlier stage of development; but it appears hardly open to doubt that *the ribs are originally continuous with the hæmal processes,* and that the indications of a separation between those two parts at this stage are not due to a secondary fusion, but to a commencing segmentation.

It further appears, as MÜLLER, GEGENBAUR and others have stated, that the ribs and hæmal processes of the tail are serially homologous structures; but that the view maintained by GÖTTE in his very valuable memoirs on the Vertebrate skeleton is also correct to the effect that *the hæmal arches of the tail are homologous throughout the series of Fishes.*

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believe only arisen from the fact that GEGENBAUR had not the young larvæ to work with by which alone this question could be settled.

To this subject we shall return again at the end of the section.

Before leaving the hæmal arches it may be mentioned that behind the region of the ventral caudal fin the two hæmal processes merge into one, and form an unpaired knob resting on the ventral side of the notochord, and not perforated by a canal.

There are now present well-developed intervertebral rings of cartilage, each of which eventually becomes divided into two parts, and converted into the adjacent faces of the contiguous vertebræ. These rings are united with the neural and hæmal arches of the vertebræ in front and behind.

Each ring, as shown by the transverse section (Plate 28, fig. 71), is not uniformly thick, but exhibits four projections, two dorsal and two ventral. These four projections are continuous with the bases of the neural and hæmal arches of the adjacent vertebræ, and afford presumptive evidence of the derivation of the intervertebral rings from the neural and hæmal arches; in that had they so originated, it would be natural to anticipate the presence of four thickenings indicating the four points from which the cartilage had spread, while if the rings had originated independently, it would not be easy to give any explanation of the presence of such thickenings. **Gegenbaur** (No. 6), from the investigation of a much older larva than that we are now describing, also arrived at the conclusion that the intervertebral cartilages were derived from the neural and hæmal arches; but as doubts have been thrown upon this conclusion by **Götte**, and as it obviously required further confirmation, we have considered it important to attempt to settle this point. From the description given above, it is clear that we have not, however, been able absolutely to trace the origin of this cartilage, but at the same time we think that we have adduced weighty evidence in corroboration of **Gegenbaur's** view.

As shown in longitudinal section (Plate 28, fig. 69, *iv.r.*), the intervertebral rings are thicker in the middle than at the two ends. In this thickened middle part the division of the cartilage into two parts to form the ends of two contiguous vertebræ is subsequently effected. The curved line which this segmentation will follow is, however, already marked out, and from surface views it might be supposed that this division had actually occurred.

The histological structure of the intervertebral cartilage is very distinct from that of the cartilage of the bases of the arches, the nuclei being much more closely packed. In parts, indeed, the intervertebral cartilage has almost the character of fibro-cartilage. On each side of the line of division separating two vertebræ it is invested by a superficial osseous deposit.

The next oldest larva we have had was 11 centims. in length. The filamentous dorsal lobe of the caudal fin still projected far beyond the permanent caudal fin (Plate 21, fig. 16).

The vertebral column was considerably less advanced in development than that dissected by **Gegenbaur**, though it shows a great advance on the previous stage. Its features are illustrated by two transverse sections, one through the median plane of a vertebral region (Plate 29, fig. 78) and the other through that of an intervertebral region (Plate 29, fig. 79), and by a horizontal section (Plate 29, fig. 77).

In the last stage the notochord was only constricted vertebrally. Now, however, by the great growth of intervertebral cartilage there have appeared (Plate 29, fig. 77) very well-marked *intervertebral* constrictions, by the completion of which the vertebræ of *Lepidosteus* acquire their unique character amongst Fishes.

These constrictions still, however, coexist with the earlier, though at this stage relatively less conspicuous, vertebral constrictions.

The gelatinous body of the notochord retains its earlier condition. The sheath has, however, undergone some changes. In the vertebral regions there is present in any section of the sheath—(1) externally, the membrana elastica externa (*m.el.*); then (2) the external layer of the sheath (*sh.*), which is, however, less thick than before, and exhibits a very faint form of radial striation; and (3) internally, a fairly thick and concentrically striated layer. The whole thickness is, on an average, 0·18 millim.

In the intervertebral regions the membrana elastica externa is still present in most parts, but has become absorbed at the posterior border of each vertebra, as shown in longitudinal section in Plate 29, fig. 77. It is considerably puckered transversely. The sheath of the notochord within the membrana elastica externa is formed of a concentrically striated layer, continuous with the innermost layer of the sheath in the vertebral regions. It is puckered longitudinally. Thus, curiously enough, the membrana elastica externa and the sheath of the notochord in the intervertebral regions are folded in different directions, the folds of the one being only visible in transverse sections (Plate 29, fig. 79), and those of the other in longitudinal sections (Plate 29, fig. 77).

The osseous and cartilaginous structures investing the notochord may conveniently be dealt with in the same order as before, viz.: the neural arches, the hæmal arches, and the intervertebral cartilages.

The cartilaginous portions of the neural arches are still unossified, and form (Plate 29, fig. 78, *n.a.*) small wedge-shaped masses resting on the sheath of the notochord. They are invested by a thick layer of bone prolonged upwards to meet the dorsal processes (*n.a'.*), which are still formed of cartilage invested by bone.

It will be remembered that in the last stage there was no key-stone closing in the neural arch above. This deficiency is now however supplied, and consists of (1) two bars of cartilage repeated for each vertebra, but intervertebrally placed, which are directly differentiated from the ligamentum longitudinale superius, into which they merge above; and (2) two osseous plates placed on the outer sides of these cartilages, which are continuous with the lateral osseous bars of the neural arch. The former of these elements gives rise to the cartilaginous elements above the osseous bridge of the neural arch in the adult. The two osseous plates supporting these cartilages clearly form what we have called in our description of the adult the osseous roof of the spinal canal.

A comparison of the neural arch at this stage with the arch in the adult, and in the stage last described, shows that the greater part of the neural arch of the adult is formed of membrane-bone, there being preformed in cartilage only a small basal part, a dorsal process, and paired key-stones below the ligamentum longitudinale superius.

The hæmal arches (Plate 29, fig. 78) are still largely cartilaginous, and rest upon the sheath of the notochord. They are invested by a thick layer of bone. The bony layer investing the neural and hæmal arches is prolonged to form a continuous investment round the vertebral portions of the notochord (Plate 29, fig. 78). This investment is at the sides prolonged outwards into irregular processes (Plate 29, fig. 78), which form the commencement of the outer part of the thick but cellular osseous cylinder forming the middle part of the vertebral body.

The intervertebral cartilages are much larger than in the earlier stage (Plate 29, figs. 77 and 79), and it is by their growth that the intervertebral constrictions of the notochord are produced. They have ceased to be continuous with the cartilage of the arches, the intervening portion of the vertebral body between the two being only formed of bone. They are not yet divided into two masses to form the contiguous ends of adjacent vertebræ.

Externally, the part of each cartilage which will form the hinder end of a vertebral body is covered by a tube of bone, having the form of a truncated funnel, shown in longitudinal section in Plate 29, fig. 77, and in transverse section in Plate 29, fig. 79.

At each end, the intervertebral cartilages are becoming penetrated and replaced by beautiful branched processes from the homogeneous bone which was first of all formed in the perichondrium (Plate 29, fig. 77).

This constitutes the latest stage which we have had.

GEGENBAUR (No. 6) has described the vertebral column in a somewhat older larva of 18 centims.

The chief points in which the vertebral column of this larva differed from ours are: (1) the disappearance of all trace of the primitive vertebral constriction of the notochord; (2) the nearly completed constriction of the notochord in the intervertebral regions; (3) the complete ossification of the vertebral portions of the bodies of the vertebræ, the terminal so-called intervertebral portions alone remaining cartilaginous; (4) the complete ossification of the basal portions of the hæmal and neural processes included within the bodies of the vertebræ, so that in the case of the neural arch all trace of the fact that the greater part was originally not formed in cartilage had become lost. The cartilage of the dorsal spinous processes was, however, still persistent.

The only points which remain obscure in the later history of the vertebral column are the history of the notochord and of its sheath. We do not know how far these are either simply absorbed or partially or wholly ossified.

GÖTTE in his memoir on the formation of the vertebral bodies of the Teleostei attempts to prove (1) that the so-called membrana elastica externa of the Teleostei is not a homogeneous elastica, but is formed of cells, and (2) that in the vertebral regions ossification first occurs in it.

In *Lepidosteus* we have met with no indication that the membrana elastica externa is composed of cells; though it is fair to GOTTE to state that we have not examined such isolated portions of it as he states are necessary in order to make out its structure. But further than this we have satisfied ourselves that during the earlier stage of ossification this membrane is not ossified, and indeed in part becomes absorbed in proximity to the intervertebral cartilages; and GEGENBAUR met with no ossification of this membrane in the later stage described by him.

Summary of the development of the vertebral column and ribs.

A mesoblastic investment is early formed round the notochord, which is produced into two dorsal and two ventral ridges, the former uniting above the neural canal. Around the cuticular sheath of the notochord an elastic membrane, the membrana elastica externa, is next developed. The neural ridges become enlarged at each intermuscular septum, and these enlargements soon become converted into cartilage, thus forming a series of neural processes riding on the membrana elastica externa, and extending about two-thirds of the way up the sides of the neural canal. The hæmal processes arise simultaneously with, and in the same manner as, the neural. They are small in the trunk, but at the front end of the anal fin they suddenly enlarge and extend ventralwards. Each succeeding pair of hæmal arches becomes larger than the one in front, each arch finally meeting its fellow below the caudal vein, thus forming a completely closed hæmal canal. These arches are moreover produced into long spines supporting the fin-rays of the caudal fin, which thus differs from the other impaired fins in being supported by parts of the vertebral column, and not by separately formed skeletal elements.

In the next stage which we have had the opportunity of studying (larva of 5½ centims.), a series of very well-marked *vertebral* constrictions are to be seen in the notochord. The sheath is now much thicker in the vertebral than in the intervertebral regions: this is due to a special differentiation of a superficial part of the sheath, which appears more granular than the remainder. This granular part of the sheath thus forms a cylinder in each vertebral region. Between it and the gelatinous tissue of the notochord there remains a thin unmodified portion of the sheath, which is continuous with the intervertebral parts of the sheath. The neural and hæmal arches are seen to be continuous with a cartilaginous tube embracing the intervertebral regions of the notochord, and continuous from one vertebra to the next. A delicate layer of bone, developed in the perichondrium, invests the cartilaginous neural arches, and this bone grows upwards so as to unite above with the osseous investment of separately developed bars of cartilage, which are directed obliquely backwards. These bars, or dorsal processes, may be reckoned as parts of the neural arches. Between the dorsal processes of the two sides is placed a median rod of cartilage, which is developed separately from the true neural arches, and which constitutes the median spinous element of the adult. Immediately below this rod is placed the ligamentum longitudinale superius. There is now a commencement of separation between the dorsal and ventral parts of the hæmal arches, not only in the tail, but also in the trunk, where they pass ventralwards on each side of the body cavity, immediately outside the peritoneal membrane, along the lines of insertion of the intermuscular septa. These are obviously the ribs of the adult, and there is no break of continuity of structure between the hæmal processes of the tail and the ribs. In the anterior part of the trunk the ribs pass outwards along the intermuscular septa till they reach the epidermis. Thus the ribs are originally continuous with the hæmal processes. Behind the region of the ventral caudal fin the two hæmal processes merge into one, which is not perforated by a canal.

Each of the intervertebral rings of cartilage becomes eventually divided into two parts, and converted into the adjacent faces of contiguous vertebræ, the curved line where this will be effected being plainly marked out. These rings are united with the neural and hæmal arches of the vertebræ next in front and behind. As these rings are formed originally by the spreading of the cartilage from the primitive neural and hæmal processes, the intervertebral cartilages are clearly derived from the neural and hæmal arches. The intervertebral cartilages are thicker in the middle than at their two ends.

In our latest stage (11 centims.), the vertebral constrictions of the notochord are rendered much less conspicuous by the growth of the intervertebral cartilages giving rise to marked intervertebral constrictions. In the intervertebral regions the membrana elastica externa has become aborted at the posterior border of each vertebra, and the remaining part is considerably puckered transversely. The inner sheath of the notochord is puckered longitudinally in the intervertebral regions. The granular external layer of the sheath in the vertebral regions is less thick than in the last stage, and exhibits faint radial striations.

Two closely approximated cartilaginous elements now form a keystone to the neural arch above: these are directly differentiated from the ligamentum longitudinale superius, into which they merge above. An osseous plate is formed on the outer side of each of these cartilages. These plates are continuous with the lateral osseous bars of the neural arches, and also give rise to the osseous roof of the spinal canal of the adult.

Thus the greater part of the neural arches is formed of membrane bone. The hæmal arches are invested by a thick layer of bone, and there is also a continuous osseous investment round the vertebral portions of the notochord. The intervertebral cartilages become penetrated by branched processes of bone.

Comparison of the vertebral column of Lepidosteus *with that of other forms.*

The peculiar form of the articulatory faces of the vertebræ of *Lepidosteus* caused L. AGASSIZ (No. 2) to compare them with the vertebræ of Reptiles, and subsequent anatomists have suggested that they more nearly resemble the vertebræ of some Urodelous Amphibia than those of any other form.

If, however, GÖTTE'S account of the formation of the amphibian vertebræ is correct, there are serious objections to a comparison between the vertebræ of *Lepidosteus* and Amphibia on developmental grounds. The essential point of similarity supposed to exist between them consists in the fact that in both there is a great development of intervertebral cartilage which constricts the notochord intervertebrally, and forms the articular faces of contiguous vertebræ.

In *Lepidosteus* this cartilage is, as we have seen, derived from the bases of the arches; but in Amphibia it is held by GÖTTE to be formed by a special thickening of a cellular sheath round the notochord which is probably homologous with the cartilaginous sheath of the notochord of Elasmobranchii, and therefore with part of the notochordal sheath placed within the membrana elastica externa.

If the above statements with reference to the origin of the intervertebral cartilage in the two types are true, it is clear that no homology can exist between structures so differently developed. Provisionally, therefore, we must look elsewhere than in *Lepidosteus* for the origin of the amphibian type of vertebræ.

The researches which we have recorded demonstrate, however, in a very conclusive manner that the vertebræ of *Lepidosteus* have very close affinities with those of Teleostei.

In support of this statement we may point: (1) To the structure of the sheath of the notochord; (2) to the formation of the greater part of the bodies of the vertebræ from ossification in membrane around the notochord; (3) to the early biconcave form of the vertebræ, only masked at a later period by the development of intervertebral cartilages; (4) to the character of the neural arches.

This latter feature will be made very clear if the reader will compare our figures of the sections of later vertebræ (Plate 29, fig. 78) with GÖTTE'S* figure of the section of the vertebra of a Pike (plate 7, fig. 1). In GÖTTE'S figure there are shown similar intercalated pieces of cartilage to those which we have found, and similar cartilaginous dorsal processes of the vertebræ. Thus we are justified in holding that whether or no the opisthocœlous form of the vertebræ of *Lepidosteus* is a commencement of a type of vertebræ inherited by the higher forms, yet in any case the vertebræ are essentially built on the type which has become inherited by the Teleostei from the bony Ganoids.

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* "Beiträge zur vergl. Morphol. d. Skeletsystems d. Wirbeltheire." Archiv. f. Mikr. Anat., vol. xvi., 1879.

Part III.—*The ribs of Fishes.*

The nature and homologies of the ribs of Fishes have long been a matter of controversy; but the subject has recently been brought forward in the important memoirs of GÖTTE[*] on the Vertebrate skeleton. The alternatives usually adopted are, roughly speaking, these:—Either the hæmal arches of the tail are homologous throughout the piscine series, while the ribs of Ganoids and Teleostei are not homologous with those of Elasmobranchii; or the ribs are homologous in all the piscine groups, and the hæmal arches in the tail are differently formed in the different types. GÖTTE has brought forward a great body of evidence in favour of the first view; while GEGENBAUR[†] may be regarded as more especially the champion of the second view.

One of us held in a recent publication[‡] that the question was not yet settled, though the view that the ribs are homologous throughout the series was provisionally accepted.

It is admitted by both GEGENBAUR and GÖTTE that in *Lepidosteus* the ribs, in the transition from the trunk to the tail, bend inwards, and finally unite in the region of the tail to form the ventral parts of the hæmal arches, and our researches have abundantly confirmed this conclusion.

Are the hæmal arches, the ventral parts of which are thus formed by the coalescence of the ribs, homologous with the hæmal arches in Elasmobranchii? The researches recorded in the preceding pages appear to us to demonstrate in a conclusive manner that they are so.

The development of the hæmal arches in the tail in these two groups is practically identical; they are formed in both as simple elongations of the primitive hæmal processes, which meet below the caudal vein. In the adult there is an apparent difference between them, arising from the fact that in *Lepidosteus* the peripheral parts of the hæmal processes are only articulated with the basal portions, and not, as in Elasmobranchii, continuous with them. This difference does not, however, exist in the early larva, since in the larval *Lepidosteus* the hæmal arches of the tail are unsegmented cartilaginous arches, as they permanently are in Elasmobranchii. If, however, the homology between the hæmal arches of the two types should still be doubted, the fact that in both types the hæmal arches are similarly modified to support the fin-rays of the ventral lobe of the caudal fin, while in neither type are they modified to support the anal fin, may be pointed out as a very strong argument in confirmation of their homology.

The demonstration of the homology of the hæmal arches of the tail in *Lepidosteus* and Elasmobranchii might at first sight be taken as a conclusive argument in favour of GÖTTE's view, that the ribs of Elasmobranchii are not homologous with those of Ganoidei. This view is mainly supported by two facts:—

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[*] "Beiträge z. vergl. Morph. d. Skeletsystems d. Wirbelthiere. II. Die Wirbelsäule u. ihre Anhänge." Archiv. f. Mikr. Anat., vol. xv., 1878, and vol. xvi., 1879.

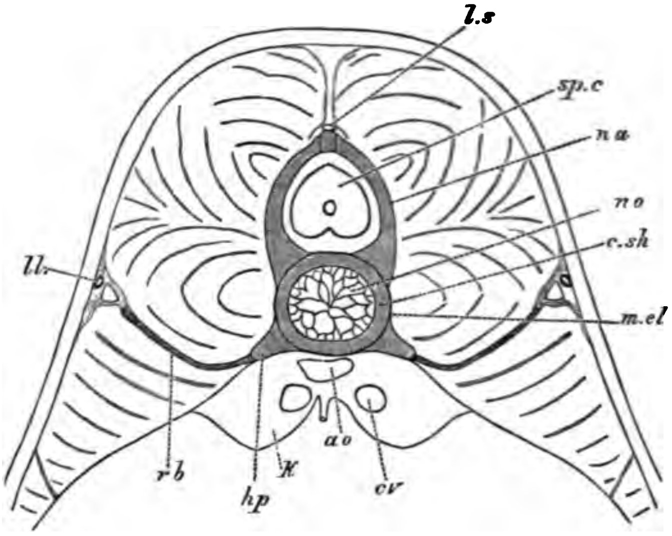
[†] "Ü. d. Entwick. d. Wirbelsäule d. *Lepidosteus*, mit. vergl. Anat. Bemerkungen." Jenaische Zeitschrift, Bd. iii., 1863.

[‡] 'Comparative Embryology,' vol. ii., pp. 462, 463.

(1) In the first place, the ribs in Elasmobranchii do not at first sight appear to be serially homologous with the ventral parts of the hæmal arches of the tail, but would rather seem to be lateral offshoots of the hæmal processes, while the hæmal arches of the tail appear to be completed by the coalescence of independent ventral prolongations of the hæmal processes.

(2) In the second place, the position of the ribs is different in the two groups. In Elasmobranchii they are situated between the dorso-lateral and ventro-lateral muscles (woodcut, fig. 1, *rb.*), while in *Lepidosteus* and other Ganoids they immediately girth the body-cavity.

Fig. 1.



Diagrammatic section through the trunk of an advanced embryo of *Scyllium*, to show the position of the ribs.

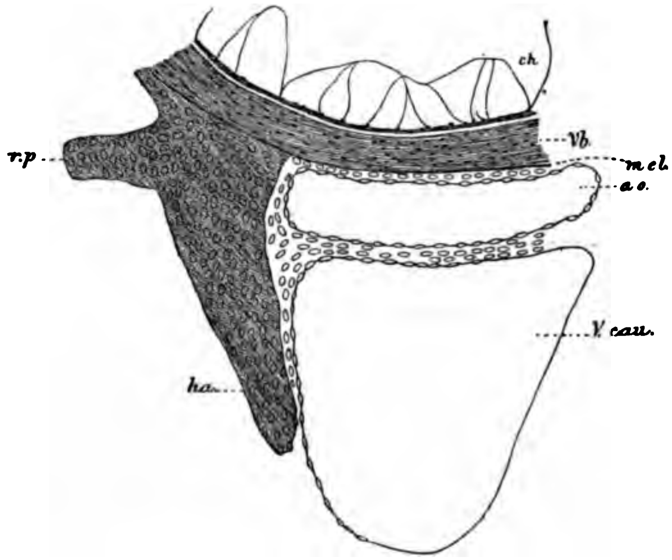
ao., aorta; *c.sh.*, cartilaginous notochordal sheath; *cv.*, cardinal vein; *hp.*, hæmal process; *k.*, kidney; *l.s.*, ligamentum longitudinale superius; *m.el.*, membrana elastica externa; *na.*, neural arch; *no.*, notochord; *ll.*, lateral line; *rb.*, rib; *sp.c.*, spinal cord.

There is much, therefore, to be said in favour of GÖTTE's view. At the same time, there is another possible interpretation of the facts which would admit the homology of the ribs as well as of the hæmal arches throughout the Pisces.

Let us suppose, to start with, that the primitive arrangement of the parts is more or less nearly that found in *Lepidosteus*, where we have well-developed ribs in the region of the trunk, girthing the body-cavity, and uniting in the caudal region to form the ventral parts of the hæmal arches. It is easy to conceive that the ribs in the trunk might somewhat alter their position by passing into the muscles, along the intermuscular septa, till they come to lie between the dorso-lateral and ventro-lateral muscles, as in Elasmobranchii. *Lepidosteus* itself affords a proof that such a change in the position of the ribs is not impossible, in that it differs from other Ganoids and from Teleostei in the fact that the free ends of the ribs leave the neighbourhood of the body-cavity and penetrate into the muscles.

If it be granted that the mere difference in position between the ribs of Ganoids and Elasmobranchii is not of itself sufficient to disprove their homology, let us attempt to picture what would take place at the junction of the trunk and tail in a type in which the ribs had undergone the above change in position. On nearing the tail it may be supposed that the ribs would gradually become shorter, and at the same time alter their position, till finally they shaded off into ordinary hæmal processes. If, however, the hæmal canal became prolonged forwards by the formation of some additional complete or nearly complete hæmal arches, an alteration in the relation of the parts would necessarily take place. Owing to the position of the ribs, these structures could hardly assist in the new formation of the anterior part of the hæmal canal, but the continuation forwards of the canal would be effected by prolongations of the hæmal processes supporting the ribs. The new arches so formed would naturally be held to be homologous with the hæmal arches of the tail, though really not so, while the true nature of the ribs would also be liable to be misinterpreted, in that the ribs would appear to be lateral outgrowths of the hæmal processes of a wholly different nature to the ventral parts of the hæmal arches of the tail.

Fig. 2.



Transverse section through the ventral part of the notochord, and adjoining structures of an advanced *Scyllium* embryo at the root of the tail.

***Vb.*, cartilaginous sheath of the notochord; *ha.*, hæmal process; *r.p.*, process to which the rib is articulated; *m.el.*, membrana elastica externa; *ch.*, notochord; *ao.*, aorta; *V.cau.*, caudal vein.**

In some Elasmobranchii, as shown in the accompanying woodcut (fig. 2), in the transitional vertebræ between the trunk and the tail, the ribs are supported by lateral outgrowths of the hæmal processes, while the wholly independent prolongations of the hæmal processes appear to be about to give rise to the hæmal arches of the tail.

This peculiar state of things led GÖTTE, and subsequently one of us, to deny for Elasmobranchs all homology between the ribs and any part of the hæmal arches of the tail; but in view of the explanation just suggested, this denial was perhaps too hasty.

We are the more inclined to take this view because the researches of GÖTTE appear to show that an occurrence, in many respects analogous, has taken place in some Teleostei.

In Teleostei, JOHANNES MÜLLER, and following him GEGENBAUR, do not admit that the hæmal arches of the tail are in any part formed by the ribs. GEGENBAUR ('Elements of Comp. Anat.,' translation, p. 431) says, "In the Teleostei, the costiferous transverse processes" (what we have called the hæmal processes) "gradually converge in the caudal region, and form inferior arches, which are not homologous with those of Selachii and Ganoidei, although they also form spinous processes."

The opposite view, that the hæmal arches of the tail in Teleostei contain parts serially homologous with the basal parts of the hæmal processes as well as with the ribs, has been also maintained by many anatomists, *e.g.*, MECKEL, AUG. MÜLLER, &c., and has recently found a powerful ally in GÖTTE.

In many cases, the relations of the parts appear to be fundamentally those found in *Lepidosteus* and *Amia*, and GÖTTE has shown by his careful embryological investigations on *Esox* and *Anguilla*, that in these two forms there is practically conclusive evidence that the ribs as well as the hæmal costiferous processes of GEGENBAUR, which support them, enter into the formation of the hæmal arches of the tail.

In a great number of Teleostei, *e.g.*, the Salmon and most Cyprinoids, &c., the hæmal arches in the region of transition from the trunk to the tail have a structure which at first sight appears to support JOHANNES MÜLLER'S and GEGENBAUR'S view. The hæmal processes grow larger and meet each other ventrally; while the ribs articulated to them gradually grow smaller and disappear.

The Salmon is typical in this respect, and has been carefully studied by GÖTTE, who attempts to show (with, in our opinion, complete success) that the anterior hæmal arches are really not entirely homologous with the true hæmal arches behind, but that in the latter, the closure of the arch below is effected by the hæmal spine, which is serially homologous with a pair of coalesced ribs, while in the anterior hæmal arches, *i.e.*, those of the trunk, the closure of the arch is effected by a bridge of bone uniting the hæmal processes.

The arrangement of the parts just described, as well as the view of GÖTTE with reference to them, will be best understood from the accompanying woodcut (fig. 3), copied from GÖTTE'S memoir.

GÖTTE sums up his own results on this point in the following words (p. 138): "It follows from this, that the half rings, forming the hæmal canal in the hindermost trunk vertebræ of the Salmon, are not (with the exception of the last) completely muscles, which may perhaps have their homologues in Teleostei and Ganoids in certain accessory processes of the vertebræ.

The other view, which we are inclined to adopt, and the arguments for which have been stated in the preceding pages, is as follows:—The Teleostei, Ganoidei, Dipnoi, and Elasmobranchii are provided with homologous hæmal arches, which are formed by the coalescence below the caudal vein of simple prolongations of the primitive hæmal processes of the embryo. The canal enclosed by the hæmal arches can be demonstrated embryologically to be the aborted body cavity.

In the region of the trunk the hæmal processes and their prolongations behave somewhat differently in the different types. In Ganoids and Dipnoi, in which the most primitive arrangement is probably retained, the ribs are attached to the hæmal processes, and are placed immediately without the peritoneal membrane at the insertions of the intermuscular septa. These ribs are in many instances (*Lepidosteus*, *Acipenser*), and very probably in all, developed continuously with the hæmal processes, and become subsequently segmented from them. They are serially homologous with the ventral parts of the hæmal arches of the tail, which, like them, are in many instances (*Ceratodus*, *Lepidosteus*, *Polypterus*, and to some extent in *Amia*) segmented off from the basal parts of the hæmal arches.

In Teleostei the ribs have the same position and relations as those in Ganoids and Dipnoi, but their serial homology with the ventral parts of the hæmal processes of the tail, is often (*e.g.*, the Salmon) obscured by some of the anterior hæmal arches in the posterior part of the trunk being completed, not by the ribs, but by independent outgrowths of the basal parts of the hæmal processes.

In Elasmobranchii a still further divergence from the primitive arrangement is present. The ribs appear to have passed outwards along the intermuscular septa into the muscles, and are placed between the dorso-lateral and ventro-lateral muscles (a change of position of the ribs of the same nature, but affecting only their ends, is observable in *Lepidosteus*). This change of position, combined probably with the secondary formation of a certain number of anterior hæmal arches similar to those in the Salmon, renders their serial homology with the ventral parts of the hæmal processes of the tail far less clear than in other types, and further proof is required before such homology can be considered as definitely established.

This is not the place to enter into the obscure question as to how far the ribs of the Amphibia and Amniota are homologous with those of Fishes. It is to be remarked, however, that the ribs of the Urodela (1) occupy the same position in relation to the muscles as the Elasmobranch ribs, (2) that they are connected with the neural arches, and (3) that they coexist in the tail with the hæmal arches, and seem, therefore, to be as different as possible from the ribs of the Dipnoi.

Part IV.—*The skeleton of the ventral lobe of the tail fin, and its bearing on the nature of the tail fin of the various types of Pisces.*

In the embryos or larvæ of all the Elasmobranchii, Ganoidei, and Teleostei which have up to this time been studied, the unpaired fins arise as median longitudinal folds of the integument on the dorsal and ventral sides of the body, which meet at the apex of the tail. The tail at first is symmetrical, having a form which has been called diphycercal or protocercal. At a later stage, usually, though not always, parts of these fins atrophy, while other parts undergo a special development and constitute the permanent unpaired fins.

Since the majority of existing as well as extinct Fishes are provided with discontinuous fins, those forms, such as the Eel (*Anguilla*), in which the fins are continuous, have probably reverted to an embryonic condition: an evolutional process which is of more frequent occurrence than has usually been admitted.

In the caudal region there is almost always developed in the larvæ of the above groups a special ventral lobe of the embryonic fin a short distance from the end of the tail. In Elasmobranchii and Chondrostean Ganoids the portion of the embryonic tail behind this lobe persists through life, and a special type of caudal fin, which is usually called heterocercal, is thus produced. This type of caudal fin appears to have been the most usual in the earlier geological periods.

Simultaneously with the formation of the ventral lobe of the heterocercal caudal fin, the notochord with the vertebral tissues surrounding it, becomes bent somewhat dorsalwards, and thus the primitive caudal fin forms a dorsally directed lobe of the heterocercal tail. We shall call this part the dorsal lobe of the tail-fin, and the secondarily formed lobe the ventral lobe.

Lepidosteus and *Amia* (WILDER, No. 15) amongst the bony Ganoids, and, as has recently been shown by A. AGASSIZ,* most Teleostei acquire at an early stage of their development heterocercal caudal fins, like those of Elasmobranchii and the Chondrostean Ganoids; but in the course of their further growth the dorsal lobe partly atrophies, and partly disappears as such, owing to the great prominence acquired by the ventral lobe. A portion of the dorsally flexed notochord and of the cartilage or bone replacing or investing it remains, however, as an indication of the original dorsal lobe, though it does not project backwards beyond the level of the end of the ventral lobe, which in these types forms the terminal caudal fin.

The true significance of the dorsally flexed portion of the vertebral axis was first clearly stated by HUXLEY,† but as A. AGASSIZ has fairly pointed out in the paper already quoted, this fact does not in any way militate against the view put forward by

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* "On the Young Stages of some Osseous Fishes.—I. The Development of the Tail," Proc. of the American Academy of Arts and Sciences, vol. xiii., 1877.

† "Observations on the Development of some Parts of the Skeleton of Fishes," Quart. Journ. of Micr. Science, vol. vii., 1859.

L. AGASSIZ that there is a complete parallelism between the embryonic development of the tail in these Fishes and the palæontological development of this organ. We think that it is moreover convenient to retain the term homocercal for those types of caudal fin in which the dorsal lobe has atrophied so far as not to project beyond the ventral lobe.

We have stated these now well-known facts to enable the reader to follow us in dealing with the comparison between the skeleton supporting the fin-rays of the ventral lobe of the caudal fin, and that supporting the fin-rays of the remaining unpaired fins.

It has been shown that in *Lepidosteus* the unpaired fins fall into two categories, according to the nature of the skeletal parts supporting them. The fin-rays of the true ventral lobe of the caudal fin are supported by the spinous processes of certain of the hæmal arches. The remaining unpaired fins, including the anal fin, are supported by the so-called interspinous bones, which are developed independently of the vertebral column and its arches.

The question which first presents itself is, how far does this distinction hold good for other Fishes? This question, though interesting, does not appear to have been greatly discussed by anatomists. Not unfrequently the skeletal supports of the ventral lobe of the caudal fin are assumed to be the same as those of the other fins.

DAVIDOFF,[*] for instance, in speaking of the unpaired fins of Elasmobranch embryos, says (p. 514): "The cartilaginous rays of the dorsal fins agreed not only in number with the spinous processes (as indeed is also found in the caudal fin of the full-grown Dog-fish)," &c.

THACHER,[†] again, in his memoir on the Median and Paired Fins, states at p. 284: "We shall here consider the skeleton of the dorsal and anal fins alone. That of the caudal fin has undergone peculiar modifications by the union of fin-rays with hæmal spines."

MIVART[‡] goes into the question more fully. He points out (p. 471) that there is an essential difference between the dorsal and ventral parts of the caudal fin in Elasmobranchs, in that in the former the radials are more numerous than the vertebræ and unconformable to them, while in the latter they are equal in number to the vertebræ and continuous with them. "This," he goes on to say, "seems to point to a difference in nature between the dorsal and ventral portions of the caudal fin, in at least most Elasmobranchs." He further points out that *Polyodon* resembles Elasmobranchs. As to Teleostei, he does not express himself decidedly except in the case of *Murœna,* to which we shall return.

MIVART expresses himself as very doubtful as to the nature of the supports of the caudal fin, and thinks "that the caudal fin of different kinds of Fishes may have arisen in different ways in different cases."

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[*] "Beiträge z. vergl. Anat. d. hinteren Gliedmassen d. Fische," Morph. Jahrbuch, vol. v., 1879.

[†] Trans. of the Connecticut Acad., vol. iii., 1877.

[‡] ST. GEORGE MIVART, "Fins of Elasmobranchs." Zool. Trans., vol. x.

An examination of the ventral part of the caudal fin in various Ganoids, Teleostei, and Elasmobranchii appears to us to show that there can be but little doubt that, in the majority of the members of these groups at any rate, and we believe in all, the same distinction between the ventral lobe of the caudal fin and the remaining unpaired fins is found as in *Lepidosteus.*

In the case of most Elasmobranchii, a simple inspection of the caudal fin suffices to prove this, and the anatomical features involved in this fact have usually been recognised; though, in the absence of embryological evidence, the legitimate conclusion has not always been drawn from them.

The difference between the ventral lobe of the caudal fin and the other fins in the mode in which the fin-rays are supported is as obvious in Chondrostean Ganoids as it is in Elasmobranchii; it would appear also to hold good for *Amia. Polypterus* we have had no opportunity of examining, but if, as there is no reason to doubt, the figure of its skeleton given by AGASSIZ ('Poissons Fossiles') is correct, there can be no question that the ventral lobe of the caudal fin is supported by the hæmal arches, and not by interspinous bones. In *Calamoicthys*, the tail of which we have had an opportunity of dissecting through the kindness of Professor PARKER, the fin-rays of the ventral lobe of the true caudal fin are undoubtedly supported by true hæmal arches.

There is no unanimity of opinion as to the nature of the elements supporting the fin-rays of the caudal fin of Teleostei.

HUXLEY,* in his paper on the development of the caudal fin of the Stickleback, holds that these elements are of the nature of interhæmal bones. He says (p. 39): "The last of these rings lay just where the notochord began to bend up. It was slightly longer than the bony ring which preceded it, and instead of having its posterior margin parallel with the anterior, it sloped from above downwards and backwards. Two short osseous plates, attached to the anterior part of the inferior surface of the penultimate ring, or rudimentary vertebral centrum, passed downwards and a little backwards, and abutted against a slender elongated mass of cartilage. Similar cartilaginous bodies occupy the same relation to corresponding plates of bone in the anterior vertebræ in the region of the anal fin; and it is here seen, that while the bony plates coalesce and form the inferior arches of the caudal vertebræ, the cartilaginous elements at their extremities become the interhæmal bones. The cartilage connected with the inferior arch of the penultimate centrum is therefore an 'interhæmal' cartilage. The anterior part of the inferior surface of the terminal ossification likewise has its osseous inferior arch, but the direction of this is nearly vertical, and though it is connected below with an element which corresponds in position with the interhæmal cartilage, this cartilage is five or six times as large, and constitutes a broad vertical plate, longer than it is deep, and having its longest axis inclined downwards and backwards. . . . .

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* "Observations on the Development of some parts of the Skeleton of Fishes." Quart. Journ. Micr. Science, vol. vii., 1859.

"Immediately behind and above this anterior hypural apophysis (as it may be termed) is another very much smaller vertical cartilaginous plate, which may be called the posterior hypural apophysis."

We have seen that MIVART expresses himself doubtful on the subject. GEGENBAUR* appears to regard them as hæmal arches.

The latter view appears to us without doubt the correct one. An examination of the tail of normal Teleostei shows that the fin-rays of that part of the caudal fin which is derived from the ventral lobe of the larva are supported by elements serially homologous with the hæmal arches, but in no way homologous with the interspinous bones of the anal fin. The elements in question formed of cartilage in the larva, become ossified in the adult, and are known as the hypural bones. They may appear in the form of a series of separate hæmal arches, corresponding in number with the primitive somites of this region, which usually, however, atrophy in the adult, or more often are from the first imperfectly segmented, and have in the adult the form of two or three or even of a single broad bony plate. The transitional forms between this state of things and that, for instance, in *Lepidosteus* are so numerous, that there can be no doubt that even the most peculiar forms of the hypural bones of Teleostei are simply modified hæmal arches.

This view of the hypural bones is, moreover, supported by embryological evidence, since AUG. MÜLLER† (p. 205) describes their development in a manner which, if his statements are to be trusted, leaves no doubt on this point.

There are a considerable number of Fishes which are not provided with an obvious caudal fin as distinct from the remaining unpaired fins, *i.e.*, Chimæra, Eels, and various Eel-like forms amongst Teleostei, and the Dipnoi. GEGENBAUR appears to hold that these Fishes ought to be classed together in relation to the structure of the caudal portion of their vertebral column, as he says on p. 431 of his 'Comparative Anatomy' (English translation): "In the Chimæræ, Dipnoi, and many Teleostei, the caudal portion of the vertebral column ends by gradually diminishing in size, but in most Fishes, &c."

For our purpose it will, however, be advisable to treat them separately.

The tail of Chimæra appears to us to be simply a peculiar modification of the typical Elasmobranch heterocercal tail, in which the true ventral lobe of the caudal fin may be recognised in the fin-fold immediately in front of the filamentous portion of the tail. In the allied genus *Callorhyncus* this feature is more distinct. The filamentous portion of the tail of Chimæra constitutes, according to the nomenclature adopted above, the true dorsal lobe, and may be partially paralleled in the filamentous dorsal lobe of the tail of the larval *Lepidosteus* (Plate 21, fig. 16).

The tail of the eel-like Teleostei is again undoubtedly a modification of the normal form of tail characteristic of the Teleostei, in which, however, the caudal fin has become very much reduced and merged into the prolongations of the anal and dorsal fins.

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* 'Elements of Comparative Anatomy.' (Translation), p. 431.

† "Beobachtungen zur vergl. Anat. d. Wirbelsäule." MÜLLER's Archiv., 1853.

This can be very clearly seen in Siluroid forms with an Eel-like tail, such as *Cnidoglanis*. Although the dorsal and ventral fins appear to be continuous round the end of the tail, and there is superficially no distinct caudal fin, yet an examination of the skeleton of *Cnidoglanis* shows that the end of the vertebral column is modified in the usual Teleostean fashion, and that the hæmal arches of the modified portion of the vertebral column support a small number of fin-rays; the adjoining ventral fin-rays being supported by independent osseous fin-supports (interspinous bones).

In the case of the Eel (*Anguilla anguilla*) HUXLEY (*loc. cit.*) long ago pointed out that the terminal portion of the vertebral column was modified in an analogous fashion to that of other Teleostei, and we have found that the modified hæmal arches of this part support a few fin-rays, through a still smaller number than in *Cnidoglanis*. The fin-rays so supported clearly constitute an aborted ventral lobe of the caudal fin.

Under these circumstances we think that the following statement by MIVART (Zool. Trans. vol. x., p. 471) is somewhat misleading :—

" As to the condition of this part (*i.e.*, the ventral lobe of the tail fin) in Teleosteans generally, I will not venture as yet to say anything generally, *except that it is plain that in such forms as Murœna, the dorsal and ventral parts of the caudal fin are similar in nature and homotypal with ordinary dorsal and anal fins.*"*

The italicized portion of this sentence is only true in respect to that part of the fringe of fin surrounding the end of the body, which is not only homotypal with, but actually part of, the dorsal and anal fins.

Having settled, then, that the tails of Chimæra and of Eel-like Teleostei are simply special modifications of the typical form of tail of the group of Fishes to which they respectively belong, we come to the consideration of the Dipnoi, in which the tail fin presents problems of more interest and greater difficulty than those we have so far had to deal with.

The undoubtedly very ancient and primitive character of the Dipnoi has led to the view, implicitly if not definitely stated in most text-books, that their tail-fin retains the character of the piscine tail prior to the formation of the ventral caudal lobe, a stage which is repeated embryologically in the pre-heterocercal condition of the tail in ordinary Fishes.

Through the want of embryological data, and in the absence of really careful histological examination of the tail of any of the Dipnoi, we are not willing to speak with very great confidence as to its nature; we are nevertheless of the opinion that the facts we can bring forward on this head are sufficient to show that the tail of the existing Dipnoi is largely aborted, so that it is more or less comparable with that of the Eel.

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* The italics are ours.

We have had opportunities of examining the structure of the tail of *Ceratodus* and *Protopterus* in dissected specimens in the Cambridge Museum. The vertebral axis runs to the ends of the tail without showing any signs of becoming dorsally flexed. At some distance from the end of the tail the fin-rays are supported by what are apparently segmented spinous prolongations of the neural and hæmal arches. The dorsal elements are placed above the longitudinal dorsal cord, and occupy therefore the same position as the independent elements of the neural arches of *Lepidosteus*. They are therefore to be regarded as homologous with the dorsal fin-supports or interspinous bones of other types. The corresponding ventral elements are therefore also to be regarded as interspinous bones.

In view of the fact that the fin-supports, whenever their development has been observed, are found to be formed independently of the neural and hæmal arches, we may fairly assume that this is also true for what we have identified as the interspinous elements in the Dipnoi.

The interspinous elements become gradually shorter as the end of the tail is approached, and it is very difficult from a simple examination of dissected specimens to make out how far any of the posterior fin-rays are supported by the hæmal arches only. To this question we shall return, but we may remark that, although there is a prolongation backwards of the vertebral axis beyond the last interspinous elements, composed it would seem of the coalesced neural and hæmal arches but without the notochord, yet by far the majority of the fin-rays which constitute the apparent caudal fin are supported by interspinous elements.

The grounds on which we hold that the tail of the Dipnoi is to be regarded as a degenerate rather than primitive type of tail are the following:—

(1) If it be granted that a diphycercal or protocercal form of tail must have preceded a heterocercal form, it is also clear that the ventral fin-rays of such a tail must have been supported, as in *Polypterus* and *Calamoicthys*, by hæmal arches, and not by interspinous elements; otherwise, a special ventral lobe, giving a heterocercal character to the tail, and provided with fin-rays supported only by hæmal arches, could never have become evolved from the protocercal tail fin. Since the ventral fin-rays of the tail of the Dipnoi are supported by interspinous elements and not by hæmal arches, this tail fin cannot claim to have the character of *that* primitive type of diphycercal or protocercal tail from which the heterocercal tail must be supposed to have been evolved.

(2) Since the nearest allies of the Dipnoi are to be found in *Polypterus* and the Crassopterygidæ of HUXLEY, and since in these forms (as evinced by the structure of the tail fin of *Polypterus*, and the transitional type between a heterocercal and diphycercal form of fin observable in fossil Crassopterygidæ) the ventral fin-rays of the caudal fin were clearly supported by hæmal arches and not by interspinous elements, it is rendered highly probable that the absence of fin-rays so supported in the Dipnoi is a result of degeneration of the posterior part of the tail.

[We use this argument without offering any opinion as to whether the diphycercal character of the tail of many Crassopterygidæ is primary or secondary.]

(3) The argument just used is supported by the degenerate and variable state of the end of the vertebral axis in the Dipnoi—a condition most easily explained by assuming that the terminal part of the tail has become aborted.

(4) We believe that in *Ceratodus* we have been able to trace a small number of the ventral fin-rays supported by hæmal arches only, but these rays are so short as not to extend so far back as some of the rays attached to the interspinous elements in front. These rays may probably be interpreted, like the more or less corresponding rays in the tail of the Eel, as the last remnant of a true caudal fin.

The above considerations appear to us to show with very considerable probability that the true caudal fin of the Dipnoi has become all but aborted like that of various Teleostei; and that the apparent caudal fin is formed by the anal and dorsal fins meeting round the end of the stump of the tail.

From the adult forms of Dipnoi we are, however, of opinion that no conclusion can be drawn as to whether their ancestors were provided with a diphycercal or a heterocercal form of caudal fin.

The general conclusions with reference to the tail fin at which we have arrived are the following:—

(1) The ventral lobe of the tail-fin of Pisces differs from the other unpaired fins in the fact that its fin-rays are directly supported by spinous processes of certain of the hæmal arches instead of independently developed interspinous bones.

(2) The presence or absence of fin-rays in the tail fin supported by hæmal arches may be used in deciding whether apparently diphycercal tail fins are aborted or primitive.

EXCRETORY AND GENERATIVE ORGANS.

I.—*Anatomy.*

The excretory organs of *Lepidosteus* have been described by MÜLLER (No. 13) and HYRTL (No. 11). These anatomists have given a fairly adequate account of the generative ducts in the female, and HYRTL has also described the male generative ducts and the kidney and its duct, but his description is contradicted by our observations in some of the most fundamental points.

In the female example of 100·5 centims. which we dissected, the kidney forms a paired gland, consisting of a narrow strip of glandular matter placed on each side of the vertebral column, on the dorsal aspect of the body cavity. It is covered on its ventral aspect by the oviduct and by its own duct, but is separated from both of these by a layer of the tough peritoneal membrane, through which the collecting tubes pass. It extends forwards from the anus for about three-fifths of the length of the body-cavity, and in our example had a total length of about 28 centims. (Plate 26, fig. 60, *k*).

Anteriorly the two kidneys are separated by a short interval in the median line, but posteriorly they come into contact, and are so intimately united as almost to constitute a single gland.

A superficial examination might lead to the supposition that the kidney extended forwards for the whole length of the body-cavity up to the region of the branchial arches, and HYRTL appears to have fallen into this error; but what appears to be its anterior continuation is really a form of lymphatic tissue, something like that of the spleen, filled with numerous cells. This matter (Plate 26, fig. 60, *ly.*) continues from the kidney forwards without any break, and has a colour so similar to that of the kidney as to be hardly distinguishable from it with the naked eye. The true anterior end of the kidney is placed about 3 centims. in front on the left side, and on the same level on the right side as the wide anterior end of the generative duct (Plate 26, fig. 60, *od.*). It is not obviously divided into segments, and is richly supplied with malpighian bodies.

It is clear from the above description that there is no trace of head-kidney or pronephros visible in the adult. To this subject we shall, however, again return.

As will appear from the embryological section, the ducts of the kidneys are probably simply the archinephric ducts, but to avoid the use of terms involving a theory, we propose in the anatomical part of our work to call them kidney ducts. They are thin-walled widish tubes coextensive with the kidneys. If cut open there may be seen on their inner aspect the numerous openings of the collecting tubes of the kidneys. They are placed ventrally to and on the outer border of the kidneys (Plate 26, fig. 60, *sg.*). Posteriorly they gradually enlarge, and approaching each other in the median line, coalesce, forming an unpaired vesicle or bladder (*bl.*)—about 6 centims. long in our example—opening by a median pore on a more or less prominent papilla (*u.g.*) behind the anus. The dilated portions of the two ducts are called by HYRTL the horns of the bladder.

The sides of the bladder and its so-called horns are provided with lateral pockets into which the collecting tubes of the kidney open. These pockets, which we have found in two female examples, are much larger in the horns of the bladder than in the bladder itself. Similar pockets, but larger than those we have found, have been described by HYRTL in the male, but are stated by him to be absent in the female. It is clear from our examples that this is by no means always the case.

HYRTL states that the wide kidney ducts, of which his description differs in no material point from our own, suddenly narrow in front, and, perforating the peritoneal lining, are continued forwards to supply the anterior part of the kidney. We have already shown that the anterior part of the kidney has no existence, and the kidney ducts supplying it are, according to our investigations, equally imaginary.

It was first shown by MÜLLER, whose observations on this point have been confirmed by HYRTL, &c., that the ovaries of *Lepidosteus* are continuous with their ducts, forming in this respect an exception to other Ganoids.

In our example of *Lepidosteus* the ovaries (Plate 26, fig. 60, *ov.*) were about 18 centims. in length. They have the form of simple sacs, filled with ova, and attached about their middle to their generative duct, and continued both backwards and forwards from their attachment into a blind process.

With reference to these sacs MÜLLER has pointed out—and the importance of this observation will become apparent when we deal with the development—that the ova are formed in the thickness of the inner wall of the sac. We hope to show that the inner wall of the sac is alone equivalent to the genital ridge of, for instance, the ovary of *Scyllium*. The outer aspect of this wall—*i.e.*, that turned towards the interior of the sac—is equivalent to the outer aspect of the Elasmobranch genital ridge, on which alone the ova are developed.* The sac into which the ova fall is, as we shall show in the embryological section, a special section of the body-cavity shut off from the remainder, and the dehiscence of the ova into this cavity is equivalent to their discharge into the body cavity in other forms.

The oviduct (Plate 26, fig. 60, *od.*) is a thin-walled duct of about 21 centims. in length in the example we are describing, continuous in front with the ovarian sac, and gradually tapering behind, till it ends (*od'.*) by opening into the dilated terminal section of the kidney duct on the inner side, a short distance before the latter unites with its fellow. It is throughout closely attached to the ureter and placed on its inner, and to some extent on its ventral, aspect. The hindermost part of the oviduct which runs beside the enlarged portion of the kidney duct—that portion called by HYRTL the horn of the urinary bladder—is so completely enveloped by the wall of the horn of the urinary bladder as to appear like a projection into the lumen of the latter structure, and the somewhat peculiar appearance which it presents in HYRTL's figure is due to this fact. In our examples the oviduct was provided with a simple opening into the kidney duct, on a slight papilla; the peculiar dilatations and processes of the terminal parts of the oviduct, which have been described by HYRTL, not being present.

The results we have arrived at with reference to the male organs are very different indeed from those of our predecessor, in that we find *the testicular products to be carried off by a series of vasa efferentia, which traverse the mesorchium, and are continuous with the uriniferous tubuli; so that the semen passes through the uriniferous tubuli into the kidney duct and so to the exterior. We have moreover been unable to find in the male a duct homologous with the oviduct of the female.*

This mode of transportation outwards of the semen has not hitherto been known to occur in Ganoids, though found in all Elasmobranchii, Amphibia, and Amniota. It is not, however, impossible that it exists in other Ganoids, but has hitherto been overlooked.

Our male example of *Lepidosteus* was about 60 centims. in length, and was no doubt mature. It was smaller than any of our female examples, but this according to GARMAN (vide, p. 361) is usual. The testes (Plate 26, fig. 58 A, *t.*) occupied a similar position to the ovaries, and were about 21 centims. long. They were, as is frequently the case with piscine testes, divided into a series of lobes (10–12), and were suspended by a delicate mesentery (mesorchium) from the dorsal wall of the abdomen on each side of the dorsal aorta. HYRTL (No. 11) states that air or quicksilver injected between the limbs of the mesentery, passed into a vas deferens homologous with the oviduct which joins the ureter. We have been unable to find such a vas deferens; but we have found in the mesorchium a number of tubes of a yellow colour, the colour being due to a granular substance quite unlike coagulated blood, but which appeared to us from microscopic examination to be the remains of spermatozoa.* These tubes to the number of 40–50 constitute, we believe, the vasa efferentia. Along the line of suspension of the testis on its inner border these tubes unite to form an elaborate network of tubes placed on the inner face of the testis—an arrangement very similar to that often found in Elasmobranchii (vide F. M. BALFOUR, 'Monograph on the Development of Elasmobranch Fishes,' plate 19, figs. 4 and 8).

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* 'Treatise on Comparative Embryology,' vol. i., p. 43.

We have figured this network on the posterior lobe of the testis (fig. 58 B), and have represented a section through it (fig. 59 A, *n.v.e.*), and through one of the vasa efferentia (*v.e.*) in the mesorchium. Such a section conclusively demonstrates the real nature of these passages: they are filled with sperm like that in the body of the testis, and are, as may be seen from the section figured, continuous with the seminal tubes of the testis itself.

At the attached base of the mesorchium the vasa efferentia unite into a longitudinal canal, placed on the inner side of the kidney duct (Plate 26, fig. 58 A, *l.c.*, also shown in section in Plate 26, fig. 59 B, *l.c.*). From this canal tubules pass off which are continuous with the tubuli uriniferi, as may be seen from fig. 59 B, but the exact course of these tubuli through the kidney could not be made out in the preparations we were able to make of the badly conserved kidney. HYRTL describes the arrangement of the vascular trunks in the mesorchium in the following way (No. 11, p. 6): "The mesorchium contains vascular trunks, viz., veins, which through their numerous anastomoses form a plexus at the hilus of the testis, whose efferent trunks, 13 in number, again unite into a plexus on the vertebral column, which is continuous with the cardinal veins." The arrangement (though not the number) of HYRTL's vessels is very similar to that of our vasa efferentia, and we cannot help thinking that a confusion of the two may have taken place; which, in badly conserved specimens, not injected with semen, would be very easy.

We have, as already stated, been unable to find in our dissections any trace of a duct homologous with the oviduct of the female, and our sections through the kidney and its ducts equally fail to bring to light such a duct. The kidney ducts are about 19 centims. in length, measured from the genital aperture to their front end. These ducts are generally similar to those in the female; they unite about 2 centims. from the genital pore to form an unpaired vesicle. Their posterior parts are considerably enlarged, forming what HYRTL calls the horns of the urinary bladder. In these enlarged portions, and in the wall of the unpaired urinary bladder, numerous transverse partitions are present, as correctly described by HYRTL, which are similar to those in the female, but more numerous. They give rise to a series of pits, at the blind ends of which are placed the openings of the kidney tubules. The kidney duct without doubt serves as vas deferens, and we have found in it masses of yellowish colour similar to the substance in the vasa efferentia identified by us as remains of spermatozoa.

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* The females we examined, which were no doubt procured at the same time as the male, had their oviducts filled with ova: and it is therefore not surprising that the vasa efferentia should be naturally injected with sperm.

II.—*Development.*

In the general account of the development we have already called attention to the earliest stages of the excretory system.

We may remind the reader that the first part of the system to be formed is the segmental or archinephric duct (Plate 23, figs. 28 and 29, *sg.*). This duct arises, as in Teleostei and Amphibia, by the constriction of a hollow ridge of the somatic mesoblast into a canal, which is placed in contiguity with the epiblast, along the line of junction between the mesoblastic somites and the lateral plates of mesoblast. Anteriorly the duct does not become shut off from the body-cavity, and also bends inwards towards the middle line. The inflected part of the duct is the first rudiment of the pronephros, and very soon becomes considerably dilated relatively to the posterior part of the duct.

The posterior part of each segmental duct acquires an opening into the cloacal section of the alimentary tract. Apart from this change, the whole of the ducts, except their pronephric sections, remain for a long time unaltered, and the next changes we have to speak of concern the definite establishment of the pronephros.

The dilated incurved portion of each segmental duct soon becomes convoluted, and by the time the embryo is about 10 millims. in length, but before the period of hatching, an important change is effected in the relations of their peritoneal openings.*

Instead of leading into the body-cavity, they open into an isolated chamber on each side (Plate 25, fig. 51, *pr.c.*), which we will call the *pronephric chamber.* The pronephric chamber is not, however, so far as we can judge, completely isolated from the body-cavity. We have not, it is true, detected with certainty at this stage a communication between the two; but in later stages, in larvæ of from 11 to 26 millims., we have found a richly ciliated passage leading from the body-cavity into the pronephros on each side (Plate 25, fig. 52, *p.f.p.*). We have not succeeded in determining with absolute certainty the exact relations between this passage and the tube of the pronephros, but we are inclined to believe that it opens directly into the pronephric chamber just spoken of.

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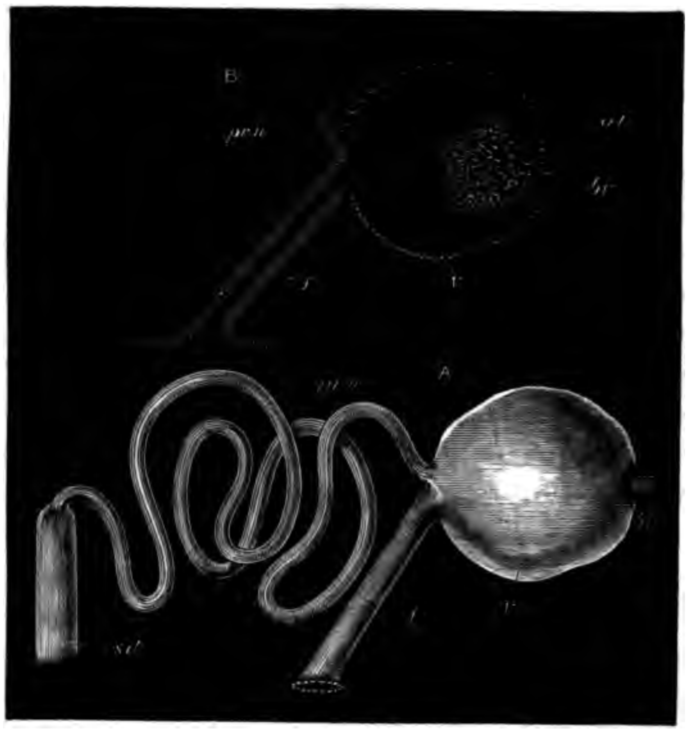
* The change is probably effected somewhat earlier than would appear from our description, but our specimens were not sufficiently well preserved to enable us to speak definitely as to the exact period.

As we hope to show, this chamber soon becomes largely filled by a vascular glomerulus. On the accomplishment of these changes, the pronephros is essentially provided with all the parts typically present in a segment of the mesonephros (woodcut, fig. 4). There is a peritoneal tube (*f.*),* opening into a vesicle (*v.*); from near the neck of the peritoneal tube there comes off a convoluted tube (*pr.n.*), forming the main mass of the pronephros, and ending in the segmental duct (*sd.*).

Fig. 4.



Diagrammatic views of the pronephros of *Lepidosteus.*

A, pronephros supposed to be isolated and seen from the side; B, section through the vesicle of the pronephros and the ciliated peritoneal funnel leading into it; *pr.n.*, coiled tube of pronephros; ***sd.***, segmental or archinephric duct; *f.*, peritoneal funnel; *v.*, vesicle of pronephros; *bv.*, blood vessel of glomerulus; *gl.*, glomerulus.

The different parts do not, however, appear to have the same morphological significance as those in the mesonephros.

Judging from the analogy of Teleostei, the embryonic structure of whose pronephros is strikingly similar to that of *Lepidosteus*, the two pronephric chambers into which the segmental ducts open are constricted off sections of the body-cavity.

With the formation of the convoluted duct opening into the isolated section of

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[*] We feel fairly confident that there is only one pronephric opening on each side, though we have no single series of sections sufficiently complete to demonstrate this fact with absolute certainty.

the body-cavity we may speak of a definite pronephros as having become established. The pronephros is placed, as can be made out in later stages, on the level of the opening of the air-bladder into the throat.

The pronephros increases in size, so far as could be determined, by the further convolution of the duct of which it is mainly formed; and the next change of importance which we have noticed is the formation of a vascular projection into the pronephric chamber, forming the glomerulus already spoken of (vide woodcut, fig. 4, *gl.*), which is similar to that of the pronephros of Teleostei. We first detected these glomeruli in an embryo of about 15 millims., some days after hatching (Plate 25, fig. 52, *gl.*), but it is quite possible that they may be formed considerably earlier.

In the same embryo in which the glomeruli were found we also detected for the first time a *mesonephros* consisting of a series of isolated segmental or nephridial tubes, placed posteriorly to the pronephros along the dorsal wall of the abdomen.

These were so far advanced at this stage that we are not in a position to give any account of their mode of origin. They are, however, formed independently of the segmental ducts, and in the establishment of the junction between the two structures, there is no outgrowth from the segmental duct to meet the segmental tubes. We could not at this stage find peritoneal funnels of the segmental tubes, though we have met with them at a later stage (Plate 25, fig. 53, *p.f.*), and our failure to find them at this stage is not to be regarded as conclusive against their existence.

A very considerable space exists between the pronephros and the foremost segmental tube of the mesonephros. The anterior mesonephric tubes are, moreover, formed earlier than the posterior.

In the course of further development, the mesonephric tubules increase in size, so that there ceases to be an interval between them, the mesonephros thus becoming a continuous gland. In an embryo of 26 millims. there was no indication of the formation of segmental tubes to fill up the space between the pronephros and mesonephros.

The two segmental ducts have united behind into an unpaired structure in an embryo of 11 millims. This structure is no doubt the future unpaired urinogenital chamber (Plate 26, figs. 58 A, and 60, *bl.*). Somewhat later, the hypoblastic cloaca becomes split into two sections, the hinder one receiving the coalesced segmental ducts, and the anterior remaining continuous with the alimentary tract. The opening of the hinder one forms the urinogenital opening, and that of the anterior the anus.

In an older larva of about 5·5 centims. the pronephros did not exhibit any marked signs of atrophy, though the duct between it and the mesonephros was somewhat reduced and surrounded by the trabecular tissue spoken of in connexion with the adult. In the region between the pronephros and the front end of the fully developed part of the mesonephros very rudimentary tubules had become established.

The latest stage of the excretory system which we have studied is in a young Fish of about 11 centims. in length. The special interest of this stage depends upon the fact that the ovary is already developed, and not only so, but the formation of the oviducts has commenced, and their condition at this stage throws considerable light on the obscure problem of their nature in the Ganoids.

Unfortunately, the head of the young Fish had been removed before it was put into our hands, so that it was impossible for us to determine whether the pronephros was still present; but as we shall subsequently show, the section of the segmental duct, originally present between the pronephros and the front end of the permanent kidney or mesonephros, has in any case disappeared.

In addition to an examination of the excretory organs *in situ*, which showed little except the presence of the generative ridges, we made a complete series of sections through the excretory organs for their whole length (Plate 26, figs. 54–57).

Posteriorly these sections showed nothing worthy of note, the excretory organs and their ducts differing in no important particular from these organs as we have described them in the adult, except in the fact that the segmental ducts are not joined by the oviducts.

Some little way in front of the point where the two segmental ducts coalesce to form the urinary bladder, the genital ridge comes into view. For its whole extent, except near its anterior part (of which more hereafter) this ridge projects freely into the body-cavity, and in this respect the young Fish differs entirely from the adult. As shown in Plate 26, figs. 56 and 57 (*g.r.*), it is attached to the abdominal wall on the ventral side of, and near the inner border of each kidney. The genital ridge itself has a structure very similar to that which is characteristic of young Elasmobranchii, and it may be presumed of young Fishes generally. The free edge of the ridge is swollen, and this part constitutes the true generative region of the ridge, while its dorsal portion forms the supporting mesentery. The ridge itself is formed of a central stroma and a germinal epithelium covering it. The epithelium is thin on the whole of the inner aspect of the ridge, but, just as in Elasmobranchii, it becomes greatly thickened for a band-like strip on the outer aspect. Here, the epithelium is several layers deep, and contains numerous primitive germinal cells (*p.o.*).

Though the generative organs were not sufficiently advanced for us to decide the point with certainty, the structure of the organ is in favour of the view that this specimen was a female, and, as will be shown directly, there can on other grounds be no doubt that this is so. The large size of the primitive germinal cells (primitive ova) reminded us of these bodies in Elasmobranchii.

In the region between the insertion of the genital ridge (or ovary, as we may more conveniently call it) and the segmental duct we detected the openings of a series of peritoneal funnels of the excretory tubes (Plate 26, fig. 57, *p.f.*), which clearly therefore persist till the young Fish has reached a very considerable size.

As we have already said, the ovary projects freely into the body-cavity for the greater part of its length. Anteriorly, however, we found that a lamina extended from the free ventral edge of the ovary to the dorsal wall of the body-cavity, to which it was attached on the level of the outer side of the segmental duct. A somewhat triangular channel was thus constituted, the inner wall of which was formed by the ovary, the outer by the lamina just spoken of, and the roof by the strip of the peritoneum of the abdominal wall covering that part of the ventral surface of the kidney in which the openings of the peritoneal funnels of the excretory tubes are placed. The structure of this canal will be at once understood by the section of it shown in Plate 26, fig. 55.

There can be no doubt that this canal is the commencing ovarian sac. On tracing it backwards we found that the lamina forming its outer wall arises as a fold growing upwards from the free edge of the genital ridge meeting a downward growth of the peritoneal membrane from the dorsal wall of the abdomen; and in Plate 26, fig. 56, these two laminæ may be seen before they have met. Anteriorly the canal becomes gradually smaller and smaller in correlation with the reduced size of the ovarian ridge, and ends blindly nearly on a level with the front end of the excretory organs.

It should be noted that, owing to the mode of formation of the ovarian sac, the outer side of the ovary with the band of thickened germinal epithelium is turned towards the lumen of the sac; and thus the fact of the ova being formed on the inner wall of the genital sac in the adult is explained, and the comparison which we instituted in our description of the adult between the inner wall of the genital sac and the free genital ridge of Elasmobranchs receives its justification.

It is further to be noticed that, from the mode of formation of the ovarian sac, the openings of the peritoneal funnels of the excretory organs ought to open into its lumen; and if these openings persist in the adult, they will no doubt be found in this situation.

Before entering on further theoretical considerations with reference to the oviduct, it will be convenient to complete our description of the excretory organs at this stage.

When we dissected the excretory organs out, and removed them from the body of the young Fish, we were under the impression that they extended for the whole length of the body-cavity. Great was our astonishment to find that slightly in front of the end of the ovary both excretory organs and segmental ducts grew rapidly smaller and finally vanished, and that what we had taken to be the front part of the kidney was nothing else but a linear streak of tissue formed of cells with peculiar granular contents supported in a trabecular work (Plate 26, fig. 54). This discovery first led us to investigate histologically what we, in common with previous observers, had supposed to be the anterior end of the kidneys in the adult, and to show that they were nothing else but trabecular tissue with cells like that of lymphatic glands. The interruption of the segmental duct at the commencement of this tissue demonstrates that if any rudiment of the pronephros still persists, it is quite functionless, in that it is not provided with a duct.

III.—*Theoretical considerations.*

There are three points in our observations on the urinogenital system which appear to call for special remark. The first of these concerns the structure and fate of the pronephros, the second the nature of the oviduct, and the third the presence of vasa efferentia in the male.

Although the history we have been able to give of the pronephros is not complete, we have nevertheless shown that in most points it is essentially similar to the pronephros of Teleostei. In an early stage we find the pronephros provided with a peritoneal funnel opening into the body-cavity. At a later stage we find that there is connected with the pronephros on each side, a cavity—the pronephric cavity—into which a glomerulus projects. This cavity is in communication on the one hand with the lumen of the coiled tube which forms the main mass of the pronephros, and on the other hand with the body-cavity by means of a richly ciliated canal (woodcut, fig. 4, p. 416).

In Teleostei the pronephros has precisely the same characters, except that the cavity in which the glomerulus is placed is without a peritoneal canal.

The questions which naturally arise in connexion with the pronephros are: (1) what is the origin of the above cavity with its glomerulus; and (2) what is the meaning of the ciliated canal connecting this cavity with the peritoneal cavity?

We have not from our researches been able to answer the first of these questions. In Teleostei, however, the origin of this cavity has been studied by ROSENBERG* and GÖTTE.† According to the account of the latter, which we have not ourselves confirmed but which has usually been accepted, the front end of the segmental duct, instead of becoming folded off from the body-cavity, becomes included in a kind of diverticulum of the body-cavity, which only communicates with the remainder of the body-cavity by a narrow opening. On the inner wall of this diverticulum a projection is formed which becomes a glomerulus. At this stage in the development of the pronephros we have essentially the same parts as in the fully formed pronephros of *Lepidosteus*, the only difference being that the passage connecting the diverticulum containing the glomerulus with the remainder of the body-cavity is short in Teleostei, and in *Lepidosteus* forms a longish ciliated canal. In Teleostei the opening into the body-cavity becomes soon closed. If the above comparison is justified, and if the development of these parts in *Lepidosteus* takes place as it is described as doing in Teleostei, there can, we think, be no doubt that the ciliated canal of *Lepidosteus*, which connects the pronephric cavity with the body-cavity, is a persisting communication between this cavity and the body-cavity; and that *Lepidosteus* presents in this respect a more primitive type of pronephros than Teleostei.

It may be noted that in *Lepidosteus* the whole pronephros has exactly the character of a single segmental tube of the mesonephros. The pronephric cavity with its

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* ROSENBERG, Untersuch. üb. d. Entwick. d. Teleostierniere. Dorpat, 1867.

† GÖTTE, Entwick. d. Unke, p. 826.

glomerulus is identical in structure with a malpighian body. The ciliated canal is similar in its relations to the peritoneal canal of such a segmental tube, and the coiled portion of the pronephros resembles the secreting part of the ordinary segmental tube. This comparison is no doubt an indication that the pronephros is physiologically very similar to the mesonephros, and so far justifies SEDGWICK'S* comparison between the two, but it does not appear to us to justify the morphological conclusions at which he has arrived, or to necessitate any modification in the views on this subject expressed by one of us.†

The genital ducts of Ganoids and Teleostei have for some time been a source of great difficulty to morphologists; and any contributions with reference to the ontogeny of these structures are of interest.

The essential point which we have made out is that the anterior part of the oviduct of *Lepidosteus* arises by a fold of the peritoneum attaching itself to the free edge of the genital ridge. We have not, unfortunately, had specimens old enough to decide how the posterior part of the oviduct is formed; and although in the absence of such stages it would be rash in the extreme to speak with confidence as to the nature of this part of the duct, it may be well to consider the possibilities of the case in relation to other Ganoids and Teleostei.

The simplest supposition would be that the posterior part of the genital duct had the same origin as the anterior, *i.e.*, that it was formed for its whole length by the concrescence of a peritoneal fold with the genital ridge, and that the duct so formed opened into the segmental duct.

The other possible supposition is that a true Müllerian duct—*i.e.*, a product of the splitting of the segmental duct—is subsequently developed, and that the open end of this duct coalesces with the duct which has already began to be formed in our oldest larva.

In attempting to estimate the relative probability of these two views, one important element is the relation of the oviducts of *Lepidosteus* to those of other Ganoids.

In all other Ganoids (vide HYRTL, No. 11) there are stated to be genital ducts in both sexes which are provided at their anterior extremities with a funnel-shaped mouth open to the abdominal cavity. At first sight, therefore, it might be supposed that they had no morphological relationship with the oviducts of *Lepidosteus*, but, apart from the presence of a funnel-shaped mouth, the oviducts of *Lepidosteus* are very similar to those of Chondrostean Ganoids, being thin-walled tubes opening on a projecting papilla into the dilated kidney ducts (horns of the urinary bladder, HYRTL). These relations seem to prove beyond a doubt that the oviduct of *Lepidosteus* is for its major part homologous with the genital ducts of other Ganoids.

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* SEDGWICK, "Early Development of the Wolffian Duct and anterior Wolffian Tubules in the Chick; with some Remarks on the Vertebrate Excretory System." Quart. Jour. of Micros. Science, vol. xxi., 1881.

† F. M. BALFOUR, 'Comparative Embryology,' vol. ii., pp. 600–603.

The relationship of the genital ducts to the kidney ducts in *Amia* and *Polypterus* is somewhat different from that in the Chondrostei and *Lepidosteus.* In *Amia* the ureters are so small that they may be described rather as joining the coalesced genital ducts than *vice versâ,* although the apparent coalesced portion of the genital ducts is shown to be really part of the kidney ducts by receiving the secretion of a number of mesonephric tubuli. In *Polypterus* the two ureters are stated to unite, and open by a common orifice into a sinus formed by the junction of the two genital ducts, which has not been described as receiving directly the secretion of any part of the mesonephros.

It has been usual to assume that the genital ducts of Ganoids are true Müllerian ducts in the sense above defined, on the ground that they are provided with a peritoneal opening and that they are united behind with the kidney ducts. In the absence of ontological evidence this identification is necessarily provisional. On the assumption that it is correct we should have to accept the second of the two alternatives above suggested as to the development of the posterior parts of the oviduct in *Lepidosteus.*

There appear to us, however, to be sufficiently serious objections to this view to render it necessary for us to suspend our judgment with reference to this point. In the first place, if the view that the genital ducts are Müllerian ducts is correct, the true genital ducts of *Lepidosteus* must necessarily be developed at a later period than the secondary attachment between their open mouths and the genital folds, which would, to say the least of it, be a remarkable inversion of the natural order of development. Secondly, the condition of our oldest larva shows that the Müllerian duct, if developed later, is only split off from quite the posterior part of the segmental duct; yet in all types in which the development of the Müllerian duct has been followed, its anterior extremity, with the abdominal opening, is split off from either the foremost or nearly the foremost part of the segmental duct.

Judging from the structure of the adult genital duct of other Ganoids they must also be developed only from the posterior part of the segmental duct, and this peculiarity so struck one of us that in a previous paper* the suggestion was put forward that the true Ganoid genital ducts were perhaps not Müllerian ducts, but enlarged segmental tubes with persisting abdominal funnels belonging to the mesonephros.

If the possibility of the oviduct of *Lepidosteus* not being a Müllerian duct is admitted, a similar doubt must also exist as to the genital ducts of other Ganoids, and we must be prepared to show that there is a reasonable ground for scepticism on this point. We would in this connexion point out that the second of the two arguments urged against the view that the genital duct of *Lepidosteus* is not a Müllerian duct applies with equal force to the case of all other Ganoids.

The short funnel-shaped genital duct of the Chondrostei is also very unlike undoubted Müllerian ducts, and could moreover easily be conceived as originating by a fold of the peritoneum, a slight extension of which would give rise to a genital duct like that of *Lepidosteus.*

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* F. M. Balfour, "On the Origin and History of the Urinogenital Organs of Vertebrates," Journ. of Anat. and Phys., vol. x., 1876.

The main difficulty of the view that the genital ducts of Ganoids are not Müllerian ducts lies in the fact that they open into the segmental duct. While it is easy to understand the genesis of a duct from a folding of the peritoneum, and also easy to understand how such a duct might lead to the exterior by coalescing, for instance, with an abdominal pore, it is not easy to see how such a duct could acquire a communication with the segmental duct.

We do not under these circumstances wish to speak dogmatically, either in favour of or against the view that the genital ducts of Ganoids are Müllerian ducts. Their ontogeny would be conclusive on this matter, and we trust that some of the anatomists who have the opportunity of studying the development of the Sturgeon will soon let us know the facts of the case. If there are persisting funnels of the mesonephric segmental tubes in adult Sturgeons, some of them ought to be situated within the genital ducts, if the latter are not Müllerian ducts; and naturalists who have the opportunity ought also to look out for such openings.

The mode of origin of the anterior part of the genital duct of *Lepidosteus* appears to us to tell strongly in favour of the view, already regarded as probable by one of us,* that the Teleostean genital ducts are derived from those of Ganoids; and if, as appears to us indubitable, the most primitive type of Ganoid genital ducts is found in the Chondrostei, it is interesting to notice that the remaining Ganoids present in various ways approximations to the arrangement typically found in Teleostei. *Lepidosteus* obviously approaches Teleostei in the fact of the ovarian ridge forming part of the wall of the oviduct, but differs from the Teleostei in the fact of the oviduct opening into the kidney ducts, instead of each pair of ducts having an independent opening in the cloaca, and in the fact that the male genital products are not carried to the exterior by a duct homologous with the oviduct. *Amia* is closer to the Teleostei in the arrangement of the posterior part of the genital ducts, in that the two genital ducts coalesce posteriorly; while *Polypterus* approaches still nearer to the Teleostei in the fact that the two genital ducts and the two kidney ducts unite with each other before they join; and in order to convert this arrangement into that characteristic of the Teleostei we have only to conceive the coalesced ducts of the kidneys acquiring an independent opening into the cloaca behind the genital opening.

The male genital ducts.—The discovery of the vasa efferentia in *Lepidosteus,* carrying off the semen from the testis, and transporting it to the mesonephros, and thence through the mesonephric tubes to the segmental duct, must be regarded as the most important of our results on the excretory system.

It proves in the first place that the transportation outwards of the genital products of both sexes by homologous ducts, which has been hitherto held to be universal in

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* F. M. Balfour, 'Comparative Embryology,' vol. ii., p. 605.

Ganoids, and which, in the absence of evidence to the contrary, must still be assumed to be true for all Ganoids except *Lepidosteus*, is a secondary arrangement. This conclusion follows from the fact that in Elasmobranchs, &c., which are not descendants of the Ganoids, the same arrangement of seminal ducts is found as in *Lepidosteus*, and it must therefore have been inherited from an ancestor common to the two groups.

If, therefore, the current statements about the generative ducts of Ganoids are true, the males must have lost their vasa efferentia, and the function of vas deferens must have been taken by the homologue of the oviduct, presumably present in the male. The Teleostei must, moreover, have sprung from Ganoidei in which the vasa efferentia had become aborted.

Considerable phylogenetic difficulties as to the relationships of Ganoidei and Elasmobranchii are removed by the discovery that Ganoids were originally provided with a system of vasa efferentia like that of Elasmobranchii.

THE ALIMENTARY CANAL AND ITS APPENDAGES.

I.—*Anatomy.*

AGASSIZ (No. 2) gives a short description with a figure of the viscera of *Lepidosteus* as a whole. VAN DER HŒVEN has also given a figure of them in his memoir on the air-bladder of this form (No. 8), and JOHANNES MÜLLER first detected the spiral valve and gave a short account of it in his memoir (No. 13). STANNIUS, again, makes several references to the viscera of *Lepidosteus* in his anatomy of the Vertebrata, and throws some doubt on MÜLLER'S determination of the spiral valve.

The following description refers to a female *Lepidosteus* of 100·5 centims. (Plate 27, fig. 66).

With reference to the mouth and pharynx, we have nothing special to remark. Immediately behind the pharynx there comes an elongated tube, which is not divisable into stomach and œsophagus, and may be called the stomach (*st.*). It is about 44·6 centims. long, and gradually narrows from the middle towards the hinder or pyloric extremity. It runs straight backwards for the greater part of its length, the last 3·8 centims., however, taking a sudden bend forwards. For about half its length the walls are thin, and the mucous membrane is smooth; in the posterior half the walls are thick, and the mucous membrane is raised into numerous longitudinal ridges. The peculiar glandular structure of the epithelium of this part in the embryo is shown in Plate 27, fig. 62 (*st.*). Its opening into the duodenium is provided with a very distinct pyloric valve (*py.*). This valve projects into a kind of chamber, freely communicating with the duodenum, and containing four large pits (*c'.*), into each of which a group of pyloric cæca opens. These cæca form a fairly compact gland (*c.*) about 6·5 centims. long, which overlaps the stomach anteriorly, and the duodenum posteriorly.

Close to the pyloric valve, on its right side, is a small papilla, on the apex of which the bile duct opens (*b.d′.*).

A small, apparently glandular, mass closely connected with the bile duct, in the position in which we have seen the pancreas in the larva (Plate 27, figs. 62 and 63, *p.*), is almost certainly a rudimentary pancreas, like that of many Teleostei; but its preservation was too bad for histological examination. We believe that the pancreas of *Lepidosteus* has hitherto been overlooked.

The small intestine passes straight backwards for about 8 centims., and then presents three compact coils. From the end of these a section, about 5 centims. long, the walls of which are much thicker, runs forwards. The intestine then again turns backwards, making one spiral coil. This spiral part passes directly, without any sharp line of demarcation, into a short and straight tube, which tapers slightly from before backwards, and ends at the anus. The mucous membrane of the intestine for about the first 3·5 centims. is smooth, and the muscular walls thin: the rest of the small intestine has thick walls, and the mucous membrane is reticulated.

A short spiral valve (*sp.v.*), with a very rudimentary epithelial fold, making nearly two turns, begins in about the posterior half of the spiral coil of the intestine, extending backwards for slightly less than half the straight terminal portion of the intestine, and ending 4 centims. in front of the anus. Its total length in one example was about 4·5 centims.

The termination of the spiral valve is marked by a slight constriction, and we may call the straight portion of the intestine behind it the rectum (*rc.*).

The posterior part of the intestine, from the beginning of the spiral valve to the anus, is *connected with the ventral wall of the abdomen by a mesentery.*

The air-bladder (*a.b.*) is 45 centims. long, and opens into the alimentary canal by a slit-like aperture (*a.b′.*) on the median dorsal line, immediately behind the epipharyngeal teeth. Each lip of this aperture is largely formed by a muscular cushion, thickest at its posterior end, and extending about 6 millims. behind the aperture itself. A narrow passage is bounded by these muscular walls, which opens dorsally into the air-bladder.

The air-bladder is provided with two short anterior cornua, and tapers to a point behind: it shows no indication of any separation into two parts. A strong band of connective tissue runs along the inner aspect of its whole dorsal region, from which there are given off on each side—at intervals of about 12 millims. anteriorly, gradually increasing to 18 millims. posteriorly—bands of muscle, which pass outwards towards its side walls, and then spread out into the numerous reticulations with which the air-bladder is lined throughout. By the contraction of these muscles the cavity of the air-bladder can doubtless be very much diminished.

The main muscular bands circumscribe a series of more or less complete chambers, which were about twenty-seven in number on each side in our example. The chambers are confined to the sides, so that there is a continuous cavity running through the central part of the organ. The whole organ has the characteristic structure of a simple lung.

The liver (*lr.*) consists of a single elongated lobe, about 32 centims. long, tapering anteriorly and posteriorly, the anterior half being on the average twice as thick as the posterior half. The gall-bladder (*g.b.*) lies at its posterior end, and is of considerable size, tapering gradually so as to pass insensibly into the bile duct. The hepatic duct (*hp.d.*) opens into the gall-bladder at its anterior end.

The spleen (*s.*) is a large, compact, double gland, one lobe lying in the turn of the intestine immediately above the spiral valve, and the other on the opposite side of the intestine, so that the intestine is nearly embraced between the two lobes.

II.—*Development.*

We have already described in detail the first formation of the alimentary tract so far as we have been able to work it out, and we need only say here that the anterior and posterior ends of the canal become first formed, and that these two parts gradually elongate, so as to approach each other; the growth of the posterior part is, however, the most rapid. The junction of the two parts takes place a very short distance behind the opening of the bile duct into the intestine.

For some time after the two parts of the alimentary tract have nearly met, the ventral wall of the canal at this point is not closed; so that there is left a passage between the alimentary canal and the yolk-sac, which forms a vitelline duct.

After the yolk-sac has ceased to be visible as an external appendage it still persists within the abdominal cavity. It has, however, by this stage ceased to communicate with the gut, so that the eventual absorption of the yolk is no doubt entirely effected by the vitelline vessels. At these later stages of development we have noticed that numerous yolk nuclei, like those met with in Teleostei and Elasmobranchii,* are still to be found in the yolk.

It will be convenient to treat the history of sections of the alimentary tract in front of and behind the vitelline duct separately. The former gives rise to the pharyngeal region, the œsophagus, the stomach, and the duodenum.

The pharyngeal region, immediately after it has become established, gives rise to a series of paired pouches. These may be called the branchial pouches, and are placed between the successive branchial arches. The first or hyomandibular pouch, placed between the mandibular and hyoid arches, has rather the character of a double layer of hypoblast than of a true pouch, though in parts a slight space is developed between its two walls. It is shown in section in Plate 24, fig. 43 (*h.m.*), from an embryo of about 10 millims., shortly before hatching. It does not appear to undergo any further development, and, so far as we can make out, disappears shortly after the embryo is hatched, without acquiring an opening to the exterior.

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* For a history of similar nuclei, vide 'Comp. Embryol.,' vol. ii., chapters iii. and iv.

It is important to notice that this cleft, which in the cartilaginous Ganoids and *Polypterus* remains permanently open as the spiracle, is rudimentary even in the embryo of *Lepidosteus*.

The second pouch is the hyobranchial pouch: its outer end meets the epiblast before the larva is hatched, and a perforation is effected at the junction of the two layers, converting the pouch into a visceral cleft.

Behind the hyobranchial pouch there are four branchial pouches, which become perforated and converted into branchial clefts shortly after hatching.

The region of the œsophagus following the pharynx is not separated from the stomach, unless a glandular posterior region (vide description of adult) be regarded as the stomach, a non-glanular anterior region forming the œsophagus. The lumen of this part appears to be all but obliterated in the stages immediately before hatching, giving rise for a short period to a solid œsophagus like that of Elasmobranchii and Teleostei.*

From the anterior part of the region immediately behind the pharynx the air-bladder arises as a dorsal unpaired diverticulum. From the very first it has an elongated slit-like mouth (Plate 27, fig. 64, *a.b'.*), and is placed in the mesenteric attachment of the part of the throat from which it springs.

We have first noticed it in the stages immediately after hatching. At first very short and narrow, it grows in succeeding stages longer and wider, making its way backwards in the mesentery of the alimentary tract (Plate 27, fig. 65, *a.b.*). In the larva of a month and a half old (26 millims.) it has still a perfectly simple form, and is without traces of its adult lung-like structure; but in the larva of 11 centims. it has the typical adult structure.

The stomach is at first quite straight, but shortly after the larva is hatched its posterior end becomes bent ventralwards and forwards, so that the flexure of its posterior end (present in the adult) is very early established. The stomach is continuous behind with the duodenum, the commencement of which is indicated by the opening of the bile duct.

The liver is the first-formed alimentary gland, and is already a compact body before the larva is hatched. We have nothing to say with reference to its development, except that it exhibits the same simple structure in the embryo that it does in the adult.

A more interesting glandular body is the pancreas. It has already been stated that in the adult we have recognised a small body which we believe to be the pancreas, but that we were unable to study its histological characters.

In the embryo there is a well-developed pancreas which arises in the same position and the same manner as in those Vertebrata in which the pancreas is an important gland in the adult.

We have first noticed the pancreas in a stage shortly after hatching (Plate 27, fig. 61, *p.*). It then has the form of a funnel-shaped diverticulum of the *dorsal* wall of the duodenum, immediately behind the level of the opening of the bile duct. From the apex of this funnel numerous small glandular tubuli soon sprout out.

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* Vide 'Comp. Embryol.,' vol. ii., pp. 50–63.

The similarity in the development of the pancreas in *Lepidosteus* to that of the same gland in Elasmobranchii is very striking.*

The pancreas at a later stage is placed immediately behind the end of the liver in a loop formed by the pyloric section of the stomach (Plate 27, fig. 62, *p.*). During larval life it constitutes a considerable gland, the anterior end of which partly envelopes the bile duct (Plate 27, fig. 63, *p.*).

Considering the undoubted affinities between *Lepidosteus* and the Teleostei, the facts just recorded with reference to the pancreas appear to us to demonstrate that the small size and occasional absence (?) of this gland in Teleostei is a result of the degeneration of this gland; and it seems probable that the pancreas will be found in the larvæ of most Teleostei. These conclusions render intelligible, moreover, the great development of the pancreas in the Elasmobranchii.

We have first noticed the pyloric cæca arising as outgrowths of the duodenum in larvæ of about three weeks old, and they become rapidly longer and more prominent (Plate 27, fig. 62, *c.*).

The portion of the intestine behind the vitelline duct is, as in all the Vertebrata, at first straight. In Elasmobranchs the lumen of the part of the intestine in which a spiral valve is present in the adult, very early acquires a more or less semilunar form by the appearance of a fold which winds in a long spiral. In *Lepidosteus* there is a fold similar in every respect (Plate 25, fig. 53, *sp.v.*), forming an open spiral round the intestine. This fold is the first indication of the spiral valve, but it is relatively very much later in its appearance than in Elasmobranchs, not being formed till about three weeks after hatching. It is, moreover, in correlation with the small extent of the spiral valve of the adult, confined to a much smaller portion of the intestine than in Elasmobranchii, although owing to the relative straightness of the anterior part of the intestine it is proportionately longer in the embryo than in the adult.

The similarity of the embryonic spiral valve of *Lepidosteus* to that of Elasmobranchii shows that STANNIUS' hesitatation in accepting MÜLLER'S discovery of the spiral valve in *Lepidosteus* is not justified.

J. MÜLLER ('Bau u. Entwick. d. Myxinoiden') holds that the so-called bursa entiana of Elasmobranchii (*i.e.*, the chamber placed between the part of the intestine with the spiral valve and the end of the pylorus) is the homologue of the more elongated portion of the small intestine which occupies a similar position in the Sturgeon. This portion of the small intestine is no doubt homologous with the still more elongated and coiled portion of the small intestine in *Lepidosteus* placed between the chamber into which the pyloric cæca, &c., open and the region of the spiral valve. The fact that the vitelline duct in the embryo *Lepidosteus* is placed close to the pyloric end of the stomach, and that the greater portion of the small intestine is derived from part of the alimentary canal behind this, shows that MÜLLER is mistaken in attempting to homologise the bursa entiana of Elasmobranchii, which is placed in front of the vitelline duct, with the coiled part of the small intestine of the above forms. The latter is either derived from an elongation of the very short portion of the intestine between the vitelline duct and the primitive spiral valve, or more probably by the conversion of the anterior part of the intestine, originally provided with a spiral valve into a coiled small intestine not so provided.

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* Vide F. M. BALFOUR, 'Monograph on Development of Elasmobranch Fishes,' p. 226.

We have already called attention to the peculiar mesentery present in the adult attaching the posterior straight part of the intestine to the ventral wall of the body. This mesentery, which together with the dorsal mesentery divides the hinder section of the body-cavity into two lateral compartments is, we believe, a persisting portion of the ventral mesentery which, as pointed out by one of us,* is primitively present for the whole length of the body-cavity. The persistence of such a large section of it as that found in the adult *Lepidosteus* is, so far as we know, quite exceptional. This mesentery is shown in section in the embryo in Plate 25, fig. 53 (*v.mt.*). The small vessel in it appears to be the remnant of the subintestinal vein.

THE GILL ON THE HYOID ARCH.

It is well known that *Lepidosteus* is provided with a gill on the hyoid arch, divided on each side into two parts. An excellent figure of this gill is given by MÜLLER (No. 13, plate 5, fig. 6), who holds from a consideration of the vascular supply that the two parts of this gill represent respectively the hyoid gill and the mandibular gill (called by MÜLLER pseudobranch). MÜLLER's views on this subject have not usually been accepted, but it is the fashion to regard the whole of the gill as the hyoid gill divided into two parts. It appeared to us not improbable that embryology might throw some light on the history of this gill, and accordingly we kept a look out in our embryos for traces of gills on the hyoid and mandibular arches. The results we have arrived at are purely negative, but are not the less surprising for this fact. The hyomandibular cleft as shown above, is never fully developed, and early undergoes a complete atrophy—a fact which is, on the whole, against MÜLLER's view; but what astonished us most in connexion with the gill in question is that we have been unable to find any trace of it even in the oldest larva whose head we have had (26 millims.), and at a period when the gills on the hinder arches have reached their full development.

We imagined the gill in question to be the remnant of a gill fully formed in extinct Ganoid types, and therefore expected to find it better developed in the larva than in the adult. That the contrary is the fact appears to us fairly certain, although we cannot at present offer any explanation of it.

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* 'Comparative Embryology,' vol. ii., p. 514.

Systematic Position of Lepidosteus.

A. Agassiz concludes his memoir on the development of *Lepidosteus* by pointing out that in spite of certain affinities in other directions this form is "not so far removed from the bony Fishes as has been supposed." Our own observations go far to confirm Agassiz' opinion.

Apart from the complete segmentation, the general development of *Lepidosteus* is strikingly Teleostean. In addition to the general Teleostean features of the embryo and larva, which can only be appreciated by those who have had an opportunity of practically working at the subject, we may point to the following developmental features* as indicative of Teleostean affinities:—

(1.) The formation of the nervous system as a solid keel of the epiblast.

(2.) The division of the epiblast into a nervous and epidermic stratum.

(3.) The mode of development of the gut (vide pp. 369, 370).

(4.) The mode of development of the pronephros; though, as shown on p. 420, the pronephros of *Lepidosteus* has primitive characters not retained by Teleostei.

(5.) The early stages in the development of the vertebral column (vide p. 388).

In addition to these, so to speak, purely embryonic characters there are not a few important adult characters:—

(1.) The continuity of the oviducts with the genital glands.

(2.) The small size of the pancreas, and the presence of numerous so-called pancreatic cæca.

(3.) The somewhat coiled small intestine.

(4.) Certain characters of the brain, *e.g.*, the large size of the cerebellum; the presence of the so-called lobi inferiores on the infundibulum; and of tori semicirculares in the mid-brain.

In spite of the undoubtedly important list of features to which we have just called attention, a list containing not less important characters, both embryological and adult, separating *Lepidosteus* from the Teleostei, can be drawn up:—

(1.) The character of the truncus arteriosus.

(2.) The fact of the genital ducts joining the ureters.

(3.) The presence of vasa efferentia in the male carrying the semen from the testes to the kidney, and through the tubules of the latter into the kidney duct.

(4.) The presence of a well-developed opercular gill.

(5.) The presence of a spiral valve; though this character may possibly break down with the extension of our knowledge.

(6.) The typical Ganoid characters of the thalamencephalon and the cerebral hemispheres (vide pp. 381 and 382).

(7.) The chiasma of the optic nerves.

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* The features enumerated above are not in all cases confined to *Lepidosteus* and Teleostei, but are always eminently characteristic of the latter.

(8.) The absence of a pecten, and presence of a vascular membrane between the vitreous humour and the retina.

(9.) The opisthocœlous form of the vertebræ.

(10.) The articulation of the ventral parts of the hæmal arches of the tail with processes of the vertebral column.

(11.) The absence of a division of the muscles into dorso-lateral and ventro-lateral divisions.

(12.) The complete segmentation of the ovum.

The list just given appears to us sufficient to demonstrate that *Lepidosteus* cannot be classed with the Teleostei; and we hold that MÜLLER's view is correct, according to which *Lepidosteus* is a true Ganoid.

The existence of the Ganoids as a distinct group has, however, recently been challenged by so distinguished an Ichthyologist as GÜNTHER, and it may therefore be well to consider how far the group as defined by MÜLLER is a natural one for living forms,* and how far recent researches enable us to improve upon MÜLLER's definitions. In his classical memoir (No. 13) the characters of the Ganoids are thus shortly stated:—

"These Fishes are either provided with plate-like angular or rounded cement-covered scales, or they bear osseous plates, or are quite naked. The fins are often, but not always, beset with a double or single row of spinous plates or splints. The caudal fin occasionally embraces in its upper lobe the end of the vertebral column, which may be prolonged to the end of the upper lobe. Their double nasal openings resemble those of Teleostei. The gills are free, and lie in a branchial cavity under an operculum, like those of Teleostei. Many of them have an accessory organ of respiration, in the form of an opercular gill, which is distinct from the pseudobranch, and can be present together with the latter; many also have spiracles like Elasmobranchii. They have many valves in the stem of the aorta like the latter, also a muscular coat in the stem of the aorta. Their ova are transported from the abdominal cavity by oviducts. Their optic nerves do not cross each other. The intestine is often provided with a spiral valve, like Elasmobranchii. They have a swimming-bladder with a duct, like many Teleostei. Their pelvic fins are abdominal.

"If we include in a definition only those characters which are invariable, the Ganoids may be shortly defined as being those Fish with numerous valves to the stem of the aorta, which is also provided with a muscular coat; with free gills and an operculum, and with abdominal pelvic fins."

To these distinctive characters, he adds in an appendix to his paper, the presence of the spiral valve, and the absence of a processus falciformis and a choroid gland.

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* We do not profess to be able to discuss this question for extinct forms of Fish, though of course it is a necessary consequence of the theory of descent that the various groups should merge into each other as we go back in geological time.

To the distinctive set of characters given by MÜLLER we may probably add the following :—

(1.) Oviducts and urinary ducts always unite, and open by a common urinogenital aperture behind the anus.

(2.) Skull hyostylic.

(3.) Segmentation complete in the types so far investigated, though perhaps *Amia* may be found to resemble the Teleostei in this particular.

(5.) A pronephros of the Teleostean type present in the larva.

(6.) Thalamencephalon very large and well developed.

(7.) The ventricle in the posterior part of the cerebrum is not divided behind into lateral halves, the roof of the undivided part being extremely thin.

(8.) Abdominal pores always present.

The great number of characters just given are amply sufficient to differentiate the Ganoids as a group; but, curiously enough, the only characters amongst the whole series which have been given, which can be regarded as peculiar to the Ganoids, are (1) the characters of the brain, and (2) the fact of the oviducts and kidney ducts uniting together and opening by a common pore to the exterior.

This absence of characters peculiar to the Ganoids is an indication of how widely separated in organisation are the different members of this great group.

At the same time, the only group with which existing Ganoids have close affinities is the Teleostei. The points they have in common with the Elasmobranchii are merely such as are due to the fact that both retain numerous primitive Vertebrate characters,* and the gulf which really separates them is very wide.

There is again no indication of any close affinity between the Dipnoi and, at any rate, existing Ganoids.

Like the Ganoids, the Dipnoi are no doubt remnants of a very primitive stock; but in the conversion of the air-bladder into a true lung, the highly specialised character of their limbs,† their peculiar autostylic skulls, the fact of their ventral nasal openings leading directly into the mouth, their multisegmented bars (interspinous bars), directly prolonged from the neural and hæmal arches and supporting the fin-rays of the unpaired dorsal and ventral fins, and their well-developed cerebral hemispheres, very unlike those of Ganoids and approaching the Amphibian type, they form a very well-defined group, and one very distinctly separated from the Ganoids.

No doubt the Chondrostean Ganoids are nearly as far removed from the Teleostei as from the Dipnoi, but the links uniting these Ganoids with the Teleostei have been so fully preserved in the existing fauna of the globe, that the two groups almost run into each other. If, in fact, we were anxious to make any radical change in the ordinary classification of Fishes, it would be by uniting the Teleostei and Ganoids, or rather constituting the Teleostei into one of the sub-groups of the Ganoids, equivalent to the Chondrostei. We do not recommend such an arrangement, which in view of the great preponderance of the Teleostei amongst living Fishes would be highly inconvenient, but the step from *Amia* to the Teleostei is certainly not so great as that from the Chondrostei to *Amia,* and is undoubtedly less than that from the Selachii to the Holocephali.

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* As instances of this we may cite (1) the spiral valve; (2) the frequent presence of a spiracle; (3) the frequent presence of a communication between the pericardium and the body-cavity; (4) the heterocercal tail.

† Vide F. M. BALFOUR, "On the Development of the Skeleton of the Paired Fins of Elasmobranchs," Proc. Zool. Soc., 1881.

LIST OF MEMOIRS ON THE ANATOMY AND DEVELOPMENT OF LEPIDOSTEUS.

1. AGASSIZ, A. "The Development of *Lepidosteus.*"—Part I. Proc. Amer. Acad. Arts and Sciences, vol. xiv., 1879.
2. AGASSIZ, L. 'Recherches s. l. Poissons Fossiles.' Neuchatel: 1833–45.
3. BOAS, J. E. "Über Herz u. Arterienbogen bei *Ceradotus* u. *Protopterus.*" Morphol. Jahrbuch, vol. vi., 1880.
4. DAVIDOFF, M. von. "Beiträge z. vergleich. Anat. d. hinteren Gliedmassen d. Fische." Morphol. Jahrbuch, vol. vi., 1880.
5. GEGENBAUR, C. 'Untersuch. z. vergleich. Anat. d. Wirbelthiere,' Heft II., Schultergürtel d. Wirbelthiere. Brustflosse der Fische. Leipzig: 1865.
6. GEGENBAUR, C. "Zur Entwick. d. Wirbelsaüle d. *Lepidosteus,* &c." Jenaische Zeitschrift, vol. iii., 1867.
7. HERTWIG, O. "Über d. Hautskelet d. Fische (*Lepidosteus* u. *Polypterus*)." Morphol. Jahrbuch, vol. v., 1879.
8. HŒVEN, VAN DER. "Über d. zellige Schwimmblase d. *Lepidosteus.*" MÜLLER's Archiv., 1841.
9. HYRTL, J. "Über d. Schwimmblase von *Lepidosteus osseus.*" Sitz. d. Wiener Akad., vol. viii., 1852.
10. HYRTL, J. "Über d. Pori abdominales, d. Kiemen-Arterien, u. d. Glandula thyroidea d. Ganoiden." Sitz. d. Wiener Akad., vol. viii., 1852.
11. HYRTL, J. 'Über d. Zussammenhang d. Geschlechts u. Harnwerkzeuge bei d. Ganoiden.' Wien: 1855.
12. KÖLLIKER, A. 'Über d. Ende d. Wirbelsäule b. Ganoiden.' Leipzig: 1860.
13. MÜLLER, J. "Über d. Bau u. d. Grenzen d. Ganoiden." Berlin Akad., 1844.
14. SCHNEIDER, H. "Über d. Augenmuskelnerven d. Ganoiden." Jenaische Zeitschrift, vol. xv., 1881.
15. WILDER, BURT G. "Notes on the North American Ganoids, *Amia, Lepidosteus, Acipenser,* and *Polyodon.*" Proc. Amer. Assoc. for the Advancement of Science, 1875.

List of Reference Letters.

a.	Anus.
a.b.	Air-bladder.
a.b′.	Aperture of air-bladder into throat.
a.c.	Anterior commissure.
a.f.	Anal fin.
al.	Alimentary canal.
ao.	Aorta.
ar.	Artery.
au.	Auditory pit.
b.	Brain.
b.c.	Body-cavity.
b.d.	Bile duct.
b.d′.	Aperture of bile duct into duodenum.
bl.	Coalesced portion of segmental ducts, forming urinogenital bladder.
br.a.	Branchial arches.
br.c.	Branchial clefts.
c.	Pyloric cæca.
c′.	Apertures of cæca into duodenum.
cb.	Cerebellum.
cd.v.	Cardinal vein.
ce.	Cerebrum : in figs. 47 A and B, anterior lobe of cerebrum.
ce′.	Posterior lobe of cerebrum.
c.f.	Caudal fin.
cn.	Centrum.
ch.	Choroidal fissure.
cr.v.	Circular vein of vascular membrane of eye.
c.sh.	Cuticular sheath of notochord.
c.v.	Caudal vein.
d.	Duodenum.
d.c.	Dorsal cartilage of neural arch.
d.f.	Dermal fin-rays.
d.l.	Dorsal lobe of caudal fin.
dl.f.	Dorsal fin.
e.	Eye.
ed.	Epidermis.
ep.	Epiblast.
f.b.	Fore-brain.
f.e.	Pyriform bodies surrounding the zona radiata of the ovum, probably the remains of epithelial cells.
g.b.	Gall-bladder.
g.d.	Genital duct.
gl.	Glomerulus.
g.r.	Genital ridge.
h.	Heart.
h.a.	Hæmal arch.
h.b.	Hind-brain.
h.c.	Head-cavity.
hp.d.	Hepatic duct.
h.m.	Hyomandibular cleft.
h.op.	Operculum.
hy.	Hypoblast ; in fig. 10, hyoid arch.
hyl.	Hyaloid membrane.
i.c.	Intercalated cartilaginous elements of the neural arches.
in.	Infundibulum.
ir.	Iris.
i.s.	Interspinous cartilage or bones.
i.v.	Sub-intestinal vein.
iv.r.	Intervertebral ring of cartilage.
k.	Kidney.
l.	Lens.
l.c.	Longitudinal canal, formed by union of the vasa efferentia.
l.in.	Lobi inferiores.
l.l.	Ligamentum longitudinale superius.

lr.	Liver.
lt.	Lateral line.
ly.	Lymphatic body in front of kidney.
m.	Mouth.
m.b.	Mid-brain.
m.c.	Medullary cord.
m.el.	Membrana elastica externa.
mes.	Mesorchium.
mn.	Mandible.
md. and *mo.*	Medulla oblongata.
ms.	Mesoblast.
n.a.	Neural arch.
n.a′.	Dorsal element of neural arch.
nc.	Notochord.
n.v.e.	Network formed by vasa efferentia on inner face of testis.
od.	Oviduct.
od′.	Aperture of oviduct into bladder.
ol.	Nasal pit or aperture.
olf.	Olfactory lobe.
op.	Optic vesicle.
op.ch.	Optic chiasma.
op.l.	Optic lobes.
op.th.	Optic thalami.
or.ep.	Oral epithelium.
ov.	Ovary.
p.	Pancreas.
pc.	Pericardium.
pc.f.	Pectoral fin.
p.ch.	Pigmented layer of choroid.
p.f.	Peritoneal funnel of segmental tube of mesonephros.
p.f.p.	Peritoneal funnel leading into pronephric chamber.
p.g.	Pectoral girdle.
pl.f.	Pelvic fin.
pn.	Pineal gland.
p.o.	Primitive germinal cells.
pr.	Mesoblastic somite.
pr.c.	Pronephric chamber.
pr.n.	Pronephros.
pr.n′.	Opening of pronephros into pronephric chamber.
pt.	Pituitary body.
py.	Pyloric valve.
p.z.	Parietal zone of blastoderm.
r.	Rostrum.
rb.	Rib.
rc.	Rectum.
s.	Spleen.
s.c.	Seminal vessels passing from the longitudinal canal into the kidney.
s.d.	Suctorial disc.
sg.	Segmental or archinephric duct.
sg.t.	Segmental tubules.
sh.	Granular outer portion of the sheath of the notochord in the vertebral regions.
s.mx.	Superior maxillary process.
s.nc.	Sub-notochordal rod.
so.	Somatic mesoblast.
sp.	Splanchnic mesoblast.
sp.n.	Spinal nerve.
sp.v.	Spiral valve.
st.	Stomach.
s.t.	Seminal tubes of the testis.
su.p.	Suctorial papillæ.
t.	Testis.
th.	Thalamencephalon.
th.l.	Lobes of the roof of the thalamencephalon.
tr.	Trabeculæ.
u.g.	Urinogenital aperture.
v.	Ventricle.
v.e.	Vasa efferentia.
v.h.	Vitreous humour.
v.l.	Ventral lobe of the caudal fin.
v.mt.	Ventral mesentery.
vn.	Vein.
vs.	Blood-vessel.

v.sh.	Vascular sheath between the hyaloid membrane and the vitreous humour.
v.th.	Vesicle of the thalamencephalon.
x.	Groove in epiblast, probably formed in process of hardening.
y.	Yolk.
z.	Commissure in front of pineal gland.
z.r.	Outer striated portion of investing membrane (zona radiata) of ovum.
z.r'.	Inner non-striated portion of investing membrane of ovum.
I.	Olfactory nerve.
II.	Optic nerve.
III.	Oculomotor nerve.
V.	Trigeminal nerve.
VIII.	Facial and auditory nerves.

Explanation of Figures.

PLATE 21.

- Figs. 1–4. Different stages in the segmentation of the ovum.
    - Fig. 1. Ovum with a single vertical furrow, from above.
    - Fig. 2. Ovum with two vertical furrows, from above.
    - Fig. 3. Side view of an ovum with a completely formed blastodermic disc.
    - Fig. 4. The same ovum as fig. 3, from below, showing four vertica furrows nearly meeting at the vegetative pole.
- Figs. 5–10. External views of embryos up to time of hatching.
    - Fig. 5. Embryo, 3·5 millims. long, third day after impregnation.
    - Fig. 6. Embryo on the fifth day after impregnation.
    - Fig. 7. Posterior part of same embryo as fig. 6, showing tail swelling.
    - Fig. 8. Embryo on the sixth day after impregnation.
    - Fig. 9. Embryo on the seventh day after impregnation.
    - Fig. 10. Embryo on the eleventh day after impregnation (shortly before hatching).
- Fig. 11. Head of embryo about the same age as fig. 10, ventral aspect.
- Fig. 12. Side view of a larva about 11 millims. in length, shortly after hatching.
- Fig. 13. Head of a larva about the same age as fig. 12, ventral aspect.
- Fig. 14. Side view of a larva about 15 millims. long, five days after hatching.
- Fig. 15. Head of a larva 23 millims. in length.
- Fig. 16. Tail of a larva 11 centims. in length.
- Fig. 17. Transverse section through the egg-membranes of a just-laid ovum.

We are indebted to Professor W. K. Parker for figs. 12, 14, and 15.

PLATE 22.

Figs. 18–22. Transverse sections of embryo on the third day after impregnation.

Fig. 18. Through head, showing the medullary keel.

Fig. 19. Through anterior part of trunk.

Fig. 20. Through same region as fig. 19, showing a groove (x.) in the epiblast, probably artificially formed in the process of hardening.

Fig. 21. Through anterior part of tail region, showing partial fusion of layers.

Fig. 22. Through posterior part of tail region, showing more complete fusion of layers than fig. 21.

Figs. 23–25. Transverse sections of an embryo on the fifth day after impregnation.

Fig. 23. Through fore-brain and optic vesicles.

Fig. 24. Through hind-brain and auditory pits.

Fig. 25. Through anterior part of trunk.

Figs. 26–27. Transverse sections of the head of an embryo on the sixth day after impregnation.

Fig. 26. Through fore-brain and optic vesicles.

Fig. 27. Through hind-brain and auditory pits.

PLATE 23.

Figs. 28–29. Transverse sections of the trunk of an embryo on the sixth day after impregnation.

Fig. 28. Through anterior part of trunk (from a slightly older embryo than the other sections of this stage).

Fig. 29. Slightly posterior to fig. 28, showing formation of segmental duct as a fold of the somatic mesoblast.

Fig. 30. Longitudinal horizontal section of embryo on the sixth day after impregnation, passing through the mesoblastic somites, notochord, and medullary canal.

Figs. 31–34. Transverse sections through an embryo on the seventh day after impregnation.

Fig. 31. Through anterior part of trunk.

Fig. 32. Through the trunk somewhat behind fig. 31.

Fig. 33. Through tail region.

Fig. 34. Further back than fig. 33, showing constriction of tail from the yolk.

Figs. 35–37. Transverse sections through an embryo on the eighth day after impregnation.

Fig. 35. Through fore-brain and optic vesicles.

Fig. 36. Through hind-brain, showing closed auditory pits, &c.

Fig. 37. Through anterior part of trunk.

Fig. 38. Section through tail of an embryo on the ninth day after impregnation.

PLATE 24.

Fig. 39. Section through the olfactory involution and part of fore-brain of a larva on the ninth day after impregnation, showing olfactory nerve.

Fig. 40. Section through the anterior part of the head of the same larva, showing pituitary involution.

Figs. 41–43. Transverse sections through an embryo on the eleventh day after impregnation.

Fig. 41. Through fore-part of head, showing the pituitary body still connected with the oral epithelium.

Fig. 42. Slightly further back than fig. 41, showing the pituitary body constricted off from the oral epithelium.

Fig. 43. Slightly posterior to fig. 42, to show olfactory involution, eye, and hyomandibular cleft.

Fig. 44. Longitudinal section of the head of an embryo of 15 millims. in length, a few days after hatching, showing the structure of the brain.

Fig. 45. Longitudinal section of the head of an embryo, about five weeks after hatching, 26 millims. in length, showing the structure of the brain. In the front part of the brain the section passes slightly to one side of the median line.

Figs. 46 A to 46 G. Transverse sections through the brain of an embryo 25 millims. in length, about a month after hatching.

Fig. 46 A. Through anterior lobes of cerebrum.

Fig. 46 B. Through posterior lobes of cerebrum.

Fig. 46 C. Through thalamencephalon.

Fig. 46 D. Through optic thalami and optic chiasma.

Fig. 46 E. Through optic lobes and infundibulum.

Fig. 46 F. Through optic lobes and cerebellum.

Fig. 46 G. Through optic lobes and cerebellum, slightly behind fig. 46 F.

PLATE 25.

Figs. 47 A, B, C. Figures of adult brain.

Fig. 47 A. From the side.

Fig. 47 B. From above.

Fig. 47 C. From below.

Fig. 48. Longitudinal vertical section through the eye of an embryo, about a week after hatching, showing the vascular membrane surrounding the vitreous humour.

Fig. 49. Diagram showing the arrangement of the vessels in the vascular membrane of the vitreous humour of adult eye.

Fig. 50. Capillaries of the same vascular membrane.

Fig. 51. Transverse section through anterior part of trunk of an embryo on the ninth day after impregnation, showing the pronephros and pronephric chamber.

Fig. 52. Transverse section through the region of the stomach of an embryo 15 millims. in length, shortly after hatching, to show the glomerulus and peritoneal funnel of pronephros.

Fig. 53. Transverse section through posterior part of the body of an embryo, about a month after hatching, showing the structure of the mesonephros, the spiral valve, &c.

PLATE 26.

Figs. 54, 55, 56, and 57 are a series of transverse sections through the genital ridge and mesonephros of one side from a larva of 11 centims.

Fig. 54. Section of the lymphatic organ which lies in front of the mesonephros.

Fig. 55. Section near the anterior end of the mesonephros, where the genital sac is completely formed.

Fig. 56. Section somewhat further back, showing the mode of formation of the genital sac.

Fig. 57. Section posterior to the above, the formation of the genital sac not having commenced, and the genital ridge with primitive germinal cells projecting freely into the body-cavity.

Fig. 58 A. View of the testis, mesorchium, and duct of the kidney of the left side of an adult male example of *Lepidosteus*, 60 centims. in length, showing the vasa efferentia and the longitudinal canal at the base of the mesorchium. The kidney ducts have been cut open posteriorly to show the structure of the interior.

Fig. 58 B. Inner aspect of the posterior lobe of the testis from the same example, to show the vasa efferentia forming a network on the face of the testis.

Figs. 59 A and B. Two sections showing the structure and relations of the efferent ducts of the testis in the same example.

Fig. 59 A. Section through the inner aspect of a portion of the testis and mesorchium, to show the network of the vasa efferentia (*n.v.e.*) becoming continuous with the seminal tubes (*s.t.*). The granular matter nearly filling the vasa efferentia and the seminal tubes represents the spermatozoa.

Fig. 59 B. Section through part of the kidney and its duct and the longitudinal canal (*l.c.*) at the base of the mesorchium. Canals (*s.c.*) are seen passing off from the latter, which enter the kidney and join the uriniferous tubuli. Some of the latter (as well as the seminal tubes) are seen to be filled with granular matter, which we believe to be the remains of spermatozoa.

Fig. 60. Diagram of the urinogenital organs of the left side of an adult female example of *Lepidosteus* 100 centims. in length. This figure shows the oviduct (*od.*) continuous with the investment of the ovary, opening at *od'.* into the dilated part of the kidney duct (segmental duct). It also shows the segmental duct and the junction of the latter with its fellow of the right side to form the so-called bladder, this part being represented as cut open. The kidney (*k.*) and lymphatic organ (*ly.*) in front of it are also shown.

PLATE 27.

Fig. 61. Transverse section through the developing pancreas (*p.*) of a larva 11 millims. in length.

Fig. 62. Longitudinal section through portions of the stomach, liver, and duodenum of an embryo about a month after hatching, to show the relations of the pancreas (*p.*) to the surrounding parts.

Fig. 63. External view of portions of the liver, stomach, duodenum, &c., of a young Fish, 11 centims. in length, to show the pancreas (*p.*).

Fig. 64. Transverse section through the anterior part of the trunk of an embryo, about a month after hatching, showing the connexion of the air-bladder with the throat (*a.b'.*).

Fig. 65. Transverse section through the same embryo as fig. 64 further back, showing the posterior part of the air-bladder (*a.b.*).

Fig. 66. Viscera of an adult female, 100 centims. in length, showing the alimentary canal with its appended glands in natural position, and the air-bladder with its aperture into the throat (*a.b'.*). The proximal part of the duodenum and the terminal part of the intestine are represented as cut open, the former to show the pyloric valve and the apertures of the pyloric cæca and bile duct, and the latter to show the spiral valve.

This figure was drawn for us by Professor A. C. HADDON.

PLATE 28.

Fig. 67. Transverse section through the tail of an advanced larva, showing the neural and hæmal processes, the independently developed interneural and interhæmal elements (*i.s.*), and the commencing dermal fin-rays (*d.f.*).

Fig. 68. Side view of the tail of a larva, 21 millims. in length, dissected so as to show the structure of the skeleton.

Fig. 69. Longitudinal horizontal section through the vertebral column of a larva, 5·5 centims. in length, on the level of the hæmal arches, showing the intervertebral rings of cartilage continuous with the arches, the vertebral constriction of the notochord, &c.

Figs. 70 and 71. Transverse sections through the vertebral column of a larva of 5·5 centims. The red represents bone, and the blue cartilage.

Fig. 70. Through the vertebral region, showing the neural and hæmal arches, the notochordal sheath, &c.

Fig. 71. Through the intervertebral region, showing the intervertebral cartilage.

Figs. 72 and 73. Transverse sections through the trunk of a larva of 5·5 centims. to show the structure of the ribs and hæmal arches.

Fig. 72. Through the anterior part of the trunk.

Fig. 73. Through the posterior part of the trunk.

PLATE 29.

Figs. 74–76. Transverse sections through the trunk of the same larva as figs. 72 and 73.

Fig. 74. Through the posterior part of the trunk (rather further back than fig. 73).

Fig. 75. Through the anterior part of the tail.

Fig. 76. Rather further back than fig. 75.

Fig. 77. Longitudinal horizontal section through the vertebral column of a larva of 11 centims., passing through the level of the hæmal arches, and showing the intervertebral constriction of the notochord, the ossification of the cartilage, &c.

Fig. 78. Transverse section through a vertebral region of the vertebral column of a larva 11 centims. in length.

Fig. 79. Transverse section through an intervertebral region of the same larva as fig. 78.

Fig. 80. Side view of two trunk vertebræ of an adult *Lepidosteus.*

Fig. 81. Front view of a trunk vertebra of adult.

In figures 80 and 81 the red does not represent bone as in the other figures, but simply the ligamentum longitudinale superius.

VIII. *On the Development of the Skull in* Lepidosteus osseus.

By William Kitchen Parker, *F.R.S.*

Received November 3,—Read December 8, 1881.

[Plates 30–38.]

In the early part of 1879 I received from Professor A. Agassiz a large series of eggs and embryos of *Lepidosteus,* which by the help of Mr. Garman he had obtained from the Black Lake the year before. These specimens, carefully preserved in fifty-four small bottles, Professor Agassiz put into my hands to be worked out by Mr. Balfour and myself. A few years before I had received from Professor Burt G. Wilder some larger young of this Fish, which extended the "stages" for us considerably; and several adults obtained by Mr. Balfour, and one which I received from Professor Flower, made up the total of our materials. The present communication is the result of my own researches into the growth of the skull and visceral arches. Mr. Balfour, assisted by my son, Mr. W. N. Parker, has prepared an elaborate memoir on the embryology of this important type, to which are added observations on the structure of various organs in the adult.

When our work was scarcely begun I prevailed upon Dr. Traquair to work out the *adult skull,* and that piece of research is, I believe, nearly finished, so that this *Holostean Ganoid* will soon have had a fair amount of attention given to it.

My observations extend over a series of embryos and young from one-third of an inch to four and a-half inches in length. The larger young are already quite like the adult; but as Dr. Traquair's paper has not yet been sent in to the Royal Society, I shall preface mine by a short description of the adult skull: it will give the same interpretation as the promised memoir by my talented friend, for we have had several discussions upon the nomenclature and meaning of the various parts, and are of one mind as to their names and nature.

BIBLIOGRAPHY.

Agassiz, Professor Alexander. "On the Development of *Lepidosteus.*" Proceedings of the American Academy of Arts and Sciences (Presented Oct. 8, 1878); pp. 65–76; plates 1–5.

Balfour, F. M.

1. 'Elasmobranch Fishes.' London: 1878.
2. 'Comparative Embryology,' vol. ii. London: 1881.

BRIDGE, Professor T. W.

1. "On the Osteology of *Amia calva*." Jour. of Anat. and Phys., vol. xi., pp. 605–622; plate 23.
2. "On the Osteology of *Polyodon folium*." Phil. Trans., 1878, Part II., pp. 683–733; Plates 55–57.

GEGENBAUR, Dr. CARL.

1. 'Untersuchgen zur vergleichenden Anatomie der Wirbelthiere.' (Drittes Heft: Das Kopfskelet der Selachier.) Leipzig: 1872.
2. 'Elements of Comparative Anatomy.' Translated by JEFFREY BELL. London: 1878.

GÜNTHER, Dr. ALBERT.

1. "On *Ceratodus*." Phil. Trans., 1872, pp. 511–571; Plates 30–42.
2. 'Study of Fishes.' Edinburgh: 1880.

HUBRECHT, Dr. A. A. W. "Beitrag zur Kentniss des Kopfskeletes der Holocephalen." Niederlandisches Archiv für Zoologie, iii., taf. xvii., pp. 1–22.

HUXLEY, Professor T. H.

1. 'Memoirs of the Geological Survey.' London: 1861.
2. 'Elements of Comparative Anatomy.' London: 1864.
3. 'Anatomy of the Vertebrated Animals.' London: 1871.
4. "On *Menobranchus lateralis*." Proc. Zool. Soc., March 17, 1874, pp. 186–204; plates 29–32.
5. "On *Ceratodus Forsteri*." Proc. Zool. Soc., Jan. 4, 1876, pp. 24–59.

MÜLLER, Dr. JOH. Vergleichende Anatomie der Myxinoiden, der Cyclostomen, mit durchbohrtem Gaumen." Berlin: 1835.

PARKER, W. K.

1. "On the Skull of the Common Frog." Phil. Trans., 1871, pp. 137–211; Plates 3–10.
2. "On the Skull of the Salmon." Phil. Trans., 1873, pp. 95–145; Plates 1–8.
3. "On the Skull of the Batrachia." Phil. Trans., 1876, Part II., pp. 601–669; Plates 54–62.
4. "On the Skull of the Urodeles." Phil. Trans., 1877, pp. 529–597; Plates 21–29.
5. "On the Skull of Sharks and Skates." Trans. Zool. Soc., vol. xii., part iv., 1878, pp. 189–234; plates 34–42.
6. "On the Skull of the Urodeles." Trans. Linn. Soc., 2nd series, Zool., vol. ii., 1881, pp. 165–212, plates 14–21.
7. "On the Skull of the Batrachia." Part III. Phil. Trans., 1881, Part I., pp. 1–266; plates 1–44.
8. "On the Skull of the Urodeles." Trans. Zool. Soc., vol. xi., part vi., pp. 171–214; plates 36–41.

TRAQUAIR, Dr. R. H.

1. "On the Cranial Osteology of *Polypterus*." Jour. of Anat. and Phys., vol. v., pp. 166–182; plate 6.
2. "On the Ganoid Fishes of the British Carboniferous Formation." Trans. Palæont. Soc., 1877 (Part I.: Palæoniscidæ), pp. 1–60; plates 1–7.
3. "On the Structure and Affinities of the Platysomidæ." Trans. Roy. Soc. Edinb., vol. xxix., pp. 343–391; plates 3–6.

WIEDERSHEIM, Dr. ROBERT. 'Das Kopfskelet der Urodelen.' Leipzig: 1877.

The skull of the adult Lepidosteus.

This skull, which belongs to the long-beaked variety, is more than a foot long, and the foremost two-thirds of its length belongs to the "rostrum," which is gradually attenuated from behind forwards, and then dilates gently at the fore end. The olfactory sacs are very small, and instead of being placed close in front of the antorbital region, are carried to the end of the snout, and have on each side a pair of holes bounded by small bony plates. The orbits are a little above half an inch across, and they are bounded by a perfect "circumorbital" series. The auditory capsules are impacted into the walls of the hind skull, which at the sides is only about one-twentieth the length of the head. The occipital condyle is behind the skull; the condyles of the quadrate are in front of the orbits. There is no fontanelle in the perfect skull—that space is entirely covered by large ganoid scutes.

A. *On the superficial plates or "scutes."*

Beginning at the occipital roof, we find the hind skull covered with two large plates that represent the parietals; here, at once, we see how variable these scutes are, which answer to *much more* than the investing bones in the higher types, for there is, on the left side (in my specimen), an irregularly four-sided "dermo-occipital," the inner edge of which passes over the mid-line a little. Thus the right parietal is much larger than the left. Outside each parietal there is a somewhat smaller scute—the squamosal; this bone finishes the roof. The two or three small, irregular scutes behind the squamosal are "post-temporals," and serve as fixing-points to the *clavicular series* of the shoulder-girdle; they are post-cranial.

Outside the hinder part of the squamosal there are two smallish additional "temporal" scutes, wedged in above the "opercular."

In front of the parietals and squamosals we see the frontals; they interdigitate by large, sharp, sutural teeth, with those bones, and then run on over the orbital region, and over the hinder two-fifths of the rostrum. They are elegantly narrow-waisted (taken together) in the orbital region, and they contract into sharp styles in front, where they embrace the next pair.

These next bones are the "ethmo-nasals," or superficial splints of the ethmoidal region; they are close together along the mid line, sharp and narrow between the fore part of the frontals, and then wider; they reach, behind, within an inch and a-half of the orbits, and then run forward up to the nasals (proper). These latter are small, short, widely-crescentic plates, that lie on the small, distal, olfactory sacs; they are the *ossa terminalia* of the upper *mucous* series. The nasals finish the skull-beak above; below, an oblique transverse plate of bone supports the prenasal pad or remnant of the larval sucking-disk; it may be called the "prenasal bone." The nasal bone sends a process obliquely across the nasal opening; this bone lies in the membranous bar, which was developed early, dividing the "nostril" into two holes. Behind and under the prenasal and nasal bones we have the two short thick premaxillaries, which carry some very large, sharp teeth, as well as a rasp of smaller denticles. These bones have a palatal as well as a dentary region; and between the two the bone is punched with holes by the foremost teeth of the mandible, as in the Crocodile; the palatine processes diverge a little, behind, to embrace the vomers.

The vomers are very long, thin styles of bone, closely co-adapted along the mid line of the palatal region of the rostrum up to its hinder fourth; they have their lower face covered with a rasp of very small teeth.

Outside these there is on each side a larger splint, which reaches from the premaxillary to the antorbital space. The outside of each bone is bevelled and helps to carry the large sub-marginal teeth; these are the *superficial* palatines. Outside these, on the margin, we have the "maxillary chain" of mucous bones, about fifteen on each side, but some of them are ankylosed together; they increase in length from before backwards. The preorbital (prequadrate) space is overlapped by the last two of this chain; these are free; the larger front bone answers to the free part of the "os mystaceum" of an Acanthopterous Teleostean; the lesser scale to its jugal. A large splint passes along by its sharp fore end, on the inside of the parosteal palatine, nearly to its middle; it broadens rapidly at the prequadrate space and turns downwards, as a rounded wing. It is notched below close in front of the quadrate condyle, and then passes backwards as a large lanceolate splint, which is closely applied to nearly all the inner face of the mandibular suspensorium; this is the pterygoid bone, the osseous counterpart of the whole palato-quadrate arcade. Above the broad hind part of that arcade, close in front of the basi-pterygoid articulation, there is a much smaller lanceolate splint, the meso-pterygoid; it lies inside, but rises somewhat above the obliquely directed suspensorium. An oblong splint, rather larger than the last, lies on the outside of the lower face of the suspensorium, and is dilated where it fits to the enlargement at the quadrate condyle; this is the "preopercular": a very different bone from its counterpart in the Teleostei, and like the lower part of a Frog's squamosal. These four—the superficial palatine, the pterygoid, the mesopterygoid, and the preopercular—are the splints of the palato-quadrate arcade; the rest of the mandibular splints are on the free ramus, and will be described soon. There is a perfect "circumorbital" series of small, thick ganoid scutes, which run before and behind into other tracts, and are not separated as a mere ring. The upper region is composed of a chain of larger bones than the rest, and this tract is continued forwards as three bones, narrowing forwards, in the *preorbital* region, over the "prequadrate space." Behind and above, these larger bones run backwards under the squamosal until they become the "post-temporals" to which the clavicular bones are attached. They are the direct cephalic continuation of what in the Teleostei are known as the "lateral line" series. A row of small scutes runs straight down in front of the orbit from the preorbital band; the lowest of these binds on the quadrate, over the hinge, and over the foot of the preopercular. From this angular bone there is an increasing number of scutes under the orbit, and those behind the orbit become a solid tesselated pavement (part of which I have just described as lying directly beneath the squamosal), and this pavement lies on the edge of the lower part of the interopercular and covers all its ascending part.

This most remarkable "interopercular" is much like the preopercular of the Teleosteans, and might easily be mistaken for it. It is a huge plate bent upon itself at a right angle; the ascending part is half the size of the lower region, and whilst covered by the facial "pavement," itself covers the hyomandibular and symplectic; that part is pointed above. The lower region of the bone is a large externally ganoid tract, pointed in front, ear-shaped behind, hollow within, and coiled inwards. Behind the facial pavement, and articulated by a cup-like facet to the "opercular process" of the hyomandibular, is the opercular bone; it is four-sided, but narrow above where it articulates with the post-temporal. Its broad lower edge is toothed and overlaps the subopercular; its hind face is free and forms the upper half of the free edge of the *operculum*. The "subopercular" forms the lower half of that edge; it is a broad plate with a rounded lower and hinder margin, and is uncinate in front and above, where it is wedged in between the interopercular and opercular. These three bones—the interopercular, the opercular, and the subopercular—belong to the hyomandibular region. The splints of the mandible are ganoid where they are exposed. The main bone is the "dentary"; it covers nearly all the outside of the ramus and the upper and lower edges within. On the inside, between those edges, MECKEL's cartilage is hidden in its foremost two-thirds by a long, thin, narrow splint, the "splenial." The rami of the mandible are close together, half-way, backwards, and then gently diverge. The coronoid region is very high, large, and incurved. The "coronoid" bone flanks the front of this part as an oblique splint, and the "supra-angulare" covers the smooth, convex outside; a short, thick, wedge of bone, the "angulare," is set on to the angle of the ramus. These five bones—the dentary, splenial, coronoid, supra-angulare, and angulare—are the normal investing bones of the mandible. In the lower part of the hyoid arch there are three narrow flattened rays, from an inch to an inch and a-half in length, attached to the outer face of the "epi-hyal"; these are the "branchiostegals," and they correspond in number to what we find in the Cyprinoid Teleosteans.

Under the skull there is a long beam of bone, reaching to the middle of the vomers, lying there somewhat above them, and then escaping, showing a free keel; this is the "parasphenoid." It runs backwards, gently broadening, and then narrowing again, and gradually losing its keel, up to the "basipterygoid processes," it flanks them with a small pair of wings, and is then alate again, more extensively, under the "sacculus" of the ear, and ends as two flat processes under the basioccipital, and nearly reaching to its end.

B. *On the endo-cranial bones.*

In the hind skull I fail to find a distinct supraoccipital under the roof, for the exoccipitals meet there, and are separated only by a narrow tract of cartilage. The basioccipital is large, and projects beyond the arch; its hollow articular face is transverse, and notched above and below. The exoccipitals are large, and finish the arch above. The auditory capsule is invested by *four* cartilage bones on each side, viz.: a large prootic, a middle sized epiotic, and a small opisthotic, and the sphenotic bone which is of the average size, and ossifies the post-orbital angle of the endocranium, where the fore part of the auditory capsules are imbedded. I can find no "pterotic" under the squamosal. There are two alisphenoids, and they ossify the basipterygoid processes, but I find no distinct basisphenoid, presphenoid, nor orbito-sphenoids; although after the brain cavity has retreated, losing much of its front part, a bony plate is found in the presphenoidal region. This plate is thin, fenestrate, and rests upon the parasphenoid. I believe that it is formed by the coalescence of the once distinct lateral ethmoids, which were at first formed in the sides of the fore part of the chondrocranium, like the primary elements of the Frog's "girdle-bone" (see Plate 38, figs. 2, 3, *l.eth.*). The rostral part of the endocranium is, I believe, unossified, even in the adult.

The quadrate and metapterygoid bones occupy only part of the mandibular suspensorium; the rest remains unossified; the metapterygoid bone takes up the part which answers to the pedicle and rudimentary otic process; a facet of cartilage covers it where it articulates with the "basipterygoid process." The hyomandibular is fenestrate and well ossified; it is only half as large as the mandibular suspensorium. A short tract of cartilage separates its upper from the slender long symplectic.

The articulare is a strong solid mass of bone, capped by a saddle-shaped condyle for articulation with the quadrate.

The small inter-hyal piece is not ossified, but the epi-, cerato-, and hypo-hyals are; the long *double* basi-hyal ("ento-glossal") does not become solid bone, but it is *endosteally* hardened. The fore part of the basi-branchial series is slightly ossified, the arches are as well ossified as in the ordinary Teleostei; thus there are in the first four arches a pharyngo, epi-, cerato-, and hypo-branchial piece, largely converted into bone; the fifth is ossified, but it only has the cerato-branchial element in it.

First Stage.—Recently hatched young of Lepidosteus, 2¾ *to* 4½ *lines long.*

At this stage (Plate 30, figs. 1, 2) the mesocephalic flexure is at its fullest development, but the angle formed by the meeting of the *para*-chordal and *pro*-chordal regions is very open. Moreover, the bend appears less than it is, for the snout is very large and dilated, and its axis is coincident with that of the cerebral hemispheres (C^{1a}.) and of the mid brain (C^2.).

The fissure, however, between the fore and hind brain (C^3.), and below the mid brain, is very large, and is turned backwards considerably, above; the notochord (*nc.*) seeks to ascend into this space, but only rises one-third of the way to the top; the third nerve descends in this cavity to reach the orbital muscles.

In the sectional figures given to illustrate this stage, very little of the primordial cranium comes into view; it is not yet formed into consistent cartilage, but the "embryonic cartilage" is sufficiently developed to show the cranial and facial rudiments.

The pituitary body (*py.*) is distinguishable from the palatal skin, and it is quite free above, the infundibulum (*inf.*) lying over it.

In both figures (Plate 30, figs. 1 and 2, cl^{1-4}.) the clefts and visceral folds are visible on their inner or *hypoblastic* aspect; in the more advanced specimen a rod of solidifying tissue was seen as cut through in six of these, namely, the mandible, hyoid, and four of the branchial arches (*mn.*, *hy.*, br^{1-4}.)—all but the *fifth*, which remains rudimentary.

The heart (*h.*) is seen impacted between the mandibular rudiment (*mn.*) and the yolk (*y.*); it thus lies as far forwards as the snout.

The notochord (*nc.*) is very large, and in the more advanced specimen (Plate 30, fig. 2) the terminal hook of this cephalic end of the rod is fixed in between the infundibulum (*inf.*) and the swelling antero-inferior part of the hind brain (C^3.).

Second Stage.—Recently hatched young of Lepidosteus, 5 *to* 5½ *lines long.*

In somewhat larger specimens (about 11 millims. long) I was able to make preparations of the whole chondrocranium (Plate 30, fig. 3): a sectional view is given of a somewhat larger specimen (Plate 30, fig. 4).

The section shows that the skull is rapidly straightening, but of course the great *clinoid fissure* is never obliterated. This is formed by the primordial flexure, and is always permanent in the Vertebrata; but the flexure itself is not so great in this type as in the *Amphibia* and the "Amniota."

The notochord (Plate 30, fig. 4, *nc.*) is not traceable so far up in the clinoid region as it was; the pituitary body (*py.*) is still independent of the infundibulum (*inf.*).

In the dissected skull (Plate 30, fig. 3) the basal and facial parts alone are chondrified, the sides and roof are entirely membranous; the cartilage is still very fragile, but quite differentiated from the surrounding connective-fibre cells. The chondrocranium proper is at present composed of two bands of cartilage, which run from the atlas to the large suctorial snout (*s.d.*); the lateral structures are the auditory capsules and the visceral arches.

The basal pair of bands (Plate 30, fig. 3, *iv.*, *tr.*) are thickest behind and at the fore end; for two-fifths of their length they embrace the notochord by their inner edge; they then diverge from each other so as to form a large spindle-shaped space (*py.*) in front of which, for an eighth of their whole length, they approximate, but do not come into contact with each other.

These dilated fore ends of the basal bands are as broad as the parachordal part behind (*iv.*), but are not so thick; they are conjugated by a tract of embryonic cartilage, the rudiment of the "intertrabecula" (*i.tr.*); they themselves are the rudiments of the cornua trabeculæ (*c.tr.*). The bowed interorbital part of the basal bands are the trabeculæ (*tr.*), they are only half the width of the hind and fore parts; these out-bent bars do not merely fence in the small pituitary body, they embrace the base of the fore-brain.

The free, ascending, blunt end of the cranial notochord (*nc.*) is twisted a little to the left—it may curve either way, but the appearance is partly due to artificial pressure; it is not invested by a large ascending "posterior clinoid" cartilage, as in the *Amniota*, for that tract is very small and very late in appearance in *Lepidosteus*. The *pro*-chordal part of the basal bands is bent somewhat upon the *para*-chordal part. Behind the exit of the 9th and 10th nerves and the hind face of the auditory capsules the investing bars (*iv.*) thin out, and end in a sharp edge in front of the 1st vertebra.

The auditory capsules (*au.*) are of a short oval shape; they are largely chondrified, but the layer of cartilage is very thin; below, there is an oval fenestra, still membranous (*au.f.*); it is about two-fifths the length of the capsule. The capsules and the investing bars are only connected together by embryonic cartilage at present; the convexity of the capsules has produced a corresponding concavity in the side of each bar. The horizontal canal bulges over the lower convex part, which contains the otolith; thus there is a rudimentary "tegmen tympani." Rudiments of all the framework of the visceral arches are now present, but the segmentation of these parts is imperfect. The skeleton of the first and second arches—mandibles and hyoid—is massive; that of the branchial arches is very delicate. The pier of the mandibular arch (*p.pg.*, *q.*, *pd.*) is not a mere "pterygo-quadrate," as in the Selachians, Teleostei, and Urodeles, but is a "palato-quadrate," as in the Anura. This arises from the primary continuity of the ethmo-palatine cartilage with the pterygoid fore-growth of the mandibular suspensorium; and this is not all, for the *palatine region* of the bar is also *primarily continuous* with the trabecula at its dilated fore end.

The "pedicle" of this compound suspensorium is not *fixed*, as in the Tadpole, but *free*, as in the metamorphosed Frog; as soon as it is sufficiently developed to articulate with the basal bar, it will then, at that part, correspond very accurately with what we find in the adult Frog. At present this palato-quadrate, or suspensorial cartilage is roughly scythe-shaped, the "blade" being behind, and lying between the auditory capsule and the narrowing basal band, whilst the "handle" is in front, and is glued to the trabecula. The blade is shorter than the handle, and on its heel it has an oval, condyloid facet, which looks forwards and a little outwards and downwards; it is pinched a little behind this facet, and then becomes an apiculated lanceolate blade. The handle is oval in section, narrow at first, and then wider as it turns inwards at its junction with the trabecula. A free bar, rather shorter than the "handle," and having its thick end reversed, or behind, articulates by a flat facet with the flat facet of the suspensorium. This is MECKEL's cartilage, or the free mandible (*mk.*).

Already, in the coronoid region, this short lower jaw is elevated into a crest; its lower edge is convex, its upper concave, and its end is blunt; it reaches nearly as far forwards as the palatine region of the suspensorium. Between the convex hind margin of the first arch (suspensorium and mandible) and the concave fore margin of the next (the hyoid) arch there is a *hypoblastic pouch* (see Plate 30, figs. 1 and 2, cl^1.), but this first cleft does not open on the outside, like the hyo-branchial and four branchials.

The hyoid arch consists already of *four* segments on each side, with the rudiment of a median conjugating bar, composed of embryonic cartilage. The pier (*hm.*) of the hyoid arch, like its counterpart in the Mammal, is anvil-shaped; this is the hyomandibular, with its symplectic process, the counterpart of the pterygoid fore-growth of the mandibular pier or suspensorium.*

At present it is only a fourth less than the pier in front of it; but it becomes relatively much less, instead of much larger, as in the Teleostei. The hyomandibular is now a solid mass of cartilage, concave above, bilobate below, and sending upwards, to the auditory capsule, a thick process, and downwards, to the articular process of the suspensorium, a sharp process. The upper process is its proximal part or head, the lower is its symplectic prolongation.

The middle part has two lobes, below, and behind; the upper of these afterwards gives off the opercular process, the lower is already scooped on its inside where it articulates with the next joint, the inter-hyal (*i.hy.*). This latter piece is a small pyriform segment, with its lesser end upwards and hooked inwards, for articulation with the hyomandibular. This is, so to speak, a supernumerary segment placed between the *antero-superior* pier and the *postero-inferior* arch. That arch is already composed of a main bar above, the cerato-hyal (*c.hy.*),—a thick rounded rod, and a short, almost globular segment below, the hypo-hyal (*h.hy.*); the two fit together by flat facets; the right and left hypo-hyals touch each other below. In front of and between these lower segments there is a tract of tissue which will harden into the basi-hyal (*b.hy.*).

The branchial arches (*br.*) are slender rods, bent in a sigmoid manner so as to form round in-turned hooks, above. The last (br^5.) is only one-fourth as large as the other four, and is permanently arrested as the "inferior pharyngeal;" it is simply a short cerato-branchial.

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* Mr. BALFOUR informs me that this bar is primarily continuous with the skull, and that after chondrification it is still unsegmented.

The four well-developed rods are not yet segmented across into the normal pieces; they are oval in section, pointed above, and rounded below, where they articulate with the common basi-branchial (*b.br.*), which is one-third thicker than the arches. This conjugating rod is somewhat flat; it is thickest in front, where it fits by a rounded end between the hypo-hyals, and flat behind, where it projects beyond the arches.

Third Stage.—Young Lepidostei, *7¼ to 8 lines long.*

In young individuals about two-thirds of an inch (16 to 17 millims.) long, the cartilage has become quite consistent, and some parts not chondrified in the last stage have become solid.

The head (Plate 30, figs. 5, 6) has straightened out considerably, and the basis cranii bulges below; but the sharp, recurved fold under the mid brain, where the fore and hind brain approximate, although a mere chink, is ineradicable.

The round and curiously mammillated snout is much smaller now, relatively; the notochord (*nc.*) has retreated considerably; the pituitary body (*py.*) is still separate from the infundibulum (*inf.*).

The azygous trabecular bar (figs. 7, 8, *i.tr.*) now shows itself in sections taken vertically through the head, but all the rest of the basal cartilage is far from the mid line. The notochord (*nc.*) is very large, and descends considerably beneath the swelling hind brain (figs. 5, 6, C^3.). Below the mouth and throat, the fore end of the mandible is seen in the *right* section (Plate 30, fig. 5, *mk.*); it contained more than half of the head above, and less than half below; thus that rod, and the rest of the post-orals (*hy.*, br^{1-4}.), are cut through a little on the right side of the middle. But in the *left* half (Plate 30, fig. 6), the median rods (*b.hy.*, br^{1-4}.) are brought into view, below; above, the junction of the trabecula with the palato-quadrate (*tr.*, *p.pg.*) is seen; this, however, had to be exposed by dissection; the sub-ocular space is seen as a crescentic gap with its convexity downwards.

The chondrocranium is figured as seen from above and below in a dissection of a somewhat larger specimen (8 lines long); it is much more perfect than in the last stage (Plate 30, figs. 3, 7, 8).

A full third of the cranial floor is membranous; the side walls are membranous in front, and are made by the auditory capsules behind; but a rudiment of the roof or "tegmen cranii" is now found, right and left.

The thick cranial notochord (*nc.*) is only half the length of the chondrocranium, now; it is somewhat moniliform, lessening by three successive stages, and is bent a little in this specimen to the right.

The narrow part is rounded at its end, and ascends but little into the clinoid fissure (Plate 30, fig. 5, *nc.*); a third part at least is not invested by the parachordal cartilage (*iv.*).

The huge so-called pituitary space (*py.*) is pyriform, has an indented outline behind, and is apiculate in front. The concave inner side of the basal bands embrace the notochord closely, and end behind in bevelled flaps; in front, where the last narrowing of the notochord takes place, they end almost transversely, but give off from their outer edge the trabecular outgrowths. They are quite confluent, externally, with the well-chondrified auditory capsules (*au.*). In their broad, proximal part the trabeculæ (*tr.*) are dilated, and are perforated by the internal carotid artery;* thence they converge steadily as scarcely arcuate bands, only one-third as large as their parachordal roots (*iv.*).

Their front confluent part is as long as their hinder separate part; and there the bands double their width, have a convex outline externally, and narrowing inwards at their extremity, are continuous then with the palato-quadrate bars (*p.pg.*). The front margin of these four bands is crenate, with three convexities, the middle enlargement being the largest; this is not, however, formed by the trabeculæ themselves, but by a pyriform wedge of newer cartilage, which has developed between and above them. This new element is the intertrabecula (*i.tr.*); in the last stage (Plate 30, fig. 3, *i.tr.*) it was merely composed of embryonic cartilage—a small, inconspicuous tract of tissue, lying between the dilated ends of the trabeculæ (*tr.*).

This new tract does not reach to the pituitary space, but already, in the upper view, can be seen projecting beyond the confluent paired bands (*tr.*, *p.pg.*). The simple, ovoidal form of the auditory capsules (*au.*) is now lost, for the semi-circular canals (*a.s.c.*, *h.s.c.*, *p.s.c.*) have developed greatly, and they have given the form of their curves and swellings to the cartilaginous capsule.

The floor of the sacculus is still largely membranous (fig. 7, *au.f.*); this circular fenestra is inside the centre of the floor; within, also, the capsule is not cartilaginous. Outside, the bulging of the horizontal canal (*h.s.c.*) has formed a rudiment of the "tegmen tympani" (fig. 7); this, however, is simply used to form the large oblong concavity for the hyomandibular (*hm.*), and never acquires any tympanic function.

The large cranial nerves have free course over the basal plate, and in front of and behind the auditory capsules they have only membrane to pass through; the space for these (Plate 30, figs. 7, 8, IX., X.) is seen to be very wide.

From the front of the auditory capsule cartilage is creeping along the superorbital region (fig. 8, *s.ob.*); this new tract is styliform, and at present only reaches one-third of the distance to the end of the chondrocranium. Below this rudiment, right and left, of the "tegmen cranii" the proximal part of each trabecula has developed an oblong facet of cartilage for articulation with the *pedicle* of the suspensorium (*pd.*); this is the first appearance of the paired "basipterygoid" processes of the basicranii. I take this to be not only the first appearance of these important processes in the development of this particular skull, but also in the order of the types; in the Amphibia, Reptiles, Birds, and Mammals these processes are seldom absent.

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* This passage is seen inside the superorbital cartilage (*s.ob.*); the line leading from the letters to this part in fig. 8 is not long enough.

The dorsal end of the suspensorium or mandibular pier has already become oblique; the true extremity is now lateral, and is the pedicle of the pier; this is an oblong tract or facet articulating with the basi-pterygoid face on the basal band.

The second part is the otic process (behind *pd.*); it is a subtriangular "ear" of cartilage, running backwards towards the pier of the hyoid arch (*hm.*), under the superorbital rudiment (*s.ob.*). Thence the suspensorium narrows gently, and a little behind the middle of the bar forms, on its outside, a jutting step; this is the gently concave quadrate hinge which articulates with the mandible (*mk.*).

The rest of the suspensorium lessens to one-half its hinder width, and is *ƒ*-shaped, bending inwards at first and then outwards before it makes its last inward bend to join the trabecula (Plate 30, figs. 7, 8, *p.pg.*, *tr.*). The extent of the coalesced part is greater than the width of the bar; both that part and the fore part of the free bar belong, not to the proper pier of the mandible, but to the palato-maxillary arcade—it is the proper "ethmo-palatine." In the Tadpole of all, and the adult of most, Batrachia, it is not differentiated from the pterygoid band. The mandible (*mk.*) has already a flattish articular facet, an angular and a coronoid process; the main bar is gently arcuate, and lessens gradually to its distal end, which is rounded, and does not touch its fellow of the opposite side.

The hyoid arch is large and highly subdivided; it has now a large forwardly-projecting basal piece (Plate 30, fig. 8, *b.hy.*).

The pier is the hyomandibular (*hm.*), with its styloid symplectic fore-growth (*sy.*); this part is free, and not, like its serial homologue the pterygoid process of the suspensorium, concrescent with the bar in front of it. This hyoid pier is still three-fourths as large as the pier of the mandible; its dorsal condyle is large and rounded, its body is swollen behind, ready to form the "opercular process," and below this knob, on the inner side, it is scooped for articulation with the inter-hyal (*i.hy.*). Here, as in the *hyostylic* types of fishes, the hyoid arch is subdivided primarily into an antero-superior and a postero-inferior bar; but in this type, as in the Teleostei, the latter is subdivided again into three segments—the inter-hyal, the cerato-hyal, and the hypo-hyal (*i.hy.*, *c.hy.*, *h.hy.*).*

The inter-hyal (*i.hy.*) is a small, short, but thickish segment which articulates with the inner face of the hyomandibular (*hm.*) above, and with the cerato-hyal (*c.hy.*) below. The cerato-hyal is nearly as long as the mandible, and is twice as thick as its distal part; it is oval in section, rather pinched in at the middle, and rounded at both ends. The lower convex end fits into the shallow cavity on the top of the hypo-hyal

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* In the Sturgeon and its congeners the symplectic is segmented off from the hyomandibular, and the same thing often occurs in the diminished and modified hyoid pier of the Batrachia, and even amongst some of the *Sauropsida*, as in the Chelonia (see 'On the Skull of *Chelone viridis*,' "Challenger" series, vol. i., part 5, plate 6, figs. 6, 6*a*).

(*h.hy.*), the thick, semi-globular, distal segment of the arch. The basal piece, *glosso-vel basi-hyal* (*b.hy.*), is wide, oblong, and three-fourths the height of the cerato-hyal; its rounded fore end is rather blunt; it is, in reality, a double bar, and its tissue is much lighter and more spongy than that of the side bars; its posterior end is doubly scooped for articulation with the hypo-hyals.

The branchial arches (Plate 30, fig. 8, *br'.*) are now beginning to divide across into four pieces on each side, namely, a pharyngo-, an epi-, a cerato-, and a hypo-branchial (see Plate 31, figs. 12, 13); a rod of cartilage of about the same thickness as the arches runs along the mid line, connecting them together; this is the basi-branchial (*b.br.*).

Transverse sections of a somewhat younger specimen (7¼ lines long) show much that is instructive, and corroborate the observations made upon *dissected* embryos.

Section 1.—The first of these (Plate 31, fig. 1) is in front of the chondrocranium, through the fore part of the nasal capsule (*ol.*). The skin is very thick, and the mucous membrane of the nasal sac is composed of large columnar cells. The upper part is much flatter than the lower, in which three of the suctorial disks (*s.d.*) are seen in section.

Section 2.—The next section (Plate 31, fig. 2), is through the middle of the nasal sacs (*ol.*) and the fore-part of the hemispheres (C^{1a}.); here the lower face is less convex and the upper more so; the fore end of each trabecula (*c.tr.*) is cut through; it is a rounded projection, the rudiment of the free cornu.

Section 3.—The third section (Plate 31, fig. 3) is through the back of the nasal sacs (*ol.*), the first third of the cerebral hemispheres (C^{1a}.), and the solid coalesced end of the triple trabecular outgrowths of the basis cranii. The outline of the trabeculæ (*tr.*) is clearly seen below; but above, these bars—which are oval in section at this part and very thick—are confluent with an upper median mass, the intertrabecula (*i.tr.*); this is wider than the paired bars, fits in and on them, and rises over them as a dilated and concave floor to the membranous floor of the cranium.

Section 4.—The fourth section (Plate 31, fig. 4) is through the hemispheres (C^{1a}.) and barely misses the eye-ball (for *ol.* read *e.*). Here the fore end of the palato-quadrate cartilages (*p.pg.*) is cut through, where these bars run into the trabeculæ (*tr.*). The intertrabecula (*i.tr.*) is wider and flatter, and the shallow sulcus between the paired trabeculæ is gone; they together form a convex mass below, at this part. The section of the palatines would be circular, but the conjugational band is very thick, and obscures their real form.

Section 5.—The fifth section (Plate 31, fig. 5) runs through the fore part of the eye-balls (*e.*). This section is at the end of the intertrabecula (*i.tr.*), and the trabeculæ and palatine bars (*tr.*, *p.pg.*) are only half as large as in the last, and this section is seen to be oval, for the connecting cells are much reduced in quantity.

Section 6.—The sixth section (Plate 31, fig. 6), a little farther back, shows an evident fissure between the trabeculæ (*tr.*), which are now flatter, like bricks, and the connecting band is so thin as almost to set the palatine band (*p.pg.*) free.

Fourth Stage.—Young Lepidosteus, *11½ to 12½ lines long: average size* 1 *inch.*

In these, the largest of those reared by Professor AGASSIZ and Mr. GARMAN, the chondrocranium is perfect; the occipital arch is beginning to ossify, and the investing bones are very numerous and quite distinct.

The cranium at this stage (Plate 32) corresponds very closely with that of a young Sturgeon *five inches* long, but has much larger membranous tracts, and is altogether a much lighter structure; in having rudimentary basi- and ex-occipital bony centres, it has already gone beyond the skull of an adult Sturgeon.

The fissure between the fore and hind brain (Plate 32, fig. 4, C^1., C^3.) is very distinct, and reaches to the base of the mid brain (C^2.); but there is no "posterior clinoid wall," such as would exist and be very massive in the skull of an embryo Sauropsidan or Mammal at the same stage.

Already the notochord (Plate 32, fig. 4, *nc.*) has retreated to a considerable distance behind the pituitary body (*py.*), which is now an appendage to the infundibulum (*inf.*); the brain well fills the whole cranial cavity up to this stage, but the hemispheres (C^{1a}.) are relatively very small.

Another thing to be noticed is this, namely, that the *pre-cerebral* growth of cartilage is almost as long as the whole cranial cavity, although it is only a fraction of the length to which it will attain. At first sight it might be thought that the mesocephalic flexure was gone, but the up-throwing of the mid brain, and the meeting of the fore and hind brain, show that the bend is very large and very sharp *at one point.* The four faces of the skull are all largely membranous, and but for the notochord (*nc.*), the floor would be open along nearly its whole length, for the cartilage only closes in at the mid line beneath the front end of the hemispheres and the olfactory lobes (C^{1a}., C^{1b}.). The thick cranial notochord (*nc.*) is receiving a bony investment between the thin, post-auditory ends of the investing mass (*iv.*); this will be seen better in the transverse sections (Plate 33). The fore ends of the basal bars (figs. 2, 3, *iv.*) diverge from the notochord (*nc.*) some distance behind its apex; in the middle part they are completely confluent with the auditory sacs.

The narrowed, diverging bars that retreat from, and then shoot on far in front of the notochord, are the trabeculæ (*tr.*); they approximate gently, and their interspace in front is sharply pointed. But the trabeculæ have not merely approximated, they are united together by the intertrabecular wedge (see Plate 31, figs. 7, 8, *i.tr.*); and this has now become a large rod, running forwards to the end of the narrow snout.

Outside this thick rod, but little of which is formed by the lateral bars, those bars grow externally into a large lanceolate leaf of cartilage, which reaches right and left nearly to the small, *distant* nasal sacs (fig. 5, *ol.*). These peculiarly *Acipenserine* outgrowths of the trabeculæ are the familiar "cornua" (*c.tr.*) curiously modified; both ends of each leafy growth are free, as rounded ears of cartilage; on the inside there is a sulcus, deepest, on both surfaces, between each cornu and the coalesced bars in the middle.

There is a squarish ear of cartilage on each side the pituitary body, growing from the trabeculæ, and looking outwards and forwards; these are the "basi-pterygoid processes" (*b.pg.*), so familiar to us in the Amniota; to these the pedicles of the suspensorium (*pd.*) are articulated. Behind these the auditory capsules are seen as very large masses, completely confluent with the chondrocranium. Below (Plate 32, fig. 2, *vb.*), the vestibule forms an almost hemispherial projection; this is caused by the sacculus, which contains the otolith (fig. 3, *ot.*).

Behind this there is a lesser eminence caused by the ampulla of the posterior canal (*p.s.c.*), between it and the larger cavity we see a shallow fossa. On the outside of this fossa the capsule projects, where it contains the horizontal canal (*h.s.c.*), and under this projection there is an oblong articular cavity for the hyomandibular.

Outside this, and between it and the superorbital band (*s.ob.c.*), there is a rounded projection. This corresponds with the lateral pre-auditory mass seen in Teleostei and ossified as the so-called post-frontal—my sphenotic ("On the Salmon's Skull," Phil. Trans., 1873, Plate 7, figs. 1–3).

Above (Plate 32, figs. 1, 5, *sp.o.*, *a.s.c.*, *h.s.c.*, *p.s.c.*), the outline of the large ear capsules is sinuous, the sinuosities being caused by the bulgings of the horizontal and posterior canals, and by the sphenotic process. Between the capsules, above, the tegmen cranii (*s.o.*, *t.cr.*) is developed both over the occipital and the post-sphenoidal regions. This roof is rather pointed behind, over the foramen magnum, and has an evenly concave margin in front; there it forms the hinder boundary of the large fontanelle (*fo.*), which is a short oval, emarginate in its narrow fore end. In front of the fontanelle there is a considerable *ethmoidal tegmen* (*t.cr.*), which covers the olfactory lobes and the small hemispheres (Plate 32, figs. 4, 5, C^{1a}., C^{1b}.); this is pointed behind in the middle, and laterally runs into the narrow arcuate superorbital band. The sides of the skull are oblique, the roof being more than thrice the width of the floor in the orbital region; these sides are mainly membranous; thus the orbito-sphenoidal cartilages are only represented by so much of the superorbital bands as belong to their territory; the alisphenoidal cartilage is merely so much of the chondrocranium as projects beyond the auditory capsule, laterally, between the basipterygoid and the sphenotic processes (*b.pg.*, *sp.o.*).

The small nasal capsules (fig. 5, *ol.*) have no separate cartilaginous roof; they are carried to the front of the snout.

The suspensorium of the mandible has retained its *primary* continuity with the ethmo-palatine cartilage, so that it is still a palato-quadrate (Plate 32, fig. 2, *p.pg.*, *pd.*); but this is quite free now from the other primary connexion, namely, that with the trabecula (*tr.*). This large arch, with its pier, foregrowth, and free mandibular bar, has undergone a similar lengthening to that of the cranium.

The pier or suspensorium is a large, oblongo-arcuate plate, ending in front in an oval sub-convex condyle, and a long, terete, pterygo-palatine process; this latter is considerably longer than the main part, and reaches as far forwards as the cornu trabeculæ (Plate 32, figs. 1, 2, *p.pg.*, *c.tr.*); it is slightly arcuate.

The main part of the suspensorium has a thick convex lower, and a sharp concave upper, margin. The upper edge has a convex enlargement behind. This is the rounded pedicle, which articulates with the basipterygoid process of the trabecula (*pd.*, *b.pg.*). The lower margin becomes concave towards the end; a postero-external, triangular process—the otic process—finishes the dorsal end of the suspensorium. The sub-convex, oval condyle (*q.c.*) looks forward and outward, and fits into the scooped hinder face of the articular region of the free mandible. Above its articular concavity the mandible sends forwards a large rounded ear of cartilage, convex outside (Plate 32, fig. 1, *cr.c.*) and concave within (fig. 2, *cr.c.*); this is the coronoid process of the mandible. The angular process is a free rounded spur below and behind the articular concavity. The rest of the rod (*mk.*) is terete, and almost straight; it is only slightly arched upwards and reaches nearly to the end of the snout.

The hyoid arch (Plate 32, figs. 1 and 2) has a pier which is curiously and suggestively like the suspensorium of the mandible; but, already, it is relatively much less, being now about half as large. It has a pedicle, a free posterior process, a fore-growing rounded rod, and an articular facet for the free, inferior arch; the only difference, here, is the absence of any *borrowed* addition at the fore end, such as the pterygoid cartilage has in the concrescent palatine.

The *pedicle* of the hyoid pier (*hm.*) is the oblong, articular head, fitting inside the oblong concavity under the auditory "tegmen;" it has no definite neck.

The free posterior process (*op.p.*) is for the opercular bone; it is short and rounded. The fore-growing rounded rod is the "symplectic" region (*sy.*); it is terete, gently curved downwards, blunt at its fore end and enlarged near its origin; it lies anteriorly behind (under) the convex edge of the suspensorium. The body of the hyomandibular is gently bilobate and fenestrate in the middle (*hm.f.*); the articular facet for the "inter-hyal" (fig. 6, *i.hy.*) is a scooping between the two convexities of the hind margin (Plate 32, fig. 2).

The rest of the hyoid arch is not in one piece like the mandible, but in *three*, and these have, also, a large double median bar conjugating them. The first of these is a small, unciform segment of cartilage, the inter-hyal (fig. 6, *i.hy.*); it is articulated to the inner face of the hyomandibular by its hooked end, obliquely, and obliquely also to the top of the cerato-hyal by its base. The latter (*c.hy.*) is half the length and twice the thickness of the mandible; it is a rounded rod, swollen near the top, and then thickened gradually to its distal end. All but the top and lower concave face is ossified.

The distal concave end is articulated to the top of a globular segment—the hypohyal (*h.hy.*); this is not ossified. The right and left segments fit into a pair of concavities on the hind face of the glossal piece, or basihyal (*b.hy.*). This is tongue-shaped, the sides are parallel, the fore end rounded; it is moderately thick, is essentially double, and is as long as the cerato-hyal. The subdivided, ossifying branchial arches will be described in the next stage.

The uniformity of the rapidly elongating intertrabecula is shown in the vertical section of the skull (Plate 32, fig. 4, *i.tr.*). The three trabeculæ are shown inside the lengthening snout, with its four rows of mucous glands in a second upper view (Plate 32, fig. 5); and in it, also, the position and relation of the sense-capsules and brain are displayed, and also how that the tegmen cranii (*t.cr.*) leaves the large mid brain (C^2.) unprotected.

A partial view of the chondrocranium, namely, the floor, from its upper face up to the end of the cranial cavity (Plate 32, fig. 3) shows the huge notochord (*nc.*), whose bony sheath is incomplete above, and the fore end of which is free, and but little attenuated; that part is curved but little upwards (see Plate 32, fig. 4). The fusion of the basal bands and auditory capsules is shown to be perfect, and the cupped tracts for the ampulla of the posterior canal (*p.s.c.*) and for the sacculus (*vb.*) are also seen. In front of these, on each side of the diverging parachordals—now to be called trabeculæ (*tr.*)—the basipterygoid peduncles (*b.pg.*) are shown. In the emargination behind these, in the fore part of the ear-capsules and in the occipital ring, thin films of bony matter are forming, which will become the alisphenoids, prootics, and exoccipitals; these will be shown better in the sections. The huge lanceolate pituitary fenestra (*py.*) is floored by the parasphenoid (*pa.s.*), which wedges in, in front, between the converging trabeculæ. In front of that part the chondrocranium is complete. The olfactory nerves (I.) escape from the bulbs (Plate 32, figs. 4, 5, C^{1b}.) and run along to the distant nasal sacs (*ol.*) between the intertrabecula and cornua trabeculæ in the deep groove between them above. The nasal branch of the ophthalmic (5′) runs forwards outside these. Some of the bony plates are shown on the chondrocranium; the foremost of these is the first of the *maxillary chain* (figs. 1, 2, *mx′.*); and on the palato-pterygoid there are three "parastoses," namely: the palatine, pterygoid, and mesopterygoid (Plate 32, figs. 1, 2, *pa′.*, *pg.*, *ms.pg.*).

Transversely vertical sections show much that is instructive in this stage also (see Plate 33, figs. 1–13).

Section 1.—In this (Plate 33, fig. 1) the fore end of the long face is seen to be convex above and somewhat concave below. The skin is very thick and glandular; the nasal sacs (*ol.*) are simple pouches, with a thick epithelium, the tissue beneath the skin has now become osseous; in this way we get the premaxillaries, nasals, maxillaries, palatines, &c.; the plates directly over the nasal sacs are the nasals, and the palatine part of the premaxillaries is shown below the sacs. In this section we see the fore end of the prenasal cartilage or intertrabecula (*i.tr.*); it is a long oval in section, with the narrow part below and the sides compressed.

Section 2.—Here (Plate 33, fig. 2) we have the prenasal (*i.tr.*) cut through behind the nasal sacs; palatine teeth attached to bony laminæ (*pa′.*) are seen below, and similar bony tracts (*p.ob.*) are seen, right and left above, enclosing the olfactory nerve (I.). Here the cartilage (*i.tr.*) is alate, each sharp wing being nearer the base than the top.

Section 3.—In this (Plate 33, fig. 3) the parts to be described are numerous, for the cornua trabeculæ (*c.tr.*) are cut through, and also the palato-quadrate (*p.pg.*) near its fore end. The intertrabecula (*i.tr.*) is oval in section, but it grows out, right and left, into wings, which thicken towards their outer edge and are as wide as the median bar; these are the cornua trabeculæ (*c.tr.*). Under their rounded end we see a small oval section of cartilage placed obliquely; this is the palato-quadrate (*p.pg.*). The fossa over each cornu trabeculæ is more scooped than that beneath it; in this lies the olfactory nerve (I.).

Beneath the intertrabecula, and following its curve, there is a thin lamina of bone; this is the parasphenoid; the oblique laminæ right and left of this are the vomers. The palatines (*pa′.*) are seen in the submarginal ridge, and one of the maxillary chain of bones (*mx′.*) in the lesser, outer ridge. Above, some of the superior (or ethmo-nasal, *et.n.*) and supero-lateral (or preorbital, *p.ob.*) scutes are cut through; the former protect the olfactory nerves (I.).

Part of this section (fig. 3A) is separate from the rest; it is through the lower jaw, in front of the tongue.

Here Meckel's cartilage (*mk.*) has a short oval section; outside it we see the dentary (*d.*) as a larger and a lesser lamina.

Section 4.—This (Plate 33, fig. 4) is behind the angle of the mouth, and close in front of the cranial cavity, where the three bars (*i.tr.*, *c.tr.*) are thickest. The middle part is one-half higher than in the last section, and is broader above and below. The side bars (*c.tr.*) are twice as thick here, and are shorter, and upturned; the olfactory nerve (I.) grooves both the bars, and is more than half enclosed in cartilage. Under these thick rounded wings each palato-quadrate (*p.pg.*) is seen; it is twice as thick as in the last section, is circular, and is its own width below the trabecular cornu.

As this is close behind the gape the mandible is in two sections; the upper is small—it is the fore end of the coronoid process; at a good distance below this part the main rod (*mk.*) is severed; it is oval, with the narrow end above. Bony laminæ belonging to the preorbital series of scutes are seen supero-laterally; below the intertrabeculæ the parasphenoid (*pa.s.*) is shown; a hooked, zigzag line of bone is seen propping up the palato-quadrate cartilage; this is the pterygoid, whilst outside the mandible the dentary (*d.*) is visible.

Section 5.—This (Plate 33, fig. 5) was made through the fore part of the hemispheres (C^{1a}.) and of the tegmen cranii. The base is formed almost entirely by the trabeculæ (*tr.*) for the middle bar dies out in this region; here they are at their thickest part; they form a crescentic mass, the horns of which grow upwards and a little outwards as the lateral ethmoidal wall. These walls pass above into the convex roof; there is a superorbital enlargement where these thickish laminæ pass into each other.

The palato-quadrate cartilages (*p.pg.*) are lesser again, flattened obliquely, and are still further from the basal bars. Behind the angle of the mouth the mandible thickens rapidly; this part (*ar.c.*) shows a section oblique and sub-reniform, close in front of the articular condyle. The large mouth-cavity (*m.*) is partly filled here with a transversely oval mass—the tongue; it has a large *double* core of a soft spongy kind of cartilage; this is the exceptionally symmetrical basi-hyal (*b.hy.*).

The same kind of surface bones are cut through above; the outer film is a superorbital (*s.ob.*)—a continuation of the same chain as the preorbitals; and the inner piece is the pointed fore end of the frontal (*f.*). The pterygoids (*pg.*) are still in section, but the mandible is cut through behind its splints.

Section 6.—The next section (Plate 33, fig. 6) is through the middle of the eye-ball (*e.*), and through the hinder wide part of the hemispheres (C^{1a}.). Here the cranium is largely membranous, for the only cartilages cut through are the superorbitals (*s.ob.c.*), mere bands running superolaterally, and the trabeculæ below. The latter (*tr.*) are just distinct at this point, and are brick-shaped—a little turned up at their outer ends. Thus at this part we have the fore end of the great fontanelle (see Plate 32, fig. 1, *fo.*), and the wide orbito-sphenoidal fenestra (fig. 2, *os.f.*).

The quadrate region (*p.pg.*) is cut through here, behind the hinge of the mandible; this is still faced on the inside by the pterygoid (*pg.*), whilst another film of bone is cut through above, namely, the mesopterygoid (*ms.pg.*). Above, the frontals (*f.*) are seen in section, and below, the tongue and basi-hyal (*b.hy.*).

Section 7.—This (Plate 33, fig. 7) is post-orbital, and is through the mid brain (C^2.), and the thalamencephalon (C^1.), near the pituitary body.

The trabeculæ (*tr.*) are very small, nearly circular in section, and at their greatest distance apart. The flattened superorbital bands (*s.ob.c.*) are thrown to the side of the membrano-cranium, which is at its weakest point here. The quadrate (*q.*), which was thin above and thick below in the last, is now thick above and thin below; the long pterygoid is still on its inside, and below its out-turned thin edge a small round rod of cartilage is seen: this is the symplectic (*sy.*).

In the root of the tongue there are three cartilages cut through, the middle bar is nearly circular, the others are flattened; these are the basi-branchial and the first hypo-branchials (*b.br.*, *h.br.*). In a fold outside and below these, a thicker round rod is seen; this is the cerato-hyal (*c.hy.*). Protecting the pituitary body, the parasphenoid (*pa.s.*) has here widened out considerably; for a short space, shown in this and the next section, the roof shows scarcely any osteoblasts.

Section 8.—This section (Plate 33, fig. 8) is through the fore part of the auditory capsules and the widest part of the mid brain (C^2.), where it turns down to join the hind brain. The anterior canal (*a.s.c.*) is cut through; here the capsule (*au.*) is imperfect within, and is beginning to ossify as the prootic. The investing mass (*iv.*) is cut through where it is shooting out into the trabeculæ, and the bands are wide apart. The superorbital band has now passed into the antero-superior angle of the auditory capsule. The pharyngeal cavity is open below, for here the great opercular fold is cut through; in it bony plates (*op.*) are forming; in its upper part the root of the symplectic (*sy.*) is severed, and below that a much larger bar with an ectosteal sheath; this is the cerato-hyal (*c.hy.*); below, the fore part of the heart (*h.*) is cut through, and above and across it the basal, distal, and cornual elements of one or two of the middle branchial arches (*b.br.*, *h.cr.*, *c.br.*). Here the parasphenoid (*pa.s.*) is of great width, undergirding not only the open basal fontanelle; but the basal plates also. The fore part of the Gasserian ganglion (5) is cut through.

Section 9.—This (Plate 33, fig. 9) is through the widest part of the hind brain (C^3.), and behind the fontanelle, for here we have the hind "tegmen" (*t.cr.*), as a thinnish layer of roof-cartilage, passing externally into the auditory capsules. At this part there is a thick band of cartilage on the inner side, in front of the large "meatus internus." Here the large ampulla of the anterior canal (*a.s.c.*) and the beginning of the horizontal canal are exposed, and outside it there is a ledge—tegmen tympani—with which the hyomandibular (*hm.*) is continuous. This is a flat cartilage; the whole of the hyoid pier is relatively small in this type.

Here the notochord is cut through, where it is enclosed by the broad, fore margin of the investing mass (*iv.*), which thins out, and curves outwards and upwards, to join the capsule. The opercular fold (*op.*) is cut through behind the lower part of the hyoid arch, which is projected forwards; the branchials (*br.*) are similar to the last, but further back in the series, and the heart (*h.*) is now cut across at its fore part.

There are several bony plates cut through in this, viz.: parietal, squamosal, opercular (*op.*).

Section 10.—Here (Plate 33, fig. 10) only the upper or main part is figured; it shows a section of the middle or widest part of the auditory capsule, where the inner wall is membranous. The anterior canal (*a.s.c.*) is cut through close to its junction with the posterior, and also the horizontal canal (*h.s.c.*) with the subdivisions of the vestibule (*vb.*). This section is behind the junction of the capsule with the investing mass (*iv.*), and shows the ganglion of the 7th and 8th nerves (VII., VIII.). The flat hyomandibular (*hm.*) is still in section, but behind its articular head; the broad parasphenoid (*pa.s.*), and the outer bony plates, are similar to those of the last section.

Section 11.—This (Plate 33, fig. 11) shows the hind part of the auditory capsule; the whole course of the posterior canal (*p.s.c.*) is seen through this back wall of cartilage. Here the capsules are some distance from the investing mass (*iv.*); above, the roof (*t.cr.*) has become deficient again, and at this part the hyomandibular (*hm.*) is cut through some distance below the condyle, and in this specimen a film of bone is seen investing the cartilage. The large notochord (*nc.*) has also a sheath of bone, and under each basal bar there is a section of the forked hinder part of the parasphenoid (*pa.s.*). One of the branchial arches (? the third) is seen in its whole extent (*p.br.*, *e.br.*, *c.br.*, *h.br.*, *b.br.*).

Section 12.—In this section (Plate 33, fig. 12) the occipital arch (*e.o.*) is cut through a little behind its upper part, and quite behind the auditory capsules. Here the 9th and 10th nerves and their ganglia (IX., X.) are laid bare, and the heart (*h.*) is cut through at its thickest part. The investing mass has become narrow and hour-glass shaped in section; it sends upwards a cartilage which thins out gradually upwards; this is the exoccipital (*e.o.*), which becomes supraoccipital above. The exoccipital bones are forming as embracing ectostoses; the notochord has its sheath, the rudiment of the basioccipital (*b.o.*), and under the investing mass the parasphenoidal forks are seen. Here there is one pharyngo-branchial (*p.br.*) and the distal parts of two or three arches, with the basal piece (*b.br.*) lying over the heart (*h.*).

Section 13.—This last section (Plate 33, fig. 13) is close in front of the first vertebra; it catches the last pharyngo-branchial (for *h.br.* read *p.br.*), and shows some pharyngeal teeth. Only half the arch comes in here, but it shows well the outer and inner laminæ of the exoccipital bone (*e.o.*). Here the notochordal bony sheath (*nc.*, *b.o.*) is thicker, and the splintery ends of the parasphenoid narrower than in the last section.

In the last three sections bony laminæ are forming beneath the skin; these and their relations will be better understood by reference to the figures and descriptions of more advanced stages.

Fifth Stage.—Young Lepidostei *2 to 2½ inches long.*

In this stage the skull is rapidly acquiring its permanent character (Plate 34, figs. 1, 2); the rostral part is now twice as long as the cranial cavity; in the last the pre-cranial part was not quite so long as the cranial.

This is mainly due to the growth of the intertrabecula (*i.tr.*), which already is considerably more than twice as long as the very long cornua (*c.tr.*).

The endocranium is now in this specimen (which is 2 inches long) nearly as perfect as it will be; the upper fontanelle (*fo.*) is a short ellipse, with the long diameter axial; it is relatively much less, through the growth of the tegmen cranii (*t.cr.*), fore and aft. The whole endocranium may be said to be pyriform, with an extremely long stalk; the "nose" of the *pear* is represented by the basioccipital (*b.o.*). The occipito-auditory region is semicircular above, with an apiculation over the foramen magnum; the orbital region is suddenly narrowed, and this gently lessens into the ethmoidal, which as gently becomes rostral, and the rostrum slowly lessens to its fore end, where it has two small wings. Near its root, however, it has two large wings—these are the cornua trabeculæ (*c.tr.*). The floor of the skull is still largely open (fig. 3), and the sides are occupied by the large orbital fenestræ (*os.f.*). The sheath of the notochord (*nc.*) is most completely ossified below; above (fig. 3), it is still membranous in front; the bony sheath is the basioccipital (*b.o.*); it runs a little, right and left, into the basal cartilage.

The auditory capsules are completely confluent with the chondrocranium; and their canals (*a.s.c.*, *h.s.c.*, *p.s.c.*) are to be seen through the cartilaginous wall. Above, the even form of the skull is not much altered by the canals within; below, the sacculus (fig. 2, *vb.*) largely swells the vestibular region; the form of this cavity is a short oval (figs. 2 and 3, *vb.*). The condition of the hind skull is best studied from above, in a preparation of the base (fig. 3); here the exoccipitals (*e.o.*) are seen to be forming broad borders of bone to the cartilage of the arch, and these are approaching the cephalostyle (basioccipital, *b.o.*).

Further forwards, in the angle between the capsule and the basal plate, under the Gasserian ganglion, the cartilage is being ossified as a prootic (*pr.o.*), and the wider wings which stand out from the front of the capsule are becoming the sphenotics (*sp.o.*). Also, still further forwards, the lower wings or basipterygoids (*b.pg.*) are getting a coating of bone; this ectostosis runs upwards into the side wall, in front of the capsule—it is the alisphenoid (*al.s.*).

In front of these last wings the trabeculæ (*tr.*) become bent, first outwards and then inwards, ready to join the median bar (*i.tr.*). They are rounded, solid rods. The basal fontanelle is now divided across, near its hinder end, by a narrow band, which is cartilaginous at its roots and fibrous in the middle; this is the *late* and feeble "post-pituitary wall" (*p.cl.*)—here a mere *bridge.*

The small triangular space behind is the posterior basi-cranial fontanelle (*p.b.c.f.*); the large, pinched, pyriform space in front is the pituitary space (*py.*) or anterior basi-cranial fontanelle. Its narrow anterior third runs up to the intertrabecula (*i.tr.*), which goes further back than the ethmoidal wall (*l.eth.*), and is, indeed, the rudiment of the "perpendicular ethmoid." Infero-laterally, the ethmoidal wall (Plate 34, fig. 2) is very restricted, for the orbito-sphenoidal fenestral (*os.f.*) is of great length, being extended equally in front of and behind the optic nerve (II.). But above (fig. 1, *t.cr.*) the tegmen cranii, in front, is as large as the long oval fontanella (*fo.*); behind that space the spheno-occipital tegmen is one-half longer, axially, and twice as wide across.

The three confluent trabecular bars combine to close in the fore part of the cranial cavity (Plate 34, figs. 1 and 3), only leaving an opening right and left for the long olfactory nerves (I.). When they have escaped from the skull they lie for the hinder half of their course in a deep rounded sulcus between the cornual extensions of the trabeculæ (*c.tr.*) and the huge middle bar (*i.tr.*). The three bars are at first nearly of the same width; the cornua are rounded where they first project as longitudinal wings, but they soon become narrow rods, and end in a pointed manner behind the middle of the pre-cranial region. Thence the intertrabecula is a gently compressed rod, only slowly lessening forwards, and ending as a slightly winged lobe in the end of the beak.

The first and second visceral arches (Plate 34, fig. 4) are elongated forwards like the skull, but the "pier" of the hyoid arch (*hm.*, *sy.*) is less than half the size now of the mandibular suspensorium (*p.pg.*, *pd.*). I have shown these with their splints attached as seen from above (Plate 34, fig. 1) and from below (fig. 2); also without the "parostoses" from their inner side (fig. 4). Their intrinsic bony centres or "ectostoses" are now clearly seen, but they are very small in proportion to the cartilage in which they appear. The suspensorium and its free bar reach from the auditory region to a small distance behind the end of the snout; these cartilages, moreover, largely overlap each other; end to end, they would be considerably longer than the entire skull. The quadrate hinge (*q.c.*) is opposite the point where the olfactory nerves (I.) emerge, a little in front of the cranial cavity. The dorsal end of the suspensorium is bilobate; the pedicle (*pd.*) is an oblong, oblique facet, looking backwards and inwards, and articulating with a similar facet on the basipterygoid (*b.pg.*); the outer lobe is free; it is the triangular otic process (*ot.p.*); it reaches almost to the ear-capsule, but is too short to articulate with it. The main part of the suspensorium runs from these hind lobes to the quadrate hinge (*q.c.*); the inner margin is first hollow and then arched; the arch runs along the free anterior process; that edge is sharp. The outer edge is thick and ribbed on its inner face (Plate 34, figs. 2, 4); it is nearly parallel with the upper, being convex behind and concave in front. The width of this large plate is equal to half its length, and it is very elegantly sigmoid.

The quadrate hinge (*q.c.*) is a small oblong saddle, the main direction of which is forwards and a little downwards; it is convex outwardly, but has a rising inner crest (Plate 34, fig. 4), which fits closely to the articular condyle (*ar.c.*). Beyond the hinge the pterygo-palatine bar is first half as wide as its root, and then losing its inner crest, it becomes a rounded, straight, slender style, which ends very close to the point of the cornu trabeculæ (Plate 34, fig. 1, *p.pg.*, *c.tr.*).

The lower or inner face of the suspensorium is gently convex; the upper or outer face, gently concave. On the convex inner face there is a large splint-bone — the pterygoid (Plate 34, fig. 2, *pg.*); it covers more than half of the broad part of the cartilage—its antero-superior part. For some distance in front of the quadrate condyle it is continued forwards, undiminished in size; it then lessens gradually into a pointed style, which runs parallel with, and a little on the outside of, the rostrum (*i.tr.*) for three-fifths of its length.

A very narrow dentigerous bone underlies the narrow fore half of the pterygoid, and then goes beyond it up to the premaxillary, or nearly to the end of the snout. It is rather broadened in its diverging hind part, and then the right and left bones gently converge forwards; these are the *parosteal* palatines (*pa'.*).

Along the inner edge of the suspensorium there is a narrow, thin, falcate splint, which reaches from the top of the broad part to the neck of the pedicle; this is the "mesopterygoid" (*ms.pg.*). The front of the neck of the pedicle is ossified as a small ectosteal patch; this is the "metapterygoid" (*mt.pg.*). The neck of the quadrate condyle also is ossified; this is the small quadrate ectostosis (*q.*).

From the dilated end of the intertrabecula (Plate 5, fig. 2, *i.tr.*) to the fore end of the palato-quadrate styles, there is an extremely delicate pair of bony threads; these are the vomers (*v.*). In their hinder third these bones underlie the styloid end of a bone more than twice their breadth; this is the parasphenoid (*pa.s.*); it is carinate below, widening as it approaches the basis cranii; it then narrows till it nearly reaches the optic nerve (for I. read II.); thence it widens, loses its keel, and becomes grooved along its middle below. This bone then trebles its width, and sends out two pairs of angular projections, the lesser to support the basipterygoid processes (*b.pg.*), and the larger farther back to support the vestibular swellings of the ear-capsules (*vb.*). The bone then becomes divided into two sharp styles, which embrace the lower part of the basioccipital. The parasphenoid is three-fourths the length of the head; it forms the only floor to the skull in the pituitary region (fig. 3, *py.*, *pa.s.*).

The bones that invest the skull above, and postero-laterally, will be described in the next stage, but partly also in the sections illustrating this. The splints of the first arch also will be described hereafter, but there is one which is figured on this Plate, namely, the preopercular. This bone (Plate 34, figs. 1, 2, *p.op.*) is a very narrow, but rather sharp splint, which is applied to, and takes the curves of, the lower edge of the suspensorium; it is the normal splint of the mandibular suspensorium, and is more like that of a Frog than that of an osseous Fish.

The length of the free mandible (Plate 34, fig. 4), as compared with that of the palato-quadrate, is as 20 to 19; it is, therefore, already a very long jaw. In its hind part it is nearly as broad as its pier, and then runs to its end as a somewhat stouter rod than the pterygo-palatine above it.

The condyle (*ar.c.*) is cylindroidal, concave as seen from the side, but somewhat convex across; there is an ectosteal "articulare" (*ar.*) in the broad part near the condyle. The angle is scarcely produced; but in front of the condyle, above, the cartilage grows into a pedunculated crescent of cartilage, the coronoid crest (*cr.c.*) The notch between this part and the lessening rod (*mk.*) is very deep; this leafy coronoid is convex outside and concave within. The long Meckelian rod (*mk.*) is gently arcuate and pointed at its fore end, where it nearly meets its fellow of the other side.

This skull is scarcely *amphistylic* even, much less *hyostylic*, for the pedicle of the mandibular pier is strong and well articulated, but the hyomandibular (*hm.*) is feeble, and the binding process or symplectic (*sq.*), feebler still.

The hyoid pier or hyomandibular (Plate 34, fig. 4, *hm.*), with its symplectic foregrowth (*sy.*), is about one-third the size of the palato-quadrate in front of it. The articular facet is a long, arched, convex condyle, with scarcely a perceptible neck; behind it is the knob for the opercular; below, this multilobate mass is sinuous, and scooped on its inner face to form the oblique condyloid facet for the interhyal. The front margin is concave, and the whole bulk suddenly lessens into a sigmoid style, bent first upwards, and then downwards, as it runs obliquely forwards to bind inside the hinder angle of the suspensorium, which is scooped to receive it. A small oval fenestra is seen in front of the middle of the hyomandibular; and around this, below the condyle, there is an ectosteal sheath. All but the front fourth of the symplectic also is ossified as a delicate shaft-bone.

The free or postero-inferior part of the hyoid arch is more evenly massive than its pier or antero-superior part. The conjugational piece, or inter-hyal (*i.hy.*), is pyriform;

it forms a loose joint inside the hyomandibular by its narrow upper end; and below, its inner face is scooped for the backwardly bent head of the cerato-hyal (*c.hy.*).

That segment is seven times the size of the inter-hyal joint; its head is bent back, its shoulder thickened, and has a short, separate ossification, the epi-hyal (*e.hy.*), and its shaft is rounded. The inferior condyle is hemispherical, and the main part of the thickest rod is ossified as the cerato-hyal (*c.hy.*).

The hypo-hyals (Plate 34, fig. 4, *h.hy.*) are sub-globular nodules, scooped above for the cerato-hyal, and ossified on their outside. They fit against the paired concavities of the basi-hyal (*b.hy.*)—a double, oblong, tongue-shaped, interglossal segment, as long, and twice as wide, as the cerato-hyal. This thickish plate is grooved above and below, is rounded and emarginate in front, and is composed of a spongy kind of cartilage, full of fibrous septa, which form a network in it.

There are four perfect, and one imperfect, branchial arches; they are less than half as solid as the hyoid (Plate 34, fig. 4, *br.*). In the mandibular arch, when the mouth is closed, the axes of the suspensorium and its free cartilage become coincident. In the hyoid arch they are *parallel*, not coincident; here, in the branchial arches, they are continuous, the upper being superimposed upon the lower element.

The relative size of these parts is greatly altered, and the subdivision does not exactly correspond with that of the mandibular and hyoid. The part answering to the suspensorium and hyomandibular is less than a fourth the size of that which corresponds to the mandible and cerato-hyal. Each "pier" is subdivided into two pieces—a pharyngo-branchial (*p.br.*) above, and an epi-branchial (*e.br.*) below. The upper piece is a little tongue of cartilage, turned inwards and forwards, and the lower has a short, bony shaft, and is a little rod turned directly downwards. These two segments *do not* correspond with the hyomandibular and symplectic; the upper piece has a double counterpart in the hyoid arch, namely, the hyomandibular and symplectic—one cartilage, with two bony centres in it.

Also, the counterpart of the cerato-hyal, the cerato-branchial (*c.br.*) articulates directly with its pier, the epibranchial; so that there is nothing in the branchial arches corresponding to the inter-hyal.

Instead of the hypo-branchials (*h.br.*) being short nodules, they are in the first two arches nearly as long as the cerato-branchials; are thicker than them below, but less ossified. The three joints between the four pieces seem to show no distinct joint-cavity, but are fibrous. Below, the rounded rods of the hypo-branchials fit into depression on the basal bar—basi-branchial (*b.br.*). This is, in front, a thickish, rounded rod of cartilage; it then thins out, and behind it is flat and emarginate. The first and second arch unite with the long, first basi-branchial segment; the third nearly reaches the short second piece, and the fourth is loosely attached to the side of the third piece, which is as long as the first. None of these pieces are ossified, and the first does not reach the basi-hyal, for the hypo-hyals are thrust between them.

The lessening third and fourth hypo-branchials (*h.br.*) are not ossified. The fifth cerato-branchial ($c.br^5$.) is less than those in front of it; it is not ossified, and there is no other segment in that arch.

The branchial arches are but little modified after this; they merely increase in size, and are always small as compared with the arches in front of them. A series of sectional views will complete the illustrations of this stage.

I have now to illustrate this stage by a series of sections made from a specimen 2½ inches long.

Section 1.—This section (Plate 33, fig. 14) is taken from near the end of the snout in front of the nasal sacs. The intertrabecula (*i.tr.*) is seen to be oval, with the larger end below. The whole snout is elliptical, with a slight convexity above, and a slight concavity below, at the mid line. The premaxillaries (*px.*) are cut through, both in their body and their palatine process (*p.px.*); they are reticulations of thin laminæ, and both above and below enclose a mucous gland.

Section 2.—The nasal sacs are cut through here (Plate 33, fig. 15, *ol.*) and the bony laminæ are more complex; the intertrabecula (*i.tr.*) has the same shape as in the last, but the whole snout is rounder above and flatter below.

Section 3.—Behind the nasal sacs other bones come into view (Plate 33, fig. 16); here the evenly elliptical rostrum (*i.tr.*) has the laminæ of the ethmo-nasal (*et.n.*) surrounding it, and on the side a mucous gland is seen in the upper half of one of the maxillary chain (*mx'.*); here the lower face of the snout is becoming convex, with a median groove; above this, right and left, vomerine teeth (*v.t.*) are seen.

Section 4.—In this section (Plate 35, fig. 1) the lower jaw also is cut through; the snout is flatter here, has a ridged lip, right and left, with sub-marginal grooves. The intertrabecula (*i.tr.*), half-way between its fore end and the cornua (Plate 34, figs. 1, 2, *c.tr.*) is circular in section, and is flanked by the layers of the thickening ethmo-nasal (*et.n.*); outside, one of the maxillary chain (*mx'.*) is seen lying over the palatine (*pa'.*), with its large tooth, and under the rostrum the two vomers (*v.*), each with a small tooth, are cut through. Below, the mandibular rods (*mk.*) are cut through at their front part, and right and left we see the solidifying substance of the dentaries (*d.*); over the cartilage towards the mid line, a small, separated style is cut through—this is the splenial (*spl.*).

Section 5.—Another section in front of the angles of the mouth (Plate 35, fig. 2) brings the tongue into view. Here the rostrum (*i.tr.*) is deep, and twice the size it had in the last section; it is flattish above, and more convex below. Here the loose reticulation of the extremely thin bony laminæ would seem to defy interpretation; but it can be classified into groups, and these groups named. Under the flat top of the beak, on each side of the rostrum, several of these thin plates are seen to be connected together, overarching a mucous gland (*m.g.*) above, and the olfactory nerve (I.) lower down. Above these is a sub-marginal groove, right and left; outside this groove the beak is convex, and from the convex part there runs inwards a thin bony flake towards the deep, palatal chink; this is the fore part of the frontal. Below this there is one of the maxillary chain (*mx'.*) cut through, protecting a gland; and inside this, on the lateral lobe of the beak, there is a wedge-shaped tract of fine diplöe, the lower part of which carries a large tooth; this tract is the palatine (*pa'.*) cut through. In the triangular median keel of the beak there are three thin plates cut through; two of them are superficial and the third is deeper, taking the form of the rostrum somewhat, but diverging externally, and having a short crus below—this is the parasphenoid (*pa.s.*). The paired laminæ running downwards and inwards, outside it, are the vomers (*v.*) in their widest part; they are overlapped by the inner end of the frontals, externally; which, at their outer end, overlap, obliquely, the sharp end of both the cornua trabeculæ and the pterygoid cartilages (*p.pg.*). Below, the thick lower jaws have in them the section of the MECKEL'S cartilage (*mk.*), large and almost circular. There is here the flat, double tongue, with its soft basi-hyal (*b.hy.*), also double. On each side of the tongue there is a deep sulcus. The splenial bone (*spl.*), over and within the cartilage, is here at its largest size, and the dentary (*d.*) takes up a large space by its reticulations; it encloses a mucous gland below.

Section 6.—This section (Plate 35, fig. 3) is through the angle of the mouth, and thus the upper and lower tracts are continuous; the lower or mandibular region is of great height, being cut through close in front of its huge coronoid region. Here the cornua trabeculæ (*c.tr.*) are at their thickest part, or middle (Plate 34, figs. 1, 2, *c.tr.*) and are continuous by a thin oblique tract, with the rostrum (*i.tr.*) which is at the thickest part in this and the next section. Here it is semi-elliptical above, and sub-carinate below, and the thin edges of the cornua ascend to their thick outer part; the olfactory nerves (I.) lie in the hollow between the bars. A little below the cornua the pterygo-palatines (*p.pg.*) are cut through; they are oval in section, their oblique position is parallel with that of the cornua, and their size is nearly as great. The mandibles (*mk.*) are oval in section, here, and twice as thick as the pterygo-palatines; the basi-hyal (*b.hy.*) is here at its widest part. The laminæ of the ethmo-nasals (*et.n.*), and of the frontals (*f.*) run close to each other, and below the pterygo-palatine there are two tracts of reticulated bone; these are the palatines (see fig. 2, *pa'.*) below, and the pterygoid (*pg.*) above. The splenial (*spl.*) is here at its widest part, and the dentary (*d.*) is composed of a large strip of bone, externally, and of a wide network, below.

Section 7.—The position of this section (Plate 35, fig. 4) is evident, for it is through the thickest part of the coronoid process of the mandible (Plate 34, fig. 4 *cr.c.*) where this remarkable crest is separated from the main rod by a large rounded notch. Here the cornua trabeculæ (*c.tr.*) are thinning out, behind, and the intertrabecula (*i.tr.*) is most solid, it is quite round above, and sub-angulate below. The pterygo-palatines (*p.pg.*) are oval and are further from the narrowed cornua (*c.tr.*)—they are twice as near to the coronoid cartilage (*cr.c.*) Below that crest,—which is placed obliquely across the face a little tilted upwards, and the section of which is oblong, but hooked inside—the main bar (*mk.*) is a large nearly vertical ellipse; here the basi-hyal (*b.hy.*) is very wide. The ethmo-nasals (*et.n.*) are less, and the frontals (*f.*) are larger; the palatine is gone, and the pterygoid (*pg.*) is becoming a much thicker bone. Over the coronoid process (*cr.c.*) the supra-angular bone (*s.ag.*) is seen, the dentary (*d.*) is very extensive, and is helping the supra-angulare to cover the coronoid cartilage, and growing down the outside of and coming beneath the main rod (*mk.*) The coronoid bone (under *cr.c.*) now appears inside the mandible.

Section 8.—This slice (Plate 35, fig. 5) is a little in front of the hinge of the lower jaw, and behind the outspread wings of the trabeculæ; hence, the rostrum appears to be single, although it has the trabeculæ confluent with it in its lower half. About the middle there is a slight hollow; above, it is rounded, and at its base somewhat mammillate in section; this part has the parasphenoid (*pa.s.*) fitting to it, which is thus convex instead of carinate.

The pterygo-palatine (*p.pg.*) is oblique and oval; it is nearer the rostrum than the mandible; that part (*ar.c.*) is larger than the rostrum, and is irregularly spindle-shaped in section, with its upper half slightly incurved; in its inner face the "articular" centre (*ar.*) has appeared. The basi-hyal (*b.hy.*) is now wider, and thicker; flat above, and convex behind. The ethmo-nasal bone (*et.n.*) is narrower, and the frontal (*f.*) wider; the parasphenoid (*pa.s.*) has lost its keel, and become convex and alate. The pterygoid (*pg.*) has now more diplöe above, is growing far down as a thin lamina inside the angle of the mouth; the dentary (*d.*) lies on both sides of the lower half of the cartilage; above, the supra-angulare (*s.ag.*) lies over it, and below, the angulare (*ag.*) flanks it.

Section 9.—A little further back (Plate 35, fig. 6) we get a similar section to the last, but the pterygoid (below *p.pg.*) is still more complex, above, and the articular cavity of the hinge of the lower jaw (*q.c.*, *ar.c.*) is laid open, and has a piece of the quadrate in its hinder face.

Section 10.—This is through the fore part of the eye-ball (Plate 35, fig. 7, *e.*); and here, the upper part of the chondrocranial mass is thicker; for it is in the ethmoidal region, and the olfactory nerves (I.) now run through tunnels in the closed-in skull. At this part the suspensorium is cut through in the quadrate region (*q.c.*); it appears as a sigmoid tract; thin above, thicker and rounded below, and with its upper, slightly out-turned, edge not far from the cranial axis. Below, the basi-hyal (*b.hy.*) is becoming more solid. Here each frontal is mainly a flat lamina, becoming complex externally; some of the complex outer part, however, belongs to a circumorbital. Inside the suspensorium the pterygoid (*pg.*) is a sigmoid tract of diplöe, and below the cartilage a small, triradiate tract of bone is cut through; this is the preopercular (*p.op.*).

Section 11.—Here (Plate 35, fig. 8) the cranium is cut through where the olfactory lobes (C^{1b}.) lie; it is therefore behind the proper septal portion of the intertrabecula, and shows the beginning of the tegmen cranii (*t.cr.*). The other parts are similar to those exposed in the last section, but, here, the quadrate bone (*q.c.*) is seen in the lower part of the suspensorium, as an enclosing ectosteal plate.

Section 12.—A little further back (Plate 35, fig. 9) the section is through the hemispheres (C^{1a}.), and is behind the quadrate bone*;* the other parts are similar to those in the last two sections.

Section 13.—Here (Plate 35, fig. 10) the hemispheres (C^{1a}.) are wider, and with them the cranial cavity, whose walls are thinner, and laterally are partly ossified; these bones right and left, are the lateral ethmoids (*l.eth.*); they were not seen in the dissection of the lesser specimen of this stage; (2 inches long, Plate 34; the one sectioned was 2½ inches); but these bones are figured in the dissection of the next stage (Plate 38, figs. 2, 3).

Looking at the base of the cranial axis in this and the last three or four sections, we see that the parasphenoid (*pa.s.*) fits to a cartilaginous mass having a *trilobate outline* below; this arises from the fact that it was formed by the coalescence of three cartilages, viz.: the trabeculæ and the intertrabecula.

Section 14.—The hemispheres in this section (Plate 35, fig. 11, C^{1a}.) are rapidly widening, and the cranial walls are now deficient, the sides being membranous—the orbito-sphenoidal fenestra—in the lower half. The tegmen cranii (*t.cr.*) runs down the sides half way, and is grooved above; the lower edge of the cartilage just touches the optic nerve (II.). Here the base of the skull has lost its height; it is concave above, and scooped below. Here is the hinder end of the intertrabecula, and the trabeculæ (*tr.*) are each of them crested below. The suspensorium is very flat here, especially towards the top; it thickens out again above, and is surmounted there by a small extraneous bony plate—the mesopterygoid (*ms.pg.*, see also Plate 34, figs. 1, 2); here the pterygoid (*pg.*) has become a thin plate. The essentially double nature of the basi-hyal (*b.hy.*) is clearly seen in this section.

Section 15.—The hinder part of the hemispheres (Plate 35, fig. 12, C^{1a}.) are now cut through, and the tegmen cranii (*t.cr.*) is now a thin, sinuous awning thrown over the brain-cavity; it is hollow above, and convex at the sides. The frontals (*f.*) are becoming thin, and the hinder superorbitals (*s.ob.*) are thick and large. The trabeculæ (*tr.*) only are seen in this section, which is through the fore part of the long pituitary space (see Plate 34, fig. 3, *py.*); they are oval in section, and a space equal to their width is filled up between them by the parasphenoid (*pa.s.*).

The suspensorium (*q.c.*) has thickened again, and still the same bones are applied to it, namely: the mesopterygoid, pterygoid, and preopercular (*ms.pg.*, *pg.p.*, *p.op.*). Below, the section was made behind the basi-hyal, and through the first basi- and hypo-branchials (*b.br.*, *h.br′.*); outside, we see the cerato-hyal (*c.hy.*), with its bony sheath, cut through.

Section 16.—In this section (Plate 36, fig. 1) the skull is cut through close in front of the basi-pterygoids (Plate 34, figs. 1–3, *b.pg.*); in this specimen there must have been some little projection backwards from the front tegmen (*t.cr.*), not seen in the one dissected; this would have made the fontanelle heart-shaped, instead of circular.*

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* This projection from the front "tegmen" was seen in the last stage (Plate 32, fig. 4, *t.cr.*), where the fontanelle has a similar shape to that of a young Salmon of the 2nd week. ("Salmon's Skull," Plate 4, figs. 1, 2.)

Here the cavity of the skull is at its widest part; further back the width of the head is due to the addition of the auditory capsules, but the hind brain (C^3.) is only half as wide as the mid brain (C^2.). Here the razor passed from the front of the fontanelle, above, to the middle of the fontanelle, below, just where the trabeculæ pass into the investing mass (*iv.*), and behind the optic foramina (see Plate 35, figs. 11, 12); the section is therefore somewhat oblique, backwards and downwards. The slight projection from the front tegmen (*t.cr.*) is wide apart from the alisphenoidal region, or lateral band of cartilage (*al.s.*), which is thickish, convexo-concave, and occupies more than half of the side wall.

Below, under the thalamencephalon, the parasphenoid (*pa.s.*) is thick and narrow, and is strongly wedged in between the narrowest part of the basal bars (see also Plate 34, fig. 3, *tr.*, *iv.*, *pa.s.*), which are oval in section, and slightly tilted outside. Between them and the alisphenoidal tract (*al.s.*), part of the trigeminal nerve (V.) is seen. Above, the parietals (*p.*) are cut through, and also a lateral bone—the squamosal (*sq.*). Here the suspensorium (*q.*) is, in section, like a drumstick, but feeble below and out-turned, for it thins down towards its lower thickening, and there bends outwards over the small, round symplectic (*sy.*). Above, it is round, very solid, and sheathed at the very top with a bony tract; this is the "metapterygoid" (*mt.pg.*) which is cut through.

Below, the preopercular (*p.op.*) is seen as a small tract of diploë, enclosing a mucous gland; inside it, the cerato-hyal, with its ectostosis (*c.hy.*), is severed, and further inwards the basi-branchial and the first and second hypo-branchials (*b.br.*, *h.br*^{1}., *b.br*^{2}.).

Section 17.—We have now a section (Plate 36, fig. 2) close in front of the pituitary body, but missing it; it is through the infundibulum (*inf.*) below, and the mid brain (C^2.) above, and catches the terminal point of the small tegminal projection; this thin slice was the next to the last, which is not always the case in those which are figured.

Here the basipterygoid processes (for *q.* read *b.pg.*) are seen as thick wings growing from the front part of the investing mass;* they are partly ossified by the lower edge of the alisphenoidal centre; they are thin at their root, thick outside, and they dip a little; above them, the 5th nerve (V.) is cut through. Here the parasphenoid (*pa.s.*) sends out its first pair of angular projections (Plate 34, fig. 2), so that it forms a bony floor to the skull. Above, the parietals and squamosals (*p.*, *sq.*) are seen in section, and also one of the numerous post-orbital scales. The symplectic (*sy.*) is cut off where it has a bony sheath, as it passes forwards to the suspensorium. The cerato-hyal (*c.hy.*) is trilobate here in section; outside, it is the interopercular (*i.op.*); the branchial sections are of the same bars as in the last.

Section 18.—This (Plate 36, fig. 3) is through the fore part of the hind tegmen (*t.cr.*), and where the mid brain passes into the hind brain (C^3.). Here the head is slightly concave above; the roof-cartilage is rather thin, and forms part of a nearly complete cincture, for it is confluent here with the fore part of the auditory capsule (*au.*), which in turn passes into the investing mass (*iv.*) below. There the cartilage is a thick wedge, right and left, and the space between the two wedges is filled in by the parasphenoid (*pa.s.*); in front of the apex of the notochord. Where these basal plates pass into the capsules there the facial nerve (VII.) is severed, and further outwards the capsule has a groove under it in which the fore part of the head of the hyomandibular (*hm.*) is seen; it is partly ossified above and also below as the symplectic. A neat lip, the rudimentary "tegmen tympani," is seen outside the rod, and above it the ampulla of the anterior canal (*a.s.c.*), whose arch also is severed higher up. The bony tracts below are parts of the large, infolded, angular interopercular (*i.op.*, see also Plate 37, fig. 4, *i.op.*).

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* Two errors escaped me in the lettering of this figure,—for *q.* read *b.pg.*, and for *mt.pg.* read *iv.*

The second, third, and fourth hypo-branchials and the middle of the basi-branchial ($h.br^{2-4}$., *b.br.*) are severed, and the gills are seen depending over the fore part of the heart (*h.*).

Section 19.—The roof is now in the superoccipital region (Plate 36, fig. 4, *t.cr.*); it is twice as thick and only two-thirds the width at this part, as compared with the last. Here the hind brain (C^3.) has only membranous sheets interposed between it and the auditory labyrinth, which is deficient in its cartilaginous wall in this, the region of the "meatus internus," where the auditory nerves (VIII.) enter. The section of the hyomandibular (*hm.*) is here behind the symplectic; above its head the horizontal canal (*h.s.c.*) is cut through and the end of the arch of the anterior canal (*a.s.c.*) is seen to come close to the great inner "fenestra" of the capsule. Here, as in Sharks and Skates, the basal plate (*iv.*) projects beyond the capsules, and in the angular space the 7th nerve (VII.) is cut through.

Below, this section is still in front of the notochord, but the parachordals (*iv.*) bend down very near to each other; they are supported by the parasphenoid (*pa.s.*). Here the branchial arches (*h.br.*, *b.br.*) are cut through near their hinder part; below, their gills (*g.p.*) are seen hanging over the heart (*h.*), and the interopercular is seen outside the hyomandibular (*hm.*).

Section 20.—In this slice (Plate 36, fig. 5) the apex of the notochord (*nc.*) is cut through, and here the auditory capsules have recovered their inner wall; the end of the horizontal (*h.s.c.*) and the part leading to the common sinus of the anterior (*a.s.c.*) and posterior canals are cut across. The vestibule (*vb.*) is lessening here, where the back of the "sacculus" is shown. The hyomandibular is severed behind its head, and outside it the interopercular and subopercular are shown in section in the fold (*op.*). Here a ganglionic mass belonging to the 9th and 10th nerves (X.) is brought into view; and above, the parietals and one of the temporal series (*s.t.*) are severed.

Section 21.—The back of the auditory capsule, with the ampulla and hind part of the posterior canal (Plate 36, fig. 6, *p.s.c.*), are now severed; here the double passage for the 9th and 10th nerves and the nerves themselves (IX., X.) are seen; the bony laminæ are severed that are finding their way into the back of the auditory capsule; in the cavity of the ampulla, and from the foramen over the top of the oblique lower part of the occipital arch, there are parts of the spreading exoccipitals (*e.o.*). The opisthotic and epiotic are formed over the posterior canal *later* (see Plate 38).

The two halves of the investing mass, below, form a rest for the oblique plates of the upper part or arch, and are only slightly tilted upwards outside; they are thick inside and flattish on the outside, and have the notochord (*nc.*) between them; this rod is, here, ensheathed in bone, and this thick bony sheath has sent a wing, right and left, over the basal part of the moieties of the investing mass, which are curling over their external edge, as far as to the edges of the underlying parasphenoid (*pa.s.*); this ectosteal growth is the basioccipital (*b.o.*). The top of two of the branchial arches (*p.br.*), the parietal (at its end), and a supra-temporal bone are also seen. In this section the occipital arch is seen to be two-winged, right and left; in the last stage (Plate 33, figs. 12, 13, *iv.*) the section, here, was like an hour-glass.

Section 22.—The roof is now (Plate 36, fig. 7, *s.o.*) very thick, and here the vagus nerve only (X.) is seen with its ganglion; the back wall of the auditory capsules nearly meet above; the basal plate (*iv.*), between the halves of which the osseous sheath of the notochord (*nc.*) is seen, with its right and left basioccipital wings (*b.o.*), is surmounted by the side plates of the arch (*e.o.*). Behind the horizontal canal the combined occipital arch and auditory capsules form large thick shoulders of cartilage; whilst, above, the arch has three roundish crests (*s.o.*). Here the basal plate (*iv.*) is thicker, and as in the last, the parasphenoid is corrugated; the wings of the basioccipital bone (*b.o.*) do not yet invest the lower face of the cartilage; the opercular (*op.*), and a post-temporal scute, are seen in section, as also some parts of the hinder branchial arches (*br.*), with their pectinate gills and grooving vessels. Here the peculiar *four-fold* nature of the occipital arch is well seen, the oblique sides resting on a projecting threshold, through which the notochord, with its bony sheath, runs.

Section 23.—The last of the sections (Plate 36, figs. 8, 9) figured is through the thinner hind edge of the occipital ring, which is lozenge-shaped, and somewhat winged, right and left, for the side-walls have the same obliquity as the halves of the archway above; here the threshold is not so wide as in the last section; it is narrowing towards the end of the projecting basioccipital (see Plate 34, figs. 1–3, *b.o.*).

The exoccipital ectosteal plate (*e.o.*) is seen inside the converging arch, the right and left plates nearly meet above, and there is no key-stone piece or supraoccipital bone in the rounded median part. Here the notochord (*nc.*) lies impacted between the basal plates and their ascending arch; it is the core of a strong basioccipital bone (*b.o.*), which strongly encloses it, the soft tissue spreading in radiating lobes in the thickening bone-substance. Laterally, the bone has spread so as to enclose the halves of the investing mass, and runs beyond these parts; it grows as a right and left sharp plate. Here the wide corrugated parasphenoid (*pa.s.*) is in two parts, for it is forked behind (see also Plate 34, fig. 2, *pa.s.*).

Sixth Stage.—Young Lepidosteus, 4 *inches* 5 *lines long.*

In this stage the dermal scutes are so well developed that they can be named and classified; I shall describe them first, and the endocranium afterwards.

Notwithstanding the size of this specimen it had still the remains of the embryonic suctorial disk at the end of the snout (Plate 37, figs. 1–3), forming a pad on the end of the premaxillaries (*px.*), and lying in a horizontal plane. The lower jaw just reaches this part, the disk, itself, overlapping it.

The rostral region of the head is twice as long as the cranial; the opercular bones (*op.*, *s.op.*), pass behind the projecting basioccipital (*b.o.*)

The mandibles are two-thirds the length of the head, and as in embryo Frogs, are articulated to the quadrate *in front of the eye-ball* (*e.*); in old Frogs the condyle of the quadrate may reach as far backwards as the opercular folds do in this Fish.

The bony scutes of the hinder part of the head and face do not differ much from those covering the body, except in size; but in the rostral region, both above and below, but especially below, many of the bony plates are styles of great length and tenuity; this is a specialization quite like that which is seen in the skulls of longirostral Birds, and in some extinct Sauropsida, *e.g.*, the *Ichthyosaurus.*

Most of the bones of the roof are not difficult to decipher, for the eye detects quickly the parietals, frontals, and squamosals (Plate 37, fig. 1, *p.*, *f.*, *sq.*); but nearly the whole extent of the rostrum has to be traversed before we reach the true nasals. These bones (*n.*) are small crescentic scutes that cover the small, distal olfactory capsules (*ol.*) But along the top of the rostrum, from the ethmoidal region, where the frontals diverge nearly to the nasal roofs, two long, narrow styles of bone are seen; these I propose to call "ethmo-nasals" (*et.n.*); they are manifestly separate centres that correspond to the elongated hinder part of the nasals of a Bird.

The olfactory sac, in both Ganoids and Teleosteans, is devoid of a proper paraneural roof, and the bone covering it is merely one of the many "slime-bones" seen in the skulls of these Fishes; still, that scute which directly covers the olfactory organ has the first claim to be called the nasal. The frontals (*f.*) in their foremost third are divarcated and styloid, embracing the hind end of the ethmo-nasals (*et.n.*); they are wide where they meet over the antorbital region, become pinched up to their hinder fourth, and then widen most where they are overlapped by the parietals. These latter bones (fig. 2, *p.*) are large and oblong, covering the skull well from the middle of the eye-balls to the back of the ear capsule; they are flanked and overlapped by the temporal series.

The principal temporal bone is the squamosal (*sq.*); it is a long and irregular triangle, with its sharp end foremost; its broad end is overlapped by the second large temporal (*s.t.*), which covers the hinder part of the parietal as a rounded scale. Under it there is a lesser pair, and under these upper, larger bones, there is a considerable patch of small scutes margined by the circumorbital series in front, and the angulated interopercular (*i.op.*), behind. The circumorbital series is a very perfect ring of small scutes round the eye-socket; of these the antero-superior scales are the largest. A short chain of three or four small scutes runs forwards from the *superorbital* part of the ring; these may be called *preorbitals* (*p.ob.*); they are tilted up and form a sort of "eave" to the large convex coronoid part of the mandible. There, indeed, in front of the eye-balls, the skull is pinched inwards, and set, as in a vice, between the high hinder part of the lower jaw, whose steep, almost vertical, hind margin chafes, so to speak, right and left, against each circumorbital ring (Plate 37, fig. 1).

Below the tilted preorbitals there is another short, feeble chain of three or four scutes; the last but one of these (*m.x″.*) is as long as the others together, and has all the relations of the free part of the edentulous "os mystacum," or maxillary of typical Teleosteans; the little scute behind it (*j.*) shows the same relations as the small malar (jugal) of many Teleostei.

Outside of and protecting the sub-marginal row of mucous glands, there is a long chain of bones (see Plate 32, fig. 5, *m.c.g.*, and Plate 37, figs. 1, 2, *m.x′.*); this series of scutes is the continuation of the *mystaceum* series (*m.x″.*, *j.*) but thrice their width; this may be called the *maxillary chain.* This is composed of about fourteen or fifteen very similar scutes; they are oblong, their width being about half their length. In front of these the small premaxillaries (*px.*) are seen to be distinct, right and left. Each moiety (or centre) is pointed in front, has a small palatine plate and a dentary edge with sharp teeth; these rows of teeth (fig. 3, *px.*) meet in front at an acute angle.*

Behind the palatine plate of the premaxillaries, right and left, there is a long bone in close contact with its fellow of the opposite side, and so slender that the two together are not so wide as a single ethmo-nasal (fig. 2, *et.n.*); these "needles" are the vomers (fig. 3, *v.*); they become covered with a very fine rasp of teeth, and are nearly half the length of the entire skull.

Bounding these, along the palatal face of the rostrum, there is a pair of bones one seventh longer than the vomers, and twice as wide; these are the "parosteal palatines" (*pa′.*). These bones become invested with a rasp of teeth a degree coarser than that on the vomers.

In the long valley between these palatine splints and the maxillary chain there is a row of *large* sharp teeth, and on the edge of the chain an outermost row of *small* sharp teeth. A very long carinate, trough-like bone runs over the hind part of the two vomers for a considerable distance, and then extends to the end of the skull; on escaping from them it appears also rough, with a fine rasp of teeth. Further back these teeth cease, but the bone is carinate up to the basi-pterygoid (*b.pg.*); this is the parasphenoid (Plate 37, fig. 3, *pa.s.*) This bone is wider in the ethmoidal than in the post-orbital region, but it then widens to thrice its breadth even in the ethmoidal region. It flanks the basipterygoids (*b.pg.*) with a pair of small wings, is spread out under the auditory capsules (*au.*), and applies itself as a forked splint to the under face of the basioccipital (*b.o.*). Behind, for three-fifths of their length, the palatine splints (*pa′.*) are bound on their inner edge by a larger, but similar bone; this is the pterygoid (fig. 3, *pg.*); both these bones are seen in their relation to the suspensorium in other figures (Plate 37, fig. 4, and Plate 38, fig. 5, *pa′.*, *pg.*). The pterygoids acquire a fine bony rasp when they lie close to the palatal surface. Each bone is a long style in front, and then widens gradually so as to become a broad spatula in the orbital region.

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* In an old specimen I find a flat sub-arcuate scute binding across in front of the distant premaxillaries; this latter bone might be thought to be an edentulous azygous premaxillary and the two next behind it, not premaxillaries, but the foremost of the maxillary chain; I incline to call it a prenasal.

The styloid palato-pterygoid cartilage is applied to its outer face above, but the bone passes backwards, and invests three-fourths of the inner face of the broad suspensorium (see Plate 38, *pg.*), ending behind, with a thin, rounded margin.

When these parts are removed from the rest of the skull (Plate 37, fig. 4, and Plate 38, fig. 5) their parosteal relation to the prognathous suspensorium is clearly seen.

Over the edge of the suspensorium, in its broadest part, there is a *third* parostosis; this is the mesopterygoid (Plate 37, fig. 4, and Plate 38, fig. 5, *ms.pg.*). It is a thin, falcate bone above, one-sixth the length, and one-sixth the width, of the pterygoid.

A *fourth* splint is applied to the suspensorium, and this, like the last, is extremely small as compared with what is seen in the Teleostei; this is the preoperculum (*p.op.*); this bone is falcate, narrow, gently curved downwards, pointed behind, where it lies on the interopercular, and roughly notched in front, where it binds on the outside of the quadrate bone (*q.*); it is only one-third the length and one-third the width of the succeeding bone—the interopercular (*i.op.*); whereas in the Teleostei it is much the larger bone, as a rule.

The free part of the lower jaw, or mandible, is of great length, and the dentary bone covers it from end to end (Plate 37, fig. 4, *d.*); on the inside (Plate 38, fig. 5, *d.*) it is only seen at the edges of the jaw. Under its upper edge, on the inside, a much smaller bone, three-fourths its length, binds on the upper edge of the cartilaginous axis (*mk.*); this is the splenial (*spl.*).

Behind the splenial, on the inner side (Plate 38, fig. 5 *cr.*) the coronoid is seen as a pedate tract of bone with its heel behind; it binds on the inside of the fore part of the large cartilaginous coronoid (*cr.c.*). Under the short angular process of the articular cartilage there is a small angulare (*ag.*), and outside the large, ear-shaped coronoid cartilage, on its convex face, there is a considerable scale of an oval shape, and placed obliquely forwards and upwards; this is the supra-angulare (Plate 37, fig. 4, *s.ag.*).

The specialised "scutes" just described belong to the mandibular arch; those next to be noticed belong to the hyoid. On the knob of the hyomandibular a large oval scute is articulated by its capped fore end: this is the operculare (Plate 37, fig. 1, *op.*); below this a similar bone is seen, but of an uncinate or semi-crescentic form, with its sharp end behind, and its upper edge inside the operculare; this is the sub-operculare (*s.op.*).

Binding on the fore edge of the two last, we see an angulated sub-crescentic bone, with its concave edge above, its angulated margin below, and its front point binding under the hind point of the preoperculare. This is the interoperculare (Plate 37, fig. 4, and Plate 38, fig. 5, *i.op.*). These three bones are the hyomandibular splints; those now to be described belong to the cerato-hyal; these are the branchiostegals: they are beneath and within the great operculum (Plate 37, fig. 1, *br.s.*), and are attached in front to the cerato-hyal. Generally *seven* in number in the Teleostei, there are only *three* here, as in the Carp. These bones are narrow and falcate, with their concave margin above; the uppermost is the largest; the lower is the least.

The remainder of the bones which I have to treat of are *intrinsic* centres, or "ectostoses;" they will come under notice, now, in a description of the endocranium, most of which, however, is cartilaginous.

The endocranium at this stage differs but little from that of the adult, in which, although the bony centres become dense and relatively larger, are yet not altered, either in their number or relations, to any appreciable degree.

In this skull, the "prenasal rostrum," or intertrabecula, is as much developed as in the most specialised of the Selachians—namely, the "Pristidæ," or *Saw-fishes*—much more than in the ordinary Skate ("Raiidæ"). Here the length of this precranial region is, as compared with the cranial cavity, as 14 to 5, or nearly three times as long. In an old specimen the cranial cavity is only 2 inches long, and the whole skull 12½ inches, or 2 to 10½; the brain has, relatively, retreated. Measured from the quadrate condyle, in this young stage, we get the same proportion as the measurement of the precranial to the cranial; in the old the prequadrate region is 9 inches long, and the post-quadrate 3½ inches.

This remarkable pyriform, long-stalked skull owes its greatest expansion to the superaddition behind of the large ovoidal auditory capsules, and next to them to the greater size, in the young, of the mid brain; it soon narrows in over the small hemispheres. As in the last stage, the roof of the mid brain is membranous—this is the large circular fontanelle (Plate 38, fig. 1, *fo.*), the margins of which are very moderate bands of cartilage—the postorbital part of the superorbital bands (see Plate 30, fig. 8, *so.b.*).

Below (Plate 38, fig. 3, *py.*, *pa.s.*) there is a rather large and lanceolate pituitary fontanelle; and inside the orbits (Plate 38, fig. 2, *o.s.f.*) there are the long "orbito-sphenoidal fenestræ." The upper fontanelle (*fo.*) takes up about a third of the roof, but it is not so long as either the fore or the hind part of the tegmen cranii (*t.cr.*). Thus although this is rather a well-developed chondrocranium it has four large membranous deficiencies in it.

The basioccipital (Plate 38, figs. 1–3, *bo.*) might be taken for the centrum of the first vertebra—it projects so far behind the exoccipitals (*e.o.*). The bony matter, which did occupy much of the sheath of the cranial notochord, is now mainly confined to its hind part, and forms a four-sided mass; this mass does not run forwards into the rest of the notochord—at least on its upper surface (Plate 9, fig. 3, *nc.*).

This is a very feeble rudiment of the thick and high wall, which is developed in this part in most of the "Amniota," where it runs up in the deep fissure under the mid brain. However, even here it divides the basi-cranial fontanelle into two parts, a large anterior (*py.*) and a small posterior space (*p.b.c.f.*).

Thus it is evident that in this, as in other kinds of Ichthyopsida, the basis-cranii is much less affected by the mesocephalic flexure than it is in the Sauropsida and Mammalia.

The main pituitary space (Plate 38, fig. 3, *py.*) is lessened by the ingrowth of the trabeculæ (*tr.*); but in front, it is filled in by the hind part of the long intertrabecula. The trabeculæ are bowed out right and left, between the 5th and 2nd nerves (V., II.); the 1st nerve (I.) escapes from the front of the enclosed end of the cranium, and runs all the distance to the nasal sacs close to the sides of the intertrabecula (*i.tr.*)

The paired trabeculæ (*tr.*) do not end where the skull has closed in; in front of the narrowed tegmen cranii (Plate 38, figs. 1, 2, *t.cr.*) the intertrabecula is seen to be narrow above, and to have narrow wings running along its sides.

These wings soon dilate, so as to give the rostral part of the skull an oval widening along the front *two-fifths* of its *hinder fifth*. These parts are the cornua trabeculæ, and although they are so short now, they were (Plate 30), once, the main part of the skull in front, and for some time came little short of the end of the snout. Now, they are like the right and left sides of a lanceolate leaf, with a huge mid-rib; only their terminal point is free, and the 1st nerve runs in a groove between them and the long rostrum.

The rostrum (*i.tr.*) is very uniform up to near the front end; it then becomes slightly alate before ending in a blunt and somewhat decurved point (*p.n.*); its section is nearly oval, the thicker end below.

The suspensorium (Plate 37, fig. 4, and Plate 38, fig. 5) retains the form it had in the last stage (Plate 34), but it is twice as large, and its bony centres are now perfect. The upper bone is the metapterygoid (*mt.pg.*), it occupies the neck of the suspensorium, leaving cartilage, however, on the concave articular facet—for the basipterygoid—and also on the short round "trochanter," below the joint; this spur is the arrested otic process (*ot.p.*).*

The quadrate (*q.*) is a bony quadrant running, at its angle, close to the articular condyle (*q.c.*); this latter is an elegant convexo-concave trochlea, with its largest convexity on the outside. The main part of the body of the suspensorium is unossified; it is a large oblong tract, with its postero-inferior angle rounded off; it is rather hollow outside and convex within, where it is invested by the pterygoid bone (*pg.*).

The pterygo-palatine rod (*p.pg.*) is unaltered since the last stage; it never ossifies, and reaches as far forwards as the cornua trabeculæ (*c.tr.*).

The articulo-Meckelian rod (*mk.*, *ar.*) has increased in size (both actually and relatively); its intrinsic centre also, the articulare, has become two separate points of bone (Plate 38, fig. 5, *ar.*). The rest of this long, subarcuate, terete rod (*mk.*), runs along the grooved inner face of the dentary (*d.*) nearly to its distal end. The coronoid crest (*cr.c.*) is a very large "ear" of cartilage; it is convex outside and hollow within; its fore part is a free lobe.

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* In Plate 38, fig. 5, below, for *pa.* read *pd.*

The hyomandibular (Plate 37, fig. 4, and Plate 38, fig. 5, *hm.*) is a remarkable bar, about half the size of its "serial homologue"—the suspensorium. Its arched, extended head is a convexo-concave condyle for articulation under the horizontal canal; behind this there is a cartilaginous knob for the opercular bone. The bony shaft is short, pinched in the middle, and has an oval fenestra near its front third. Below the shaft it swells out into a solid bilobate mass, the lesser lobe being behind. In front of the fore lobe, on the inside, there is a concavity for the inter-hyal (*i.hy.*). The bar from that point becomes the small sigmoid symplectic (*sy.*); it is bent downwards suddenly, and then runs straight forwards to lie along the inside of the hinder third of the lower edge of the suspensorium.

Its bony shaft occupies its hinder two-thirds; where it becomes straight, there it has a small bony elbow; its fore end is a blunt point (Plate 38, fig. 5, *sy.*).

The inter-hyal (*i.hy.*) is a small pyriform cartilage, its narrow end fits into the concavity in the hyomandibular, and its broad end has a cup on its inner side for the head of the cerato-hyal (*c.hy.*).

The latter segment has a "trochanter" behind its small rounded head; its shoulder is ossified as a separate epi-hyal (*e.hy.*); the main shaft (*c.hy.*) has its own centre; it is narrower in the middle, and is only separated from the stylo-hyal by a tract of cartilage.

The rounded lower end of the cerato-hyal fits into the oblique shallow cup of the sub-globular hypo-hyal (*h.hy.*); this short segment is ossified on its outer face; at present, at any rate it has no second centre, as in the Teleostei; but in these, as in *Acipenser*, it is completely segmented off from the cerato-hyal.

The basal piece (*b.hy.*) is a large "inter-glossal" plate as long as all these three segments above it; it is oblong, rather pinched in the middle, emarginate in front, thickish, and somewhat fibro-cartilaginous, having cross-bands and reticulating, connective fibres, wrought into it on its upper surface, and its hyaline cartilage somewhat softer than in the other parts.

The basal piece of the branchial system, and part of the first part of hypobranchials (*b.br.*, *h.br′.*) are figured. For the rest, I must refer to the figures and descriptions of the last stage; these parts have not altered in any important degree —except in size.

Comparison with other types, and Summary.

As soon as the primordial cranium becomes sufficiently differentiated—as hyaline cartilage—to be distinguished from the rest of the cephalic mesoblast, we find a peculiarly simple foundation for all the aftergrowths. In Stage 1, in embryos 10½ m.m. long, nearly all the parts of the chondrocranium—including in this term the visceral arches—are present; the hinder arches become broken up, afterwards, but the two first, and largest, the mandible and hyoid, are already as much segmented as they will be in the adult.

The skull-floor, only, is developed, as yet, and the rostral part, in front, is not chondrified, but its outlines can be traced, and the roof and walls of the skull are merely developments from the basal bands.

Those bands in this type lend no support to the theory of the visceral (or ventral) nature of the *pro*-chordal tracts or *trabeculæ;* they are, manifestly, mere continuations of the undivided *para*-chordal cartilages, which expand and contract in relation to the parts around and over them. They diverge from the front third of the notochord, as though their relation to it was not intimate, and show—for a long while at least—no tendency to grow up, with that axis, into the hollow of the mid brain.

I see nothing in this lyriform basal skeleton of the skull but an undivided *basineural* structure comparable to, and a *primary cephalic variation* of, the tracts that form the paired rudiments of the *neuro-central* cartilages of the spine. The cessation at the end of the notochord (mesially), and close behind the oral opening, laterally, of the hypo-blastic layer, causes all the pre-oral and pre-pituitary parts to be, in a sense, imperfect; they are developed as porches and outworks to the full and complete structure further back, but this does not destroy their homology, nor break their continuity with the parts formed from their own embryonic layer, of which they are the direct ongrowths.

Yet all parts growing out,—forwards, upwards, or downwards,—in front of the perfect axis, which ends close in front of the infundibulum, must be very cautiously named as "serial homologues" of the perfect base and its upper and lower arched growths; they are probably mere *outgrowths;* at most they are only *rudiments.*

The *primary* trabeculæ are merely direct *on*-growths of the parachordals; the cornua trabeculæ are *out*-growths of the trabeculæ.

The intermediate element, or intertrabecula, is a fresh outbreak, so to speak, of the median mesoblast of the axis, which is tubular, behind, where it encloses the notochord, but, re-appearing in front, beyond it, it shoots forth as a *solid process* of the skeletal axis.

Close to the fore end of the primary trabeculæ there arises a similar but rather smaller bar, and the two parts are so close together that they chondrify continuously; these side bars are the palatine cartilages, evidently rudimentary structures.

Here they are not distinct from the long spur (pterygoid cartilage), which shoots forwards from the dorsal element (suspensorium) of the mandibular arch; this is like what we see in the Tadpole, but unlike that which is found in Skates, Teleosteans, and Urodeles. Thus, with the palatine included, the suspensorium here is a *palato-quadrate;* in the Skate, Teleostean, and Urodele the suspensorium is a *pterygo-*

quadrate. As in the Tadpole, the fore end of this bar is fixed; as in the Teleostean Urodele, and adult Frog, the hind part, or pedicle, is free.

As in the Tadpole, the suspensorium is sub-parallel with the axis of the skull, and the free mandible (MECKEL'S cartilage) grows forwards and inwards; that condition is temporary in the Batrachian, it is permanent in *Lepidosteus.*

As to the development of the basal bands of the skull, this type agrees with the Selachians and Teleosteans (*Salmo*) in the synchronism of the *para-* and *pro-*chordal tracts; but in Batrachians, Urodeles, and Marsipobranchs, the *trabeculæ are developed first;* they embrace the fore end of the notochord closely, and are both *para-* and *pro-*chordal; afterwards the hinder parachordal region is chondrified, separately in Urodeles, and continuously in Batrachians and Marsipobranchs.

The development of the complex hyoid arch is very different in this *Holostean* Ganoid, and in the *Chondrostean* Sturgeon, from what we find in Teleosteans, Batrachians, and Urodeles.

In the Salmon the primary bar breaks up into two long bands, with a short segment below; the foremost is the larger, retains its connexion with the ear-capsule, widens above as the hyomandibular, and narrows, antero-inferiorly, as the symplectic region.

The narrower, hind band becomes postero-inferior in position, keeps the small distal segment, and acquires a new, small segment, above, by which it becomes attached to the space between the hyomandibular and symplectic; the *late,* small upper segment is the inter-hyal, the long bar the epi-ceratohyal, and the short, distal segment the hypo-hyal.

In the lowest Urodele, *Proteus,* the hyoid arch is composed of two massive segments, one short, the upper or hyomandibular, and the other, the long, lower bar, the cerato-hyal; this is like that which obtains in Sharks.

In the larger Urodeles (*Menopoma, Cryptobranchus*) there is but little difference (apparently) in the time of development of the segments, but the upper part breaks up into two segments corresponding to the hyomandibular and symplectic segments in the Sturgeon; the cerato-hyal is large, and the hypo-hyal breaks up into three pieces.

In many of the Caducibranchiate Urodeles, and in some kinds of Anura (*Salamandra, Triton, Pseudophryne, Bombinator*), all but the uppermost part of the hyoid arch is suppressed; but in the Batrachia, generally, it is developed, as *two, three,* or even *four* segments; these, with the exception of the uppermost, as a rule, do not appear until two or three months after transformation, and are only developed in the Tadpole in rare cases, as in *Pseudis* and *Pipa.*

In *Lepidosteus* and in *Acipenser* the formation of the segments of the hyoid arch takes place at once during chondrification; *Lepidosteus* has the same number of cartilages as the Teleostei, but *Acipenser* has a distinct symplectic piece—a kind of segmentation which is not equivalent to the subdivision of the upper part of a branchial arch into a *pharyngo-* and an *epi-*branchial, but the epi-hyal is segmented at its distal fourth.

The basi-hyal of *Lepidosteus* is remarkable for being very long and essentially double. There are only four perfect and one imperfect branchial arches; the auditory capsules, at first, are as distinct as in the Tadpole, their basal region remains membranous for a good while, as in the Salmon.

In embryos two-thirds of an inch in length, more than one-half larger than our first stage, the chondrocranium is larger and stronger, but has few fresh things in it.

The trabeculæ and palato-quadrate cartilages are still confluent, but the former are now some distance apart, the binding cells of the former stage being converted into a pyriform mass of true cartilage, with its broad end in front and projecting beyond the paired bands or trabeculæ; this is the intertrabecula. The pedicle of the suspensorium has applied the inner side of its apex to the most curved part of the trabecula, and an oblong joint is forming.

A spike of cartilage has grown forwards from the auditory capsule over the hinder part of the superorbital region; this structure is seen temporarily in large larvæ of *Triton*, and permanently in *Siren.*

The divergence of the basal bands is now at its fullest, and the apex of the cranial notochord—one-third of the rod—is twisted, curves a little upwards, and is far from the moieties of the investing mass.

In young *Lepidostei,* one-half larger than the last (1 inch long), the chondrocranium may be said to be complete, and free from intrinsic ossification, except in the sheath of the notochord, the cerato-hyal, and some parts of the branchial arches.

The whole structure is much longer, but most of the increase in length is due to the development of the three basal cartilages in front of the cranial cavity. The occipital arch is perfect, and the tegmen from it runs well forward.

The superorbital band is now perfect, and in front it passes into an anterior tegmen round the olfactory lobes, and the hemispheres, thus the cranial box is perfect there. But there is a large pyriform fontanelle below, a larger oval fontanelle on each side in the orbital region, and a still larger membranous space, the great fontanelle, above.

The sudden and immense development of the precranial bars in so short a time is very remarkable; their relative massiveness makes this skull like that of a young Sturgeon five or six times as large. In that type the solid rostrum is formed of the two large trabecular cornua, which flank the still larger intertrabecula, like decurrent leaves. In the Sturgeon there is an antorbital expansion of the lateral ethmoidal region at the end of the rostrum, and each olfactory capsule lies close in front of the antorbital wall, as in a crypt. But in *Lepidosteus* the two capsules are carried away to near the end of the snout, and have no cartilage near them except the rostral bar, on each side of which they lie.

Whilst the fore part of the chrondrocranium is like that of a young Sturgeon 5 or 6 inches long, the cranium proper is like that of a Salmon ten or twelve days after hatching, when its length agrees with that of this stage of *Lepidosteus,* namely, about 1 inch.

In the Salmon "fry" there is a hinder and a front tegmen cranii, a pair of superorbital bands running from the auditory capsules to the anterior tegmen, a largely open roof between, open orbito-sphenoidal spaces, and an open pituitary fontanelle. Moreover, this is the "norma" according to which the skull of *Polypterus* is formed; but of course during growth it becomes more solid, and partly ossified.

The suspensorium suggests a very mixed relationship in this type; it runs forwards, parallel with the skull, as in the Tadpole, but its pedicle is now well articulated with a basipterygoid process as in the metamorphosed Frog, and more clearly than in that type prefigures the cranio-facial relation of the Sauropsida, where, as in Lizards and many Birds, the pterygoid portion of the suspensorial apparatus articulates with a basipterygoid outgrowth of the skull. The open orbito-sphenoidal spaces are seen again in Batrachians, *e.g.*, in *Acris Pickeringii* and *Rappia bicolor.*

The palatine portion of the suspensorium (or palato-quadrate cartilage) loses its ethmoidal conjugation, but retains its continuity with the pterygoid cartilage. The primarily and permanently separate palatine of the "Siluroids" runs forwards in the same manner, with no ethmo-palatine joint, such as is seen in the Salmon.

In adult Batrachians of the genus *Bufo* the continuity of the palatine cartilage is lost both with the ethmoid and the pterygoid cartilage, but it articulates with the former by a raised process as in the Salmon. The small size of the hyomandibular of *Lepidosteus*, and its distance from the mandibular pier, prepare us for the transformations of that part in the Batrachia where it becomes the "columella."

In a further stage, when it has doubled the size it had in the last instance—has become 2 inches long—the young *Lepidosteus* has fairly attained to its own characteristic type of skull, and most of the very limited osseus centres are now apparent. All the fore face is now greatly drawn out, twice as much as in the last, and the suspensorium, mandible, and lingual cartilages, have shot on forwards in like manner. The membranous spaces are only different from the last by the upper fontanelle being relatively less, and neatly circular, whilst the lower space is being divided by a *late* and *feeble* "post-pituitary" bar with a large anterior, and a small posterior space. But the type of skull seen in the young "fry" of the Salmon, and in such minute arrested Frogs as the Nearctic *Acris Pickeringii*, and the Australian *Rappia bicolor* and *Camariolius tasmaniensis*, is not departed from, nor, indeed, will be.

But even what is seen in young *Lepidostei* 2 *inches* long scarcely prepares us for what we find in specimens a little more than twice that size. At this stage, as in Sawfishes (*Pristis*), this prenasal cartilage (intertrabecula) has become *three times* as long as the whole cranial cavity, and *six times* as long as its associated cornua trabeculæ—now mere delicate leafy appendages to its base. The cranium proper has not altered in any important degree since the last stage, but the bony centres are nearly all there; all those that are seen in the Salmon, or in Teleostei, generally are found, with the exception of the super-occipital and a bone not found, I believe, in the Ganoids, namely, the "pterotic;" its suppression is correlated with the development of a special temporal scute over the same region; the "squamosal," a bone only, exceptionally present in the Teleostei among the Siluroids.

The post-pituitary band of cartilage, a feeble promise of the solid post-clinoid wall of the "Amniota," is now complete. Some things characterising this type are now well seen, namely, the small lateral ethmoidal bone in the thin, closing-in, skull wall, without any prefrontal (ali-ethmoidal) wing, the double "articular" centre, and the huge coronoid crest to the articulo-Meckelian rod; these two latter characters are also seen in *Amia calva*, as shown by Professor BRIDGE.

The very small size of the preoperculars, the form and size of the interopercular, which here so strongly resembles the preopercular of the Teleostei, are very noticeable in this skull, as also the long chain of bones interspersed between the "os mystaceum" or edentulous maxillary and the premaxillary.

The adult condition of the skull in *Polypterus* and *Amia* (TRAQUAIR, Jour. of Anat. and Phys., vol. 5, plate 6, pp. 166–182; and BRIDGE, ibid., vol. 9, plate 23, pp. 605–622) presents so many things, both in likeness and contrast, that they must be noticed in conclusion.

In *Polypterus*, as in *Lepidosteus*, the four fontanelles are permanently open; the basioccipital projects far beyond the oblique foramen magnum, and the occipital bone is single, made up evidently of a basal and two lateral pieces, without a supraoccipital; then there are no pterotics, and the olfactory capsules are sub-terminal. But in this type there are no epiotics distinct from the opisthotics.

There is a large sphenotic bone, right and left, which takes up the antero-posterior sphenoidal regions and part of the lateral ethmoidal, besides which there is a pair of lateral ethmoidals which project outwards, and an ethmo-septal bone in front.

The metapterygoid in *Polypterus* is far from the skull, in which there are no basipterygoid processes; the palatine is a small ectosteal bone; the hyomandibular has only one centre; and the preopercular is continuous with the squamosal, as in the Amphibia; there is no interopercular.

But *Amia calva* has a skull which comes much nearer to that of *Lepidosteus* in several respects, but the lateral and inferior fontanelles are filled in, in this solid skull, which comes nearer that of the Physostomous Teleosteans.

The basioccipital projects behind the oblique foramen magnum; there is no supraoccipital, nor any pterotics, and the epiotics are distinct from the opisthotics.

It has a pair of bones which are not seen in *Lepidosteus*, namely, the orbitosphenoids; and its so-called prefrontals or lateral ethmoids project, as in the Teleostei. It has a distinct pedicle to the suspensorium, capped with cartilage, but not forming a definite joint with any distinct basipterygoid.

Its palatine cartilage is ossified both endosteally and ectosteally; and the whole palato-pterygoid is almost Teleostean in its solidity.

There is a large coronoid crest, and there are *two* articular bones on each side, as in *Lepidosteus*. There is a cartilaginous inter-hyal, articulated between a distinct hyomandibular and symplectic; and the upper or styloid end of the cerato-hyal has, as in Osseous Fishes, a separate centre, whilst the hypo-hyal has *only one*, as in *Lepidosteus;* in Osseous Fishes it has *two*.

There are the four normal operculars; the "os mystaceum" is dentigerous and carries a jugal in its hinder half. Above the skull, the scutes, which seem to me to be a little misunderstood by Professor BRIDGE, correspond, in essentials, to those of *Lepidosteus.*

I should propose the term "azygous parietal" for his *dermo*-superoccipital; "squamosals" for his *parietals;* and "prerostral" for the azygous transverse bone, which, as in *Lepidosteus*, furnishes the snout in front, and which is called by him *ethmoid.* The olfactory capsules are sub-terminal, and the large *nasals*, which cover them by their notched fore margin, are evidently the nasals and "ethmo-nasals" of *Lepidosteus*, in one piece, right and left.

Amia is a true Ganoid, and it has several unmistakable diagnostics even in its skull; but it comes very near to the Physostomous Teleosteans.

DESCRIPTION OF THE PLATES.

Plate.	Fig.	Stage.		Number of times magnified.
30	1	1	Section of the head of an embryo of *Lepidosteus* 4 lines long	30
„	2	1	A similar section of an embryo 4½ lines long . .	30
„	3	2	Chondrocranium of an embryo 5 lines long; lower view.	36
„	4	2	Section of the head of an embryo 5½ lines long .	30
„	5	3	Section of the head of an embryo 7½ lines long .	22½
„	6		The other half of the same head	22½
„	7	3	Chondrocranium of an embryo ⅝-inch long; lower view.	30
„	8	3	The same skull; upper view	30
„	9	3	Head of same embryo; lower view	15
31	1–15	3	A series of vertically transverse sections through the head of an embryo 7¼ lines long	30
32	1	4	Chondrocranium of an embryo 1 inch long (nearly). Upper view	24
„	2	4	The same object; lower view	24

Plate.	Fig.	Stage.		Number of times magnified.
32	3	4	Part of basis-cranii of same skull; upper view. .	36
„	4	4	Section of the head of an embryo of the same stage .	24
„	5	4	Upper view of head of same stage. Seen partly as a transparent object	24
„	6	4	Part of hyoid arch of same	24
33	1–13	4	A series of vertically transverse section of an embryo, 1 inch long (nearly)	27
„	14–16	5	First three of a series of vertically transverse sections of a young *Lepidosteus* 2½ inches long .	20
34	1	5	Dissection of skull of young *Lepidosteus*, 2 inches long; chondrocranium with some of the parostoses attached; upper view	12
„	2	5	The same object; lower view	12
„	3	5	Part of basis cranii of same skull; upper view . .	15
„	4	5	Inferior arches of the same skull	12
35	1–12	5	4th to 15th of a series of vertically transverse sections of a young *Lepidosteus*, 2½ inches long.	20
36	1–8	5	16th to 23rd of a series of vertically transverse sections of a young *Lepidosteus*, 2½ inches long.	20
„	9	5	Part of fig. 8	60
37	1	6	Skull of a young *Lepidosteus* 4 inches 5 lines long, with superficial bones *in situ*; side view . . .	6
„	2	6	The same skull; upper view.	6
„	3	6	The same; lower view.	6
„	4	6	The inferior arches of the same skull; outer view .	7½
38	1	6	Chondrocranium of a young *Lepidosteus* 4 inches 5 lines long; upper view	6¾
„	2	6	Hind part of same; lower view	6¾
„	3	6	Part of basis-cranii of same; upper view . . .	13½
„	4	6	Part of chondrocranium of same; upper view . .	13½
„	5	6	Inferior arches of the same skull; inner view . .	7½

LIST OF ABBREVIATIONS.

The Roman figures indicate nerves or their foramina.

ag.	Angular.
al.s.	Alisphenoid.
ar.	Articular.
ar.c.	Articular cartilage.
a.s.c.	Anterior semicircular canal.
au.	Auditory capsule.
au.f.	Auditory fenestra.
b.a.	Basilar artery.
b.br.	Basi-branchial.
b.hy.	Basi-hyal.
b.o.	Basioccipital.
b.pg.	Basipterygoid.
C^1.	Fore brain.
C^2.	Mid brain.
C^3.	Hind brain.
c.br.	Cerato-branchial.
c.hy.	Cerato-hyal.
cl.	Cleft.
cr.	Coronoid.
cr.c.	Coronoid cartilage.
c.tr.	Cornua trabeculæ.
d.	Dentary.
e.	Eye.
e.br.	Epi-branchial.
e.hy.	Epi-hyal.
ep.	Epiotic.
et.n.	Ethmonasal.
f.	Frontal.
fo.	Fontanelle.
g.p. and *br.p.*	Gill processes.
h.	Heart.
h.br.	Hypo-branchial.
h.hy.	Hypo-hyal.
hm.	Hyomandibular.
hm.c.	Hyomandibular facet and condyle.
hm.f.	Hyomandibular fenestra.
h.s.c.	Horizontal semicircular canal.
i.hy.	Inter-hyal.
inf.	Infundibulum.
i.op.	Interopercular.
i.tr.	Intertrabecula.
j.	Jugal.
l.eth.	Lateral ethmoid.
m.	Mouth.
m.c.g. and *m.g.*	Mucous gland.
mk.	MECKEL's cartilage.
mn.	Mandible.
ms.pg.	Mesopterygoid.
mt.pg.	Metapterygoid.
mx'., *mx.''*	Maxillary.
my.	Myelon.
n.	Nasal.
nc.	Notochord.
ol.	Olfactory capsule.
op.	Opercular and opisthotic.
op.p.	Opercular process.
os.f.	Orbito-sphenoidal fenestra.
ot.p.	Otic process.
p.	Parietal.
pa'.	Superficial palatine.
pa.s.	Parasphenoid.
p.br.	Pharyngo-branchial.
p.cl.	Posterior clinoid.
pd.	Pedicle.
pg.	Pterygoid.
pnl.	Pineal gland.
p.ob.	Preorbital.
p.op.	Preopercular.
p.pg.	Palato-pterygoid.
p.px.	Palatine process of premaxillary.
pr.o.	Prootic.

pt.o.	Postorbital
px.	Premaxillary.
py.	Pituitary body and space.
q.	Quadrate.
q.c.	Quadrate condyle.
s.ag.	Supra-angular.
s.d.	Sucking disk.
spl.	Splenial.
s.ob.	Supraorbital.
s.ob.c. and *s.ob.*	Supraorbital cartilage.
s.op.	Subopercular.
sp.o.	Sphenotic.
s.t.	Supratemporal.
su.ob.	Suborbital.
sy.	Symplectic.
t.cr.	Tegmen cranii.
tr.	Trabecula.
v.	Vomer.
vb.	Vestibule.

IX. *On the Vibrations of a Vortex Ring, and the Action upon each other of Two Vortices in a Perfect Fluid.*

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THE following paper contains (1) a discussion of the vibrations which take place in the axis of the core of a vortex ring whose section is very small in comparison with its aperture when the axis is made to deviate slightly from the circular form; and (2) a discussion of the action upon each other of two vortex rings which move in such a way that they never approach nearer than a large multiple of the diameter of either.

The fluid in which these vortices exist is supposed to be frictionless and incompressible.

The method which I have employed is the same in both cases, and is purely kinematical. It is merely the application of the fact that if $F(x, y, z, t)=0$ be any equation to a surface which always consists of the same particles then

$$\frac{dF}{dt}+u\frac{dF}{dx}+v\frac{dF}{dy}+w\frac{dF}{dz}=0$$

where u, v, w are the velocities of the particle at (x, y, z) along the axis of x, y, z respectively, and where the differential coefficients are partial.

The surface of a vortex ring is evidently a surface of this kind, and the equation just written is the condition that $F(x, y, z, t)=0$ should be the equation to the surface of a vortex ring. I have found that this condition, joined to the ordinary expressions for the velocity due to a vortex element, is sufficient to solve the problems discussed in this paper. This is an instance of the large number of problems in vortex motion which are capable of purely kinematical solution; indeed, a vortex theory of gases would be entirely kinematical so long as we only considered the molecules of gas themselves and not their effects upon the containing vessel, &c. For example, in this theory when two atoms clash, the problem of finding their subsequent motion must be capable of solution by purely kinematical considerations, but in the ordinary theory of gases the "clash of atoms" involves dynamical considerations of very considerable complexity. This is a consequence of the vortex theory being of a much more fundamental character than the ordinary one that atoms consist of small pieces of solid matter.

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[*] Since the paper was sent into the Society it has been copied by the author with changes in the notation, introduced chiefly to facilitate the printing, but no change of any importance has been introduced into the substance of the paper.

Problem I. To find the vibrations of the circular axis of a vortex ring.

Using cylindrical coordinates let the equations to the axis of the core be

$$\rho = a + \Sigma\alpha_n \cos n\theta$$
$$z = \zeta + \Sigma\beta_n \cos n\theta$$

where α_n and β_n are small compared with a the radius of the core when undisturbed; the summation over all integer values of n between zero and infinity. The axis of z is perpendicular to the plane of the vortex, and θ is measured from the axis of x as initial line.

The velocity due to a distribution of vortices is proportional to the magnetic force produced by a system of currents arranged in exactly the same way as the vortices and of the same strength.

Now the vortex filaments we are considering are distributed uniformly (or very approximately so)* in a ring the radius of whose transverse section is very small in comparison with the radius of the aperture. Now if electric currents flow uniformly through a conductor of such a shape the magnetic action at a point outside or on the surface of the conductor is the same as if all the currents were condensed into one flowing along the axis.† Hence when finding the velocities outside the vortex ring we may suppose the vortices condensed into one at the axis of the core. If ω be the angular velocity of molecular rotation, e the radius of the transverse section of the core, then $\pi e^2\omega$ is the strength of the vortex which we must suppose placed at the axis of the core. We shall for brevity denote $\pi e^2\omega$ by m.

The components (u, v, w) of the velocity at the point (x, y, z) are given by

$$u = \frac{m}{2\pi}\int_0^{s'} \frac{1}{r^3}\left\{\frac{dz'}{ds'}(y'-y) - \frac{dy'}{ds'}(z'-z)\right\} ds'$$

$$v = \frac{m}{2\pi}\int_0^{s'} \frac{1}{r^3}\left\{\frac{dx'}{ds'}(z'-z) - \frac{dz'}{ds'}(x'-x)\right\} ds'$$

$$w = \frac{m}{2\pi}\int_0^{s'} \frac{1}{r^3}\left\{\frac{dy'}{ds'}(x'-x) - \frac{dx'}{ds'}(y'-y)\right\} ds'$$

where r is the distance of the point (x, y, z) from the point (x', y', z'), a point on the vortex whose polar coordinates are (ρ', θ); s' is an arc of the vortex ring.

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* See a note by Sir W. THOMSON at the end of HELMHOLTZ's paper on "Vortex Motion," Phil. Mag., 1867.

† MAXWELL's 'Electricity and Magnetism,' 2nd edition, §. 683.

We can determine the components of the velocity in terms of the quantities we have denoted by $A_0 \dots A_n$, we shall for the sake of clearness divide the determination up into several steps.

To determine the parts of u, v, w independent of α_n and β_n, in which we shall not suppose ζ small,

$$u=\frac{m}{2\pi}\int_0^{2\pi}\cos\theta\zeta\{A_0+A_1\cos(\theta-\psi)+\dots A_n\cos n(\theta-\psi)\}\,a\,d\theta$$

$$=\frac{ma}{2\pi}\zeta\int_0^{2\pi}A_1\cos^2\theta\cos\psi\,d\theta=\tfrac{1}{2}ma\zeta A_1\cdot\cos\psi \quad \dots\dots\dots\dots \quad (1)$$

$$v=\frac{m}{2\pi}\int_0^{2\pi}\sin\theta\zeta(A_0+A_1\cos(\theta-\psi)+\dots)\,a\,d\theta=\tfrac{1}{2}ma\zeta A_1\sin\psi \quad \dots\dots \quad (2)$$

$$w=\frac{m}{2\pi}\int_0^{2\pi}\{\cos\theta(a\cos\theta-\rho\cos\psi)+\sin\theta\,(a\sin\theta-\rho\sin\psi)\}(A_0+A_1\cos\overline{\theta-\psi}+\dots)\,a\,d\theta$$

$$=\frac{ma}{2\pi}\int_0^{2\pi}\{a-\rho\cos(\theta-\psi)\}(A_0+A_1\cos(\theta-\psi)+\dots)\,d\theta$$

$$=\frac{ma}{2\pi}(A_0a2\pi-A_1\rho\pi)$$

$$=\tfrac{1}{2}ma\{2A_0a-A_1\rho\} \quad \dots\dots\dots\dots\dots\dots \quad (3)$$

These are the velocities due to the undisturbed vortex, and in using them in the second half of the paper we require A_0, A_1 determined without supposing ζ to be small.

2nd. The values of u, v, w arising from small terms in ds'.

As far as now concerned,

$$ds'=d\theta\alpha_n\cos n\theta$$

$u=0$, $v=0$ because they involve $\zeta\alpha_n$.

$$w=\frac{m}{2\pi}\int_0^{2\pi}(a+\rho\cos(\theta-\psi))[A_0+A_1\cos(\theta-\psi)+\dots]\alpha_n\cos n\theta\,d\theta$$

$$=\frac{m}{2\pi}\int_0^{2\pi}[aA_n\cos n(\theta-\psi)-\tfrac{1}{2}\rho A_{n+1}\cos n(\theta-\psi)-\tfrac{1}{2}\rho A_{n-1}\cos n(\theta-\psi)]\alpha_n\cos n\theta\,d\theta$$

$$=\tfrac{1}{2}m\alpha_n[aA_n-\tfrac{1}{2}\rho(A_{n+1}+A_{n-1})]\cos n\psi$$

3rd. Small terms in $\frac{dx'}{ds'}, \frac{dy'}{ds'}, \frac{dz'}{ds'}$.

$$\begin{aligned}
u&=\frac{m}{2\pi}\int_0^{2\pi}-\frac{n\beta_n}{a}\sin n\theta(a\sin\theta-\rho\sin\psi)[\mathrm{A}_0+\mathrm{A}_1\cos(\theta-\psi)+\ldots]ad\theta\\
&=\frac{ma}{2\pi}\int_0^{2\pi}\tfrac{1}{2}u\beta_n(\cos(n+1)\theta-\cos(n-1)\theta)+\frac{n\beta_n}{a}\rho\sin\psi\sin n\theta[\mathrm{A}_0+\mathrm{A}_1\cos(\theta-\psi)]d\theta\\
&=\tfrac{1}{2}ma\left\{n\beta_n(\mathrm{A}_{n+1}\cos(n+1)\psi-\mathrm{A}_{n-1}\cos(n-1)\psi)+\frac{n\beta_n}{a}\rho\sin\psi\sin n\psi\mathrm{A}_n\right\}\\
&=\tfrac{1}{4}man\beta_n\left\{\cos(n+1)\psi\left(\mathrm{A}_{n+1}-\frac{\rho}{a}\mathrm{A}_n\right)-\cos(n-1)\psi\left(\mathrm{A}_{n-1}-\frac{\rho}{a}\mathrm{A}_n\right)\right\}
\end{aligned}$$

$$\begin{aligned}
v&=\frac{m}{2\pi}\int_0^{2\pi}n\beta_n\sin n\theta(a\cos\theta-\rho\cos\psi)[\mathrm{A}_0+\mathrm{A}_1\cos(\theta-\psi)-]ad\theta\\
&=\tfrac{1}{4}man\beta_n\left\{\sin(n+1)\psi\left(\mathrm{A}_{n+1}-\frac{\rho}{a}\mathrm{A}_n\right)+\sin(n-1)\psi\left(\mathrm{A}_{n-1}-\frac{\rho}{a}\mathrm{A}_n\right)\right\}
\end{aligned}$$

$$\begin{aligned}
w&=\frac{m}{2\pi}\int_0^{2\pi}\left\{-\frac{n\alpha_n}{a}\sin\theta\sin n\theta(a\cos\theta-\rho\cos\psi)\right.\\
&\qquad\qquad\left.+\frac{n\alpha_n}{a}\cos\theta\sin n\theta(a\sin\theta-\rho\sin\psi)\right\}(\mathrm{A}_0+\mathrm{A}_1\cos(\theta-\psi)-)ad\theta\\
&=\frac{m}{2\pi}n\alpha_n\int_0^{2\pi}\{\tfrac{1}{2}\rho\cos\psi(\cos(n-1)\theta-\cos(n+1)\theta)\\
&\qquad\qquad-\tfrac{1}{2}\rho\sin\psi(\sin(n+1)\theta+\sin(n-1)\theta)(\mathrm{A}_0+\mathrm{A}_1\cos(\theta-\psi))ad\theta\}\\
&=\tfrac{1}{2}mn\alpha_n[\tfrac{1}{2}\rho\cos\psi(\mathrm{A}_{n-1}\cos(n-1)\psi-\mathrm{A}_{n+1}\cos(n+1)\psi)\\
&\qquad\qquad-\tfrac{1}{2}\rho\sin\psi(\mathrm{A}_{n+1}\sin(n+1)\psi+\mathrm{A}_{n-1}\sin(n-1)\psi)]\\
&=\tfrac{1}{4}mn\alpha_n\rho(\mathrm{A}_{n-1}-\mathrm{A}_{n+1})\cos n\psi.
\end{aligned}$$

4th. Small terms in $x'-x$, $y'-y$.

$$\begin{aligned}
u&=-\frac{m}{2\pi}\int_0^{2\pi}a\cos\theta\cos n\theta\beta_n(\mathrm{A}_0+\mathrm{A}_1\cos(\theta-\psi)+\ldots)d\theta\\
&=-\frac{ma}{2\pi}\int_0^{2\pi}\tfrac{1}{2}\beta_n\{\cos(n-1)\theta+\cos(n+1)\theta\}(\mathrm{A}_0+\mathrm{A}_1\cos(\theta-\psi)+\ldots)d\theta\\
&=-\tfrac{1}{4}ma\beta_n(\mathrm{A}_{n-1}\cos(n-1)\psi+\mathrm{A}_{n+1}\cos(n+1)\psi)
\end{aligned}$$

$$\begin{aligned}
v&=\frac{m}{2\pi}\int_0^{2\pi}-a\sin\theta\cos n\theta\beta_n(\mathrm{A}_0+\mathrm{A}_1\cos(\theta-\psi)+\ldots)d\theta\\
&=\tfrac{1}{4}ma\beta_n(\mathrm{A}_{n-1}\sin(n-1)\psi-\mathrm{A}_{n+1}\sin(n+1)\psi)
\end{aligned}$$

$$\begin{aligned}
w&=\frac{m}{2\pi}\int_0^{2\pi}(\cos\theta(\alpha_n\cos n\theta\cos\theta)+\sin\theta(\alpha_n\cos n\theta\sin\theta)\}(\mathrm{A}_0+\mathrm{A}_1\cos(\theta-\psi))ad\theta\\
&=\frac{m}{2\pi}\int_0^{2\pi}\alpha_n\cos n\theta(\mathrm{A}_0+\mathrm{A}_1\cos(\theta-\psi)+\ldots)ad\theta\\
&=\tfrac{1}{2}ma\alpha_n\mathrm{A}_n\cos n\psi.
\end{aligned}$$

5th. Small terms arising from C_n containing $\cos n\theta$. These are

$$u=0,\ v=0$$

$$w=\frac{m}{2\pi}\int_0^{2\pi}(a-\rho\cos(\theta-\psi))\left(\alpha_n\cos n\theta\frac{dA_0}{da}+-\alpha_n\cos n\theta\frac{dA_m}{da}\cos m(\theta-\psi)\ldots\right)ad\theta$$

$$=\tfrac{1}{2}m\left\{a^2\alpha_n\frac{dA_n}{da}\cos n\psi-\frac{\rho a\alpha_n}{2}\left(\frac{dA_{n+1}}{da}+\frac{dA_{n-1}}{da}\right)\cos n\psi\right\}$$

$$=\tfrac{1}{2}m\alpha_n\left\{a^2\frac{dA_n}{da}-\tfrac{1}{2}\rho a\left(\frac{dA_{n+1}}{da}+\frac{dA_{n-1}}{da}\right)\right\}\cos n\psi.$$

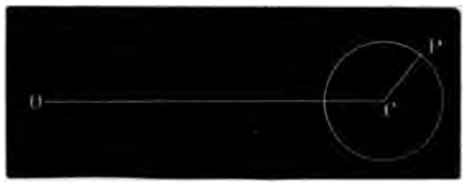
Collecting the terms we find

$$u=\tfrac{1}{2}ma\Big\{\zeta A_1\cos\psi+\tfrac{1}{2}\beta_n(n-1)A_{n+1}\cos(n+1)\psi-(n+1)A_{n-1}\cos(n-1)\psi)$$
$$+\tfrac{1}{2}n\beta_n A_n\frac{\rho}{a}(\cos(n-1)\psi-\cos(n+1)\psi)\Big\}\quad .\quad (4)$$

$$v=\tfrac{1}{2}ma\Big\{\zeta A_1\sin\psi+\tfrac{1}{2}\beta_n((n-1)A_{n+1}\sin(n+1)\psi+(n+1)A_{n-1}\sin(n-1)\psi)$$
$$-\tfrac{1}{2}n\beta_n A_n\frac{\rho}{a}(\sin(n+1)\psi+\sin(n-1)\psi)\Big\}\quad .\quad (5)$$

$$w=\tfrac{1}{2}ma\Big\{2aA_0-\rho A_1+\Big(2\alpha_n A_n+\tfrac{1}{2}\alpha_n\frac{\rho}{a}((n-1)A_{n-1}-(n+1)A_{n+1})\Big)\cos n\psi$$
$$+\alpha_n\left[a\frac{dA_n}{da}-\rho\left(\frac{dA_{n+1}}{da}+\frac{dA_{n-1}}{da}\right)\right]\cos n\psi\Big\}\quad .\quad (6)$$

Fig. 1.



Let the figure represent a section of the vortex ring by a plane through its straight axis. Let ϕ be the angle which the radius vector drawn from C the centre of the section of the core to any point P on the surface of the ring makes with the straight axis of the ring. Let C P$=e$.

Then the equations to the surface of the core are

$$\rho=a+\Sigma\alpha_n\cos n\psi+e\sin\phi$$
$$z=\mathfrak{z}+\Sigma\beta_n\cos n\psi+e\cos\phi$$

Since the vortex rings always consists of the same particles if $F(\rho, \theta, \phi)=0$ be an equation to its surface, we must have

$$\frac{dF}{dt}+\frac{dR}{d\rho}.R+\frac{dF}{d\psi}.\Psi+\frac{dF}{d\phi}.\Phi=0$$

when the differential coefficients are partial.

R is the velocity in the direction in which ρ is measured, Ψ the angular velocity round the axis of z, and Φ the angular velocity of C P round a normal to the plane containing the axis of z and O C.

Applying this equation to the first of the equations to the core, we get

$$\Sigma\dot{\alpha}_n \cos n\psi-R-\Sigma n\alpha_n \sin n\psi.\Psi+e \cos \phi.\Phi=0$$

or

$$R=\Sigma\dot{\alpha}_n \cos n\psi-\Sigma n\alpha_n \sin n\psi.\Psi+e \cos \phi.\Phi$$

and in a similar way we find

$$w=\dot{\mathfrak{z}}+\Sigma(\dot{\beta}_n \cos n\psi-n\beta_n \sin n\psi\Psi)-e \sin \phi.\Phi$$

where w is the velocity of a point on the surface of the core parallel to the axis of z. Now Ψ is zero when α_n and β_n are both zero, and it will be small in this case since α_n and β_n are both small, hence neglecting the squares of small quantities, these equations become

$$R=\Sigma\dot{\alpha}_n \cos n\psi+e \cos \phi.\Phi \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

$$w=\dot{\mathfrak{z}}+\Sigma\dot{\beta}_n \cos n\psi-e \sin \phi.\Phi \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (8)$$

But $R=u \cos \psi+v \sin \psi$.

Substituting for u and v the values given in equations (4) and (5), we get

$$R=\tfrac{1}{2}ma\{\zeta A_1+\tfrac{1}{2}\Sigma\beta_n \cos n\psi((n-1)A_{n+1}-(n+1)A_{n-1})\}$$

Since the A's are multiplied by the small quantities ζ, α_n, β_n, we may suppose since we neglect quantities of the order α_n^2 that the A's are found on the supposition that α_n and β_n are zero, or that the A's are the same as if the vortex was undisturbed.

Let us denote the value of the A's for the undisturbed vortex by German letters.

Equating the two expressions for R and putting

$$\zeta=\Sigma\beta_n \cos n\psi=e \cos \phi$$

we get

$$\tfrac{1}{2}ma\Sigma\{(\beta_n \cos n\psi+e \cos \phi)\mathfrak{A}_1+\tfrac{1}{2}\beta_n \cos n\psi((n-1)\mathfrak{A}_{n+1}-(n+1)\mathfrak{A}_{n-1})\}$$
$$=\Sigma\dot{\alpha}_n \cos n\psi+e \cos \phi.\Phi$$

equating the coefficients of $\cos \phi$ and $\cos n\psi$ we get

$$\tfrac{1}{2}ma\mathfrak{A}_1=\Phi \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (9)$$

$$\tfrac{1}{2}ma\beta_n\{\mathfrak{A}_1+\tfrac{1}{2}((n-1)\mathfrak{A}_{n+1}-(n+1)\mathfrak{A}_{n-1})\}=\dot{\alpha}_n \quad . \quad . \quad . \quad . \quad . \quad (10)$$

If we equate the value of w from equation (6) to that given by equation (7) we get

$$\tfrac{1}{2}ma\Big(2a\mathrm{A}_0-\rho\mathrm{A}_1+\Sigma\tfrac{1}{2}\frac{\rho}{a}.\alpha_n[(n-1)\mathrm{A}_{n-1}-(n+1)\mathrm{A}_{n+1}]\cos n\psi$$

$$+\left\{2\Sigma a_n\mathrm{A}_n+\Sigma\alpha_n\left[\frac{ad\mathrm{A}_n}{da}-\rho\left(\frac{d\mathrm{A}_{n+1}}{da}+\frac{d\mathrm{A}_{n-1}}{da}\right)\right]\right\}\cos n\psi\Big)$$

$$=\dot{\mathfrak{z}}+\Sigma\dot{\beta}_n\cos n\psi-e\sin\phi.\Phi$$

Since the term $2a\mathrm{A}_0-\rho\mathrm{A}_1$ is not multiplied by any small quantity we cannot suppose the A's to have the same value as for the undisturbed vortex, we must substitute for $2a\mathrm{A}_0-\rho\mathrm{A}_1$

$$2a\mathfrak{A}_0-a\mathfrak{A}_1-\alpha_n\cos n\psi\mathfrak{A}_1+\alpha_n\cos n\psi\frac{d}{d\rho}(2a\mathfrak{A}_0-a\mathfrak{A}_1)-e\sin\phi\mathfrak{A}_1$$

Since the other terms are multiplied by small quantities, we may substitute for the A's their undisturbed values. Making these substitutions we get

$$\tfrac{1}{2}ma\Big\{2a\mathfrak{A}_0-a\mathfrak{A}_1-e\sin\phi\mathfrak{A}_1+\Sigma\alpha_n\cos n\psi\Big[\frac{d}{d\rho}(2a\mathfrak{A}_0-a\mathfrak{A}_1)$$

$$+2\mathfrak{A}_n-\mathfrak{A}_1+\tfrac{1}{2}((n-1)\mathfrak{A}_{n-1}-(n+1)\mathfrak{A}_{n+1})+\frac{ad\mathfrak{A}_n}{da}-\frac{a}{2}\left(\frac{d\mathfrak{A}_{n+1}}{da}+\frac{d\mathfrak{A}_{n-1}}{da}\right)\Big]\Big\}$$

$$=\dot{\mathfrak{z}}-e\sin\phi\Phi+\Sigma\dot{\beta}_n\cos n\psi$$

Equating constant terms and the coefficients of sin ϕ and cos $n\psi$, we get

$$\tfrac{1}{2}ma^2(2\mathfrak{A}_0-\mathfrak{A}_1)=\dot{\mathfrak{z}}\quad.\quad.\quad.\quad.\quad.\quad.\quad.\quad.\quad.\quad(11)$$

$$\tfrac{1}{2}ma\mathfrak{A}_1=\Phi$$

$$\tfrac{1}{2}ma\alpha_n\Big\{\frac{d}{d\rho}(2a\mathfrak{A}_0-a\mathfrak{A}_1)+2\mathfrak{A}_n-\mathfrak{A}_1+\tfrac{1}{2}((n-1)\mathfrak{A}_{n-1}-(n+1)\mathfrak{A}_{n+1}$$

$$+a\frac{d\mathfrak{A}_n}{da}-\tfrac{1}{2}a\left(\frac{d\mathfrak{A}_{n+1}}{da}+\frac{d\mathfrak{A}_{n-1}}{da}\right)\Big\}=\dot{\beta}_n\quad.\quad.\quad.\quad.\quad.\quad.\quad.\quad.\quad.\quad(12)$$

The first of these equations gives the velocity of translation of an undisturbed circular vortex ring, the second is the same as the one we previously obtained for Φ.

We must now proceed to find the values of the $\mathfrak{A}$'s supposing the transverse section of the vortex core to be small compared with its aperture.

Since

$$\frac{1}{\{a^2+\rho+\zeta^2-2\rho a\cos(\theta-\psi)\}^{\frac{1}{2}}}=\mathfrak{A}_0+\mathfrak{A}_1\cos(\theta-\psi)+\ldots\mathfrak{A}_n\cos n(\theta-\psi)$$

$$\mathfrak{A}_n=\frac{1}{\pi}\int_0^{2\pi}\frac{\cos n\chi.d\chi}{(a^2+\rho^2+\zeta^2-2\rho a\cos\chi)^{\frac{1}{2}}}$$

except when $n=0$ when

$$\mathfrak{A}_0=\frac{1}{2\pi}\int_0^{2\pi}\frac{d\chi}{(a^2+\rho^2+\zeta^2-2\rho a\cos\chi)^{\frac{3}{2}}}$$

Now

$$a^2+\rho^2+\zeta^2-2\rho a\cos\chi=(a+\rho)^2+\zeta^2-4\rho a\cos^2\frac{\chi}{2}=((a+\rho)^2+\zeta^2)\left(1-\kappa^2\cos^2\frac{\chi}{2}\right)$$

where

$$\kappa^2=\frac{4\rho a}{(a+\rho)^2+\zeta^2}$$

Now in the case we are considering ρ is very nearly equal to a and ζ is very small, hence κ is very nearly equal to unity

$$1-\kappa^2\cos^2\frac{\chi}{2}=\kappa_1{}^2+\kappa^2\sin^2\frac{\chi}{2}$$

where

$$\kappa_1{}^2=1-\kappa^2=\frac{(\rho-a)^2+\zeta^2}{(\rho+a)^2+\zeta^2}$$

and is very small

Therefore $$\mathfrak{A}_n=\frac{1}{\pi}\int_0^{2\pi}\frac{\cos n\chi d\chi}{((a+\rho)^2+\zeta^2)^{\frac{3}{2}}\left(\kappa_1{}^2+\kappa^2\sin^2\frac{\chi}{2}\right)^{\frac{3}{2}}}$$

Now since κ_1 is very small

$$\int_0^{2\pi}\frac{\cos n\chi d\chi}{\left(\kappa_1{}^2+\kappa^2\sin^2\frac{\chi}{2}\right)^{\frac{3}{2}}}$$

will be very large, and the large part will arise from very small values of χ, or from values of χ very nearly equal to 2π, the parts arising from small values of χ and from values near to 2π will evidently be equal; hence the integral will be approximately

$$\int_0^{\epsilon}\frac{\left(1-\frac{n^2\chi^2}{2}\right)d\chi}{\left(\kappa_1{}^2+\frac{\chi^2}{4}\right)^{\frac{3}{2}}}$$

where ϵ is large compared with κ_1, this integral

$$=\frac{4(1+2n^2\kappa_1{}^2)}{\kappa_1{}^2}\left[\frac{\chi}{(\kappa_1{}^2+\chi^2)^{\frac{1}{2}}}\right]_0^{\epsilon}-8n^2\left[\log\left(\chi+\sqrt{\kappa_1{}^2+\chi^2}\right)\right]_0^{\epsilon}$$

or, since ϵ is large compared with κ_1, this is approximately

$$\frac{4(1+2n^2\kappa_1{}^2)}{\kappa_1{}^2}-8n^2\log\frac{2\epsilon}{\kappa_1}$$

or retaining only the more important terms

$$=\frac{4}{\kappa_1{}^2}+8n^2\log\kappa_1$$

hence

$$\mathfrak{A}_n=\left(\frac{4}{\pi\kappa_1^2}+\frac{8n^2}{\pi}\log\kappa_1\right)\div((a+\rho)^2+\zeta^2)^{\frac{1}{2}} \quad . \quad . \quad . \quad . \quad . \quad . \quad (13)$$

except when $n=0$ and

$$\mathfrak{A}_0=\frac{2}{\pi\kappa_1^2}\div((a+\rho)^2+\zeta^2)^{\frac{1}{2}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (14)$$

If we substitute these values for the $\mathfrak{A}$'s the equation

$$\tfrac{1}{2}ma\mathfrak{A}_1=\Phi$$

gives

$$\tfrac{1}{2}ma\left(\frac{4}{\pi\kappa_1^2}+\frac{8}{\pi}\log\kappa_1\right)\div 8a^3=\Phi$$

or, since κ_1 is approximately $e^2/4a^2$, we get if we substitue this value for κ_1 and $\pi e^2\omega$ for m

$$\omega-\tfrac{1}{2}\omega\frac{e^2}{a^2}\log\frac{2a}{e}=\Phi \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (15)$$

The second term on the left-hand side of this equation being small compared with the first, we get as a rougher approximation

$$\omega=\Phi. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (16)$$

The equation

$$\tfrac{1}{2}ma\beta_n\{\mathfrak{A}_1+\tfrac{1}{2}((n-1)\mathfrak{A}_{n+1}-(n+1)\mathfrak{A}_{n-1})\}=\dot{\alpha}_n$$

gives on substitution

$$-\frac{\omega e^2}{2a^2}n^2\log\frac{2a}{e}\beta_n=\dot{\alpha}_n \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (17)$$

Substituting for $\mathfrak{A}_0$, $\mathfrak{A}_1$ in equation (11) we find

$$\dot{\mathfrak{z}}=\frac{\omega e^2}{2a}\log\frac{2a}{e} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (18)$$

This agrees to the degree of approximation we are working to with the value for the velocity of translation of a circular vortex found by Sir W. THOMSON and given in Professor TAIT's translation of HELMHOLTZ's paper on "Vortex Motion" (Phil. Mag., June, 1867).

The value of ϕ given by equation (16) is also the same as that obtained by Sir W. THOMSON.

Substituting for the $\mathfrak{A}$'s in equation (9) we find

$$\tfrac{1}{2}ma\alpha_n\left\{-\left(\frac{d}{da}+\frac{d}{d\rho}\right)\left[\frac{4}{\pi}\log\frac{(a-\rho)^2+\zeta^2}{(a+\rho)^2+\zeta^2}\Big/((a+\rho)^2+\zeta^2)^{\frac{1}{2}}\right]\right.$$
$$\left.-\frac{4}{\pi}(n^2+2)\log\frac{(a-\rho)^2+\zeta^2}{(a+\rho)^2+\zeta^2}\Big/((a+\rho)^2+\zeta^2)^{\frac{1}{2}}\right\}=\dot{\beta}_n$$

or neglecting terms on the left-hand side which are not multiplied by the large quantity $\log\frac{\overline{a-\rho}^2+\zeta^2}{(a+\rho)^2+\zeta^2}=\log\frac{e^2}{4a^2}$ we find

$$\tfrac{1}{2}ma\alpha_n\frac{8}{\pi}\frac{(n^2-1)}{8a^2}\log\frac{2a}{e}=\dot{\beta}_n$$

or

$$\tfrac{1}{2}\omega\frac{e^2}{a^2}(n^2-1)\log\frac{2a}{e}\cdot\alpha_n=\dot{\beta}_n \quad \ldots\ldots\ldots \quad (19)$$

Differentiating equation (15) with respect to the time and substituting for $\dot{\beta}_n$ from (16) we get

$$-\left(\tfrac{1}{2}\omega\frac{e^2}{a^2}\log\frac{2a}{e}\right)^2(n^2-1)n^2\alpha_n=\alpha''_n \quad \ldots\ldots\ldots \quad (20)$$

or

$$\alpha_n=\mathrm{A}\cos\left\{\frac{\omega e^2}{2a^2}\log\frac{2a}{e}\,n.\sqrt{n^2-1}t+\beta\right\} \quad \ldots\ldots \quad (21)$$

$$\beta_n=\mathrm{A}\frac{\sqrt{n^2-1}}{n}\sin\left\{\frac{\omega c^2}{2a^2}\log\frac{2a}{e}\cdot n\sqrt{n^2-1}.t+\beta\right\} \quad \ldots\ldots \quad (22)$$

where A and β are arbitrary constants.

These equations show that the circular vortex ring of indefinitely small section is stable for all displacements of its circular axis, and that the time of vibration for a displacement expressed by

$$\rho=a+\alpha_n\cos n\theta$$

is

$$2\pi\Big/\frac{\omega e^2}{2a^2}\log\frac{2a}{e}.n.\sqrt{n^2-1} \quad \ldots\ldots\ldots\ldots \quad (23)$$

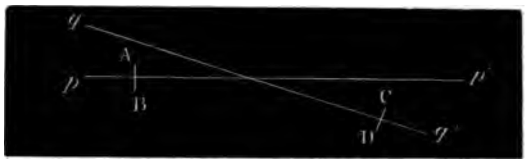
If V be the velocity of translation of the vortex, viz.: $\omega e^2\log\frac{2a}{e}\Big/2a$, the time of vibration is $2\pi a/\mathrm{V}.n\sqrt{n^2-1}$.

Sir W. THOMSON has proved that the circular vortex ring is stable for all alterations in the shape of the cross section. If we combine this with the result just obtained we see that the circular vortex ring is stable for all possible displacements. Sir W. THOMSON has also proved that for a displacement of the n^{th} order in the shape of the cross section of the vortex arc the time of vibration $=2\pi/(n-1)\omega$; hence these vibrations begin by being much quicker than those we have been considering, but since for large values of n the latter are proportional to n^2 whilst the former are only proportional to n, the vibrations of a higher order will be quicker for the circular axis than for the core. When n is very great $\sqrt{n^2-1}=n$, thus the amplitude of α_n is equal to the amplitude of β_n and $\alpha_n^2+\beta_n^2=\mathrm{A}^2$ a constant quantity; thus each point on the arc describes a circle about its mean position with an angular velocity $\omega e^2n^2\log 2a/e\Big/a^2$.

Problem II. To find the action upon each other of two vortex rings which move so as never to approach nearer than a large multiple of the diameter of either.

For the sake of simplicity we shall suppose that the normals to the planes of the vortices intersect.

Fig. 2.



Let the plane of the paper contain $p\,p'$ and $q\,q'$ the normals to the two vortices, let A B be the vortex moving along $p\,p'$, C D the vortex moving along $q\,q'$.

Let the figure of the circular axis of the vortex A B be given by

$$\rho' = a' + \Sigma\alpha_n' \cos n\theta'$$
$$z' = \mathfrak{z}' + \Sigma\beta_n' \cos n\theta'$$

where z' is measured along and ρ' perpendicular to $p\,p'$. Since the vortices never approach near to one another α_n' and β_n' will be small compared with a'; they will be functions of the time which we shall have to find.

Let the figure of the circular axis of C D be given by

$$\rho = a + \Sigma\alpha_n \cos n\theta$$
$$z = \mathfrak{z} + \Sigma\beta_n \cos n\theta$$

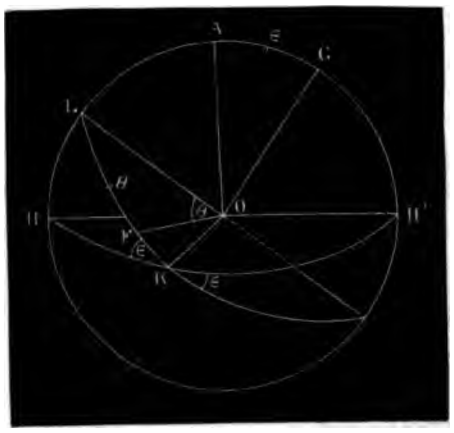
where z and ρ are measured respectively along and perpendicular to $q\,q'$. For the same reason as before α_n and β_n will be small compared with a. To find how the vortex C D is affected by the vortex A B we shall have to find the velocities of the fluid along z and ρ due to the vortex A B; in doing this we may as a first approximation assume that the axis of A B is circular and in one plane, *i.e.*, we may calculate the velocities as if α_n' and β_n' were both zero.

Let ϵ denote the angle between $p\,p'$ and $q\,q'$. Let $p\,p'$ be taken as the axis of z', the perpendicular to $p\,p'$ drawn upwards through the centre of the vortex A B, being the axis of x'.

Let l, m, n be the direction cosines referred to these axes of a radius vector in the plane of the vortex ring C D, drawn from the centre of the vortex ring and making an angle θ with C D the intersection of the plane of the vortex ring with the plane of the paper.

To find l, m, n through the centre of a sphere draw planes parallel to the two vortex rings and let these be H K H′, L K H′, the former being parallel to the ring A B and the latter to C D. Let A G be the poles of these great circles.

Fig. 3.



Then O H, K O, O A are parallel to our axes of x', y', z' respectively. The angle A G or K is ϵ, and if F is parallel to the radius vector above referred to, F L is equal to θ. Then

$$
\begin{aligned}
l &= \cos \mathrm{HF} = \cos\theta\cos\epsilon \\
m &= -\cos \mathrm{FK} = -\sin\theta \\
n &= \cos \mathrm{FA} = \cos\theta\sin\epsilon
\end{aligned}
$$

The velocity γ along the axis of z' due to the vortex A B is by formula 3 given by

$$\gamma=\tfrac{1}{2}m'(2a'^2\mathrm{A}_0-a'\rho'\mathrm{A}_1)$$

where m' is the strength of the vortex A B

$$\mathrm{A}_0=\frac{1}{2\pi}\int_0^{2\pi}\frac{d\theta}{(a'^2+\rho'^2+\zeta'^2-2\rho'a'\cos\theta)^{\frac{3}{2}}}$$

$$\mathrm{A}_1=\frac{1}{2\pi}\int_0^{2\pi}\frac{\cos\theta.d\theta}{(a'^2+\rho'^2+\zeta'^2-2\rho'a'\cos\theta)^{\frac{3}{2}}}$$

Now since the vortex rings never approach nearer than a large multiple of their diameter, a'^2 will be small compared with $\rho'^2+\zeta'^2$, if we neglect small quantities of a higher order than $a'^2/\rho'^2+\zeta'^2$, we find

$$\mathrm{A}_0=\frac{1}{(\rho'^2+\zeta'^2)^{\frac{3}{2}}}+\frac{3}{4}\frac{a'^2(3\rho'^2-2\zeta'^2)}{(\rho'^2+\zeta'^2)^{\frac{7}{2}}}$$

$$\mathrm{A}_1=\frac{3a'\rho'}{(\rho'^2+\zeta'^2)^{\frac{5}{2}}}$$

Let the coordinates of the centre of the vortex ring C D be f, 0, h, then for a point on the vortex ring

$$
\begin{aligned}
x &= f+al = f+a\cos\epsilon\cos\theta \\
y &= am = -a\sin\theta \\
z &= h+an = h+a\sin\epsilon\cos\theta
\end{aligned}
$$

Substituting in the expressions for A_0 and A_1, we find on neglecting small quantities of an order higher than a'^2/f^2+h^2

$$A_0=\frac{1}{(f^2+h^2)^{\frac{1}{2}}}-\frac{3}{2}\frac{a^2}{(f^2+h^2)^{\frac{3}{2}}}+\frac{15}{4}\frac{a^2(h\sin\epsilon+f\cos\epsilon)^2}{(f^2+h^2)^{\frac{5}{2}}}+\frac{3}{4}\frac{a'^2(3f^2-2h^2)}{(f^2+h^2)^{\frac{5}{2}}}$$

$$-\cos\theta\,\frac{3a(h\sin\epsilon+f\cos\epsilon)}{(f^2+h^2)^{\frac{3}{2}}}+\cos 2\theta\,\frac{15}{4}\frac{a^2(h\sin\epsilon+f\cos\epsilon)^2}{(f^2+h^2)^{\frac{5}{2}}}$$

$$\rho' A_1=\frac{3a'\rho'^2}{(\rho'^2+\zeta'^2)^{\frac{5}{2}}}$$

$$=\frac{3a'f^2}{(f^2+h^2)^{\frac{5}{2}}}-\frac{15}{2}\frac{a'a^2f^2}{(f^2+h^2)^{\frac{7}{2}}}+\frac{105}{4}\frac{a'a^2f^2(h\sin\epsilon+f\cos\epsilon)^2}{(f^2+h^2)^{\frac{9}{2}}}+\frac{3a'a^2(1-\frac{1}{2}\sin^2\epsilon)}{(f^2+h^2)^{\frac{5}{2}}}$$

$$-\frac{15a'a^2f(h\sin\epsilon+f\cos\epsilon)}{(f^2+h^2)^{\frac{7}{2}}}+\cos\theta\left\{\frac{6aa'f\cos\epsilon}{(f^2+h^2)^{\frac{5}{2}}}-\frac{15aa'f^2(h\sin\epsilon+f\cos\epsilon)}{(f^2+h^2)^{\frac{7}{2}}}\right\}$$

$$+\cos 2\theta\left\{\frac{105}{4}\frac{a'a^2f^2(h\sin\epsilon+f\cos\epsilon)^2}{(f^2+h^2)^{\frac{9}{2}}}-\frac{15a'a^2f\cos\epsilon(h\sin\epsilon+f\cos\epsilon)}{(f^2+h^2)^{\frac{7}{2}}}-\frac{3}{2}\frac{a'a^2\sin^2\epsilon}{(f^2+h^2)^{\frac{5}{2}}}\right\}$$

Although for reference we give the complete values of A_0 and A_1 to the order of approximation we are working to, yet when we have in the expressions for the velocities a coefficient consisting of terms of differerent orders, we shall only retain the largest term.

If we do this we find

$$\gamma=\frac{1}{2}\frac{m'a'^2}{(f^2+h^2)^{\frac{5}{2}}}(2h^2-f^2)$$

$$+\frac{3}{2}\cos\theta\frac{m'a'^2}{(f^2+h^2)^{\frac{7}{2}}}\left(f^2(f\cos\epsilon+3h\sin\epsilon)-2h^2(2f\cos\epsilon+h\sin\epsilon)\right)$$

$$+\frac{3}{2}\cos 2\theta\frac{m'a'^2a^2}{(f^2+h^2)^{\frac{7}{2}}}\left\{\frac{5}{2}(f\cos\epsilon+h\sin\epsilon)^2-\frac{35}{4}\frac{f^2(f\cos\epsilon+h\sin\epsilon)^2}{(f^2+h^2)}+5f\cos\epsilon(f\cos\epsilon+h\sin\epsilon)\right.$$

$$\left.+\frac{1}{2}\sin^2\epsilon(f^2+h^2)\right\}$$

From the formulæ (1) and (3) we find the velocity along ρ'

$$=\frac{1}{2}m'a'\zeta' A_1$$

$$=\frac{3}{2}\frac{m'a'^2\rho\zeta'}{(\rho'^2+\zeta'^2)^{\frac{5}{2}}}$$

Hence α, the velocity along x' at the vortex C D due to the vortex A B,

$$=\frac{3}{2}\frac{m'a'^2\zeta'(f+a\cos\epsilon\cos\theta)}{(\rho'^2+\zeta'^2)^{\frac{5}{2}}}$$

$$=\frac{3m'a'^2}{2}\frac{fh}{(f^2+h^2)^{\frac{5}{2}}}$$

$$+\cos\theta.\frac{3}{2}\frac{m'a'^2a}{(f^2+h^2)^{\frac{5}{2}}}.\left\{(h\cos\epsilon+f\sin\epsilon)-\frac{5fh(h\sin\epsilon+f\cos\epsilon)}{f^2+h^2}\right\}$$

$$+\cos 2\theta.\frac{3}{2}\frac{m'a'^2a^2}{(f^2+h^2)^{\frac{5}{2}}}\left\{\frac{35}{4}\frac{(h\sin\epsilon+f\cos\epsilon)^2fh}{(f^2+h^2)^2}-\frac{5}{4}\frac{(h\cos\epsilon+f\sin\epsilon)(h\sin\epsilon+f\cos\epsilon)}{(f^2+h^2)}+\frac{1}{2}\sin\epsilon\cos\epsilon\right\}$$

In this expression for α we have only retained the highest terms in each coefficient. If β be the velocity parallel to the axis of y', we have

$$\begin{aligned}\beta &= -\tfrac{3}{2}\frac{m'a'^2a\zeta'\sin\theta}{(\rho'^2+\zeta^2)^{\frac{5}{2}}}\\ &= -\tfrac{3}{2}\frac{m'a'^2a}{(f^2+h^2)^{\frac{5}{2}}}\left(h\sin\theta-\frac{5ah\sin\theta\cos\theta(h\sin\epsilon+f\cos\epsilon)}{f^2+h^2}+a\sin\theta\cos\theta\sin\epsilon\right)\end{aligned}$$

The velocity perpendicular to the plane of the vortex C D

$$=\gamma\cos\epsilon-\alpha\sin\epsilon$$

$$\left.\begin{aligned}&=\tfrac{1}{2}\frac{m'a'^2}{(f^2+h^2)^{\frac{5}{2}}}\{(2h^2-f^2)\cos\epsilon-3fh\sin\epsilon\}\\ &+\cos\theta.\tfrac{3}{2}\frac{m'a'^2a}{(f^2+h^2)^{\frac{7}{2}}}\{\cos 2\epsilon(f^3-4fh^2)+\tfrac{1}{2}\sin 2\epsilon(7f^2h-3h^3)\}\\ &+\cos 2\theta.\tfrac{3}{2}\frac{m'a'^2a^2}{(f^2+h^2)^{\frac{7}{2}}}\left\{\tfrac{5}{2}(h\sin\epsilon+f\cos\epsilon)(h\sin 2\epsilon+f\cos 2\epsilon+2f)-\tfrac{35}{4}\frac{f^2(h\sin\epsilon+f\cos\epsilon)^2}{f^2+h^2}\right\}\\ &+\dots\end{aligned}\right\}(24)$$

The velocity along the radius vector of the vortex ring C D due to the vortex A B

$$\begin{aligned}&=\alpha l+\beta m+\gamma n\\ &=\alpha\cos\epsilon\cos\theta-\beta\sin\theta+\gamma\sin\epsilon\cos\theta\end{aligned}$$

or substituting for α, β, γ their values

$$\left.\begin{aligned}&=\tfrac{3}{4}\frac{m'a'^2a}{(f^2+h^2)^{\frac{5}{2}}}\left\{\tfrac{1}{2}h+\tfrac{1}{2}\cos 2\epsilon\frac{(3h^3-7hf^2)}{f^2+h^2}+\sin 2\epsilon\frac{(f^3-4fh^2)}{f^2+h^2}\right\}\\ &+\tfrac{1}{2}\cos\theta\frac{m'a'^2}{(f^2+h^2)^{\frac{5}{2}}}\{(2h^2-f^2)\sin\epsilon+3fh\cos\epsilon\}\\ &+\tfrac{3}{4}\cos 2\theta\frac{m'a'^2a}{(f^2+h^2)^{\frac{5}{2}}}\left\{-\tfrac{3}{2}h+\tfrac{1}{2}\cos 2\epsilon\frac{(3h^3-7f^2h)}{f^2+h^2}+\sin 2\epsilon\frac{(f^3-4fh^2)}{f^2+h^2}\right\}\end{aligned}\right\}.\quad(25)$$

These expressions will enable us to find the effect of one vortex on another. We have, for example, expressed the velocity perpendicular to the plane of C D due to the vortex ring A B in the form A+B cos θ+C cos 2θ+ . . . , in Problem I. we expressed the velocity in the same direction due to the vortex C D itself in the same form, hence the total velocity perpendicular to C D can be expressed in this form, but by formula (8) the velocity perpendicular to the plane of C D is

$$\Sigma\frac{d\beta_n}{dt}\cos n\theta$$

hence $\frac{d\beta_n}{dt}$= coefficient of $\cos n\theta$ in the expression for the velocity perpendicular to the plane of the vortex C D. To find β_n we must therefore express the velocity due to the vortex A B as a function of the time. In order to make the work as simple as possible we shall suppose that the vortices in their undisturbed states had equal strengths and radii. In the small terms which express the velocity at the vortex C D due to the vortex A B we may as a first approximation calculate the quantities on the supposition that the motion is undisturbed. In order to make the expressions as simple as possible, let us measure the time from the instant when the distance between the centres of the vortices has its least value (it is easy to see that this will be when the line joining the centres of the vortices is parallel to the line bisecting the angle between their directions of motion), then the square of the distance between their centres will be expressible in the form $c^2+\beta t^2$ when c is the least distance between the centres, t the time that has elapsed since the centres were this distance apart; let v be the velocity of translation of either vortex when undisturbed, then

$$f=-c\sin\tfrac{1}{2}\epsilon-v\sin\epsilon.t$$

$$h=\quad c\cos\tfrac{1}{2}\epsilon-v(1-\cos\epsilon)t$$

Therefore

$$f^2+h^2=c^2+4v^2\sin^2\tfrac{1}{2}\epsilon.t^2$$

Making these substitutions we find that the velocity perpendicular to the plane of the vortex C D

$$=\tfrac{1}{2}\frac{m'a'^2}{(c^2+4v^2\sin^2\frac{1}{2}\epsilon.t^2)^{\frac{5}{2}}}\{\tfrac{1}{2}c^2(3+\cos\epsilon)+2v^2\sin^2\tfrac{1}{2}\epsilon(1-3\cos\epsilon)t^2\}$$

$$+\cos\theta\tfrac{3}{2}\frac{m'a'^2a}{(c^2+4v^2\sin^2\frac{1}{2}\epsilon.t^2)^{\frac{7}{2}}}\{At^3+Bt^2+Ct+D\}$$

$$+\cos 2\theta\tfrac{3}{2}\frac{m'a'^2a^2}{(c^2+4v^2\sin^2\frac{1}{2}\epsilon.t^2)^{\frac{9}{2}}}\{A't^4+B't^3+C't^2+D't+E'\}$$

where

$$\left.\begin{aligned}A&=8v^3\sin^3\tfrac{1}{2}\epsilon(\cos\tfrac{3}{2}\epsilon-5\cos\tfrac{1}{2}\epsilon)\\B&=cv^2\sin^2\tfrac{1}{2}\epsilon(15\sin\tfrac{1}{2}\epsilon-\sin\tfrac{3}{2}\epsilon)\\C&=c^2v\sin\tfrac{1}{2}\epsilon(15\cos\tfrac{1}{2}\epsilon+\cos\tfrac{3}{2}\epsilon)\\D&=-\tfrac{1}{4}c^3(5\sin\tfrac{1}{2}\epsilon+\sin\tfrac{3}{2}\epsilon)\end{aligned}\right\}\quad\ldots\ldots\ldots\quad(26)$$

$$\left.\begin{aligned}A'&=-10v^4\sin^4\tfrac{1}{2}\epsilon(2\cos 3\epsilon+\tfrac{30}{8}\cos 2\epsilon+7\cos\epsilon+\tfrac{42}{8})\\B'&=\quad 10cv^3\sin^3\tfrac{1}{2}\epsilon(3\sin\epsilon-\tfrac{1}{2}\sin 2\epsilon)\\C'&=-10c^2v^2\sin^2\tfrac{1}{2}\epsilon\cos 3\epsilon\\D'&=-\tfrac{5}{8}c^3v\sin\epsilon(14\sin\epsilon+\sin 2\epsilon)\\E'&=\quad\tfrac{5}{8}c^4\sin^2\tfrac{1}{2}\epsilon(11+\cos\epsilon+8\cos 2\epsilon)\end{aligned}\right\}\quad\ldots\quad(27)$$

This expression vanishes when $t=-\infty$; it begins by being positive but soon changes sign; it is negative when $t=0$ and remains negative for all greater values of t.

The last terms are the only ones that do not vanish when $t=\infty$. Putting $t=\infty$ we find

$$\begin{aligned}\beta_{,}&=-\tfrac{1}{4}\frac{m'a'^3a}{c^3v\sin\frac{1}{2}\epsilon}(\sin\tfrac{1}{2}\epsilon+\sin\tfrac{3}{2}\epsilon)\\&=-\frac{m'a'^3a}{c^3v}\cos^2\tfrac{1}{2}\epsilon\end{aligned}$$

Now $\beta_{,}/a$ is the angle through which the plane of the vortex is turned, and since the vortex moves at right angles to its plane this will be the angle through which the direction of motion of the vortex ring is turned.

Since $\beta_{,}$ is negative, the part of the vortex ring C D where $\cos\theta$ is positive is tilted backwards; now as we have taken it, $\cos\theta$ is positive for the upper part of the vortex, hence this part of the vortex is tilted backwards, and the normal to its plane, which is the direction in which the vortex ring moves, is bent towards pp', the direction of motion of the vortex ring A B through an angle whose circular measure

$$=\frac{m'a^3}{vc^3}\cos^2\tfrac{1}{2}\epsilon.$$

Thus the deflection, other things being the same, varies inversely as the cube of the least distance between the vortices.

Let us now consider the effect of the vortex ring C D on the vortex A B. Let us take the perpendicular to the plane of C D as the new axis of z, the perpendicular to this drawn upwards in the plane of the paper as the new axis of x. The work we went through before consisted in finding expressions for the velocities along the axes of coordinates due to one vortex at a point on the other in terms of f, h, l, m, n, and then finding the velocities perpendicular to the plane of the vortex and along its radius vector in terms of the time by substituting from the equations

$$\left.\begin{aligned}f&=-c\sin\tfrac{1}{2}\epsilon-v\sin\epsilon.t\\h&=c\cos\tfrac{1}{2}\epsilon-v(1-\cos\epsilon)t\end{aligned}\right\}$$

$$\left.\begin{aligned}l&=\cos\epsilon\cos\theta\\m&=-\sin\theta\\n&=\sin\epsilon\sin\theta\end{aligned}\right\}$$

velocity perpendicular to the plane of the vortex $=\gamma\cos\epsilon-\alpha\sin\epsilon$, velocity along the radius vector $=\alpha\cos\epsilon\cos\theta-\beta\sin\theta+\gamma\sin\epsilon\cos\theta$.

Now in finding the effect of the vortex ring C D on the vortex A B, the general expressions giving the velocities will be the same as before, as we have taken corresponding axes of coordinates. The difference in the work will come in when we substitute for the quantities involved their expressions in terms of the time; if we denote corresponding quantities in this case by affixing dashes to the symbols, denoting them in the previous one, we easily find

$$\left.\begin{aligned} f' &= -c\sin\tfrac{1}{2}\epsilon + v\sin\epsilon . t \\ h' &= -c\cos\tfrac{1}{2}\epsilon - v(1-\cos\epsilon)t \end{aligned}\right\}$$

$$\left.\begin{aligned} l' &= \cos\epsilon\cos\theta' \\ m' &= -\sin\theta' \\ n' &= -\sin\epsilon\cos\theta' \end{aligned}\right\}$$

velocity perpendicular to the plane of the vortex A B

$$=\gamma\cos\epsilon+\alpha\sin\epsilon$$

velocity along the radius vector $=\alpha\cos\epsilon\cos\theta'-\beta\sin\theta'-\gamma\sin\epsilon\cos\theta'$.

It will be seen that we can get the expressions for the quantities denoted by the accented letters from those for the quantities denoted by the unaccented letters by writing $2\pi-\epsilon$ instead of ϵ, hence the value of $\beta_{,}'$ will be got by writing in the expression for $\beta_{,}$ (equation (30)) $2\pi-\epsilon$ instead of ϵ, and interchanging a and a'.

Hence the value of $\beta_{,}'$ when $t=\infty$

$$=-\frac{ma^2a'}{vc^3}\cos^2\frac{2\pi-\epsilon}{2}=-\frac{ma^2a'}{vc^3}\cos^2\tfrac{1}{2}\epsilon$$

Now this being negative shows that the parts of the vortex ring A B where $\cos\theta'$ is positive are tilted backwards, now $\cos\theta'$ is positive in the upper half of the vortex ring A B, therefore the direction of motion of the vortex A B, which is perpendicular to the plane of the vortex, is turned away from the direction of motion of the vortex C D through an angle whose circular measure is $ma^2\cos^2\frac{1}{2}\epsilon/vc^3$; but since in the case we are considering $a=a'$, $m=m'$, this angle is the same as that through which the path of the vortex C D is turned towards the path of the vortex A B. We may express the results we have obtained by saying that the direction of motion of the vortex which is in front when the vortices are nearest together, is bent towards the direction of motion of the one which is behind, that the direction of motion of the latter is bent through an equal amount in the same direction, and that the amount of this bending is $ma^2\cos^2\frac{1}{2}\epsilon/vc^3$.

Let us now consider the effect the collision has on the size of the vortices.

The equation giving the increase in radius is

$\frac{da_0}{dt}=$the part independent of θ in the expression for the velocity along the radius vector of C D.

A reference to equation (17) will show that the vortex ring C D itself contributes nothing to this term, therefore

$\frac{d\alpha_0}{dt}$=the part independent of θ in the expression for the velocity due to the vortex ring A B along the radius vector of C D.

Hence from equation (28) we have

$$\frac{d\alpha_0}{dt}=\frac{3}{4}\frac{m'a'^2a}{(c^2+4v^2\sin^2\frac{1}{2}\epsilon.t^2)^{\frac{5}{2}}}\{\mathrm{F}t^3+\mathrm{G}t^2+\mathrm{H}t+\mathrm{K}\}$$

If we integrate this equation, substitute for F, G, H, K their values as given in equation (28) and determine the arbitrary constant introduced by the integration, so that $\alpha_0=0$ when $t=-\infty$, we find

$$\alpha_0=\frac{3}{4}m'a'^2a\left\{\frac{-c^3}{2v(c^2+4v^2\sin^2\frac{1}{2}\epsilon.t^2)^{\frac{3}{2}}}+\frac{(1+\sin^2\frac{1}{2}\epsilon)}{6v(c^2+4v^2\sin^2\frac{1}{2}\epsilon.t^2)^{\frac{3}{2}}}+\frac{c\cos\frac{1}{2}\epsilon.t}{(c^2+4v^2\sin^2\frac{1}{2}\epsilon.t^2)^{\frac{3}{2}}}\right.$$
$$\left.+\frac{(\cos\frac{3}{2}\epsilon+3\cos\frac{1}{2}\epsilon)t}{12c(c^2+4v^2\sin^2\frac{1}{2}\epsilon.t^2)^{\frac{3}{2}}}+\frac{3\cos\frac{1}{2}\epsilon+\cos\frac{3}{2}\epsilon}{6c^3}\left(\frac{\epsilon}{(c^3+4v^2\sin^2\frac{1}{2}\epsilon.t^2)^{\frac{1}{2}}}+\frac{1}{2v\sin\frac{1}{2}\epsilon}\right)\right\}$$

This expression vanishes when $t=-\infty$; it begins by being negative, so that the radius of C D is diminished at first when $t=0$, the sign of α_0 depends upon the value of ϵ, if ϵ be less than 60° it is certainly positive when $t=0$; when $t=\infty$ α_0 is positive, and its value is

$$\frac{m'a'^2a}{8vc^3\sin\frac{1}{2}\epsilon}(3\cos\frac{1}{2}\epsilon+\cos\frac{3}{2}\epsilon)$$
$$=\frac{1}{2}\frac{m'a'^2a}{vc^3\sin\frac{1}{2}\epsilon}\cos^3\frac{1}{2}\epsilon.$$

As this is positive, the vortex ring C D is bigger after the collision. The effect of the vortex ring C D on the vortex A B can be got as we saw before by writing $2\pi-\epsilon$ for ϵ in the formula given above. We have thus the ultimate increase α_0' in the radius of A B given by

$$\alpha_0'=\frac{1}{2}\frac{ma^2a'\cos^3\frac{1}{2}(2\pi-\epsilon)}{c^3\sin\frac{1}{2}(2\pi-\epsilon)}$$

or since $a=a'$, $m=m'$

$$\alpha_0=-\frac{1}{2}\frac{m'a'^2a}{c^3\sin\frac{1}{2}\epsilon}\cos^3\frac{1}{2}\epsilon$$

Hence the radius of the vortex ring A B is diminished by the collision. The effect of the collision on the size of the vortices is thus to increase the radius of the one which is in front when the vortex rings are nearest together, and decrease that of the one in the rear by $m'a^3\cos^3\frac{1}{2}\epsilon/2vc^3\sin\frac{1}{2}\epsilon$. Hence the alteration in the radius is, *cœteris paribus*, inversely proportional to the cube of the shortest distance between the vortices.

We can now find the force resultant of the impulse after collision. The impulse for a vortex ring with a very fine core equals the strength multiplied by the area. Let ω be the small angle through which the direction of motion of the vortices is deflected; let δa be the alteration in the radius of either, for the vortex ring in front δa will be positive, for the one in the rear it will be negative but of equal numerical value.

The resolved part of the force resultant of the impulse along the line bisecting their original direction of motion after collision

$$
\begin{aligned}
&=\pi m(a+\delta a)^2 \cos\left(\frac{\epsilon}{2}-\omega\right)+\pi m(a-\delta a)^2 \cos\left(\frac{\epsilon}{2}+\omega\right)\\
&=2\pi m a^2 \cos \tfrac{1}{2}\epsilon
\end{aligned}
$$

the same as before collision.

The component perpendicular to the bisector of the angle between their directions of motion

$$
\begin{aligned}
&=\pi m(a+\delta a)^2 \sin(\tfrac{1}{2}\epsilon-\omega)-\pi m(a-\delta a)^2 \sin(\tfrac{1}{2}\epsilon+\omega)\\
&=\pi m(4a\delta a \sin \tfrac{1}{2}\epsilon-2\omega a^2 \cos \tfrac{1}{2}\epsilon)
\end{aligned}
$$

Substituting for δa and ω the values $ma^3 \cos^3 \frac{1}{2}\epsilon/2vc^3 \sin \frac{1}{2}\epsilon$, and $ma^2 \cos^2 \frac{1}{2}\epsilon/vc^3$ respectively, we find that the component of the impulse perpendicular to the bisector of the angle between the directions of motion vanishes, as it did before the collision; hence we see that the force resultant of the impulse is not altered by the collision, a result which we know is true.

We pass on to consider the terms α_2 and β_2. We know that

$\frac{d\alpha_2}{dt}=$ coefficient of $\cos 2\theta$ in the expression for the velocity along the radius vector of the vortex ring C D.

Now the vortex ring C D itself, as we see from equation (17), contributes to the expression for the velocity along its radius vector the term

$$-\cos 2\theta.\frac{2m'}{a^2}\log\frac{2a}{e}.\beta_2$$

The vortex ring A B contributes as we see from equation (29) the term

$$-\cos 2\theta.\tfrac{3}{4}\frac{m'a'^2a}{(c^2+4v^2\sin^2\frac{1}{2}\epsilon.t^2)^{\frac{7}{2}}}(\mathrm{F}'t^3+\mathrm{G}'t^2+\mathrm{H}'t+\mathrm{K}')$$

say

$$\cos 2\theta f(t)$$

Thus

$$\frac{d\alpha_2}{dt}=-\frac{2m}{a^2}\log\frac{2a}{e}.\beta_2+f(t)$$

Now

$\frac{d\beta_2}{dt}=$ the coefficient of $\cos 2\theta$ in the expression for the velocity perpendicular to the plane of the vortex C D.

The vortex C D itself contributes to this coefficient the term

$$\tfrac{3}{2}\frac{m}{a^2}\log\frac{2a}{e}.\alpha_2$$

The vortex A B contributes the term

$$\tfrac{3}{2}\frac{m'a^2a'^2}{(c^2+4v^2\sin^2\tfrac{1}{2}\epsilon.t^2)^{\frac{5}{2}}}(A't^4+B't^3+C't^2+D't+E')$$

say

$$F(t)$$

Thus

$$\frac{d\beta_2}{dt}=\tfrac{3}{2}\frac{m}{a^2}\log\frac{2a}{e}\alpha_2+F(t)$$

Eliminating β_2 we find

$$\frac{d^2\alpha_2}{dt^2}+\frac{3m^2}{a^4}\left(\log\frac{2a}{e}\right)^2\alpha_2=f'(t)-\frac{2m}{a^2}\log\frac{2a}{e}.F(t)$$

say

$$=\chi(t)$$

or writing n^2 for

$$\frac{3m^2}{a^4}\left(\log\frac{2a}{e}\right)^2$$

the equation takes the form

$$\frac{d^2\alpha_2}{dt^2}+n^2\alpha_2=\chi(t)$$

The solution of this differential equation is

$$\alpha_2=A\cos nt+B\sin nt+\frac{\cos nt}{n}\int^t\chi(t')\sin nt'dt'$$
$$-\frac{\sin nt}{n}\int^t\chi(t')\cos nt'dt'$$

or choosing the arbitrary constants so that α_2 and $\frac{d\alpha_2}{dt}$ both vanish when $t=-\infty$ we find

$$\alpha_2=\frac{\cos nt}{n}\int_{-\infty}^t\chi(t')\sin nt'dt'-\frac{\sin nt}{n}\int_{-\infty}^t\chi(t')\cos nt'dt'$$

The complete value of $\chi(t)$ is given by the equation

$$\chi(t)=\tfrac{3}{4}\frac{m'a'^2(3F't^2+2G't+H')}{(c^2+4v^2\sin^2\tfrac{1}{2}\epsilon.t^2)^{\frac{5}{2}}}$$
$$-\frac{21m'a'^2av^2\sin^2\tfrac{1}{2}\epsilon(F't^4+G't^3+H't^2+K't)}{(c^2+4v^2\sin^2\tfrac{1}{2}\epsilon.t^2)^{\frac{7}{2}}}$$
$$-\frac{2m}{a^2}\log\frac{2a}{e}.\tfrac{3}{2}\frac{m'a'^2a^2(A't^4+B't^3+C't^2+D't+E')}{(c^2+4v^2\sin^2\tfrac{1}{2}\epsilon.t^2)^{\frac{5}{2}}}$$

Thus the coefficients of cos nt and sin nt in this expression for α_2 will involve integrals of the type

$$\int_{-\infty}^{t} \frac{\cos nt dt}{(a^2+t^2)^{\frac{2P+1}{2}}}$$

I have not succeeded in evaluating this integral; it is evident however that the more important part of these integrals will be produced during the time the vortices are nearly at their minimum distance apart. During the time they are far apart they will not contribute anything appreciable to this integral, so that soon after the vortex rings have passed their minimum distance the equation may without sensible error be written

$$\alpha_2 = P\frac{\cos nt}{n} - Q\frac{\sin nt}{n}$$

where P and Q are constants and

$$P = \int_{-\infty}^{+\infty} \chi(t) \cos nt.dt$$

$$Q = \int_{-\infty}^{+\infty} \chi(t) \sin nt.dt$$

There will be a similar expression for β_2. Thus the vortex rings are thrown by the collision into a state of vibration about their circular form.

We can find the action of two unequal vortices on each other by means of work of a very similar character to that just given. The only difference is that instead of the former values for f and h we must substitute the values

$$f = -c \sin \alpha - v \sin \epsilon.t$$

$$h = c \cos \alpha + (v \cos \epsilon - w)t$$

where v is the velocity of the vortex which is in front when they are nearest together, w the velocity of the one in the rear; α is the angle between the line joining their centres when they are nearest together, and the direction of motion of the vortex in the rear, β is the angle between this line and the direction of motion of the vortex in front, ϵ is the angle between the direction of motion of the vortices; α and β are given by the equations

$$w \cos \alpha = v \cos \beta$$

$$\alpha + \beta = \epsilon$$

I shall not trouble the reader with the expressions for the velocities perpendicular to the plane of either vortex and along the radius vector, but confine myself to quoting the most important consequences to be got from these expressions.

I find that after the collision the direction of motion of C D (the vortex which is in front when they are nearest together) is deflected towards the direction of motion of the other vortex A B through an angle whose circular measure is

$$\frac{m'a'^2 \cos\alpha \sin 2\beta}{\kappa c^2} = \frac{2m'a'^2 \sin^2 \epsilon vw(v - w\cos\epsilon)}{\kappa^4 c^3}$$

where m', m are the strengths of the vortices A B and C D respectively, and a' and a their radii, κ is the relative velocity of the two vortices, viz.:

$$(v^2 + w^2 - 2vw\cos\epsilon)^{\frac{1}{2}}$$

The direction of motion of A B is deflected from that of C D through an angle whose circular measure is

$$\frac{ma^2 \cos\beta \sin 2\alpha}{\kappa c^3} = \frac{2ma^2 \sin^3 \epsilon vw(w - v\cos\epsilon)}{\kappa^4 c^3}$$

The radius of the vortex C D is increased by

$$\frac{m'a'^2 a \cos\alpha \cos^2\beta}{\kappa c^3} = \frac{m'a'^2 a \sin^3 \epsilon vw^2}{\kappa^4 c^3}$$

The radius of the vortex A B is diminished by

$$\frac{ma^2 a' \cos\beta \cos^2\alpha}{\kappa c^3} = \frac{ma^2 a' \sin^3 \epsilon v^2 w}{\kappa^4 c^3}$$

The velocity of the vortex C D is diminished by

$$\frac{mm'}{2\pi a} \frac{\cos\alpha \cos^2\beta}{\kappa c^3} a'^2 \log\frac{2a}{e}$$

$$= \frac{mm'a'^2 \sin^3 \epsilon vw^2}{2\pi a \kappa^4 c^3} \log\frac{2a}{e}$$

The velocity of the vortex A B is increased by

$$\frac{mm'a^2 \sin^3 \epsilon wv^2}{2\pi a' \kappa^4 c^3} \log\frac{2a'}{e'}$$

where e, e' are the radii of the cross sections of the vortices C D, A B respectively.

The kinetic energy of the vortex C D is increased by

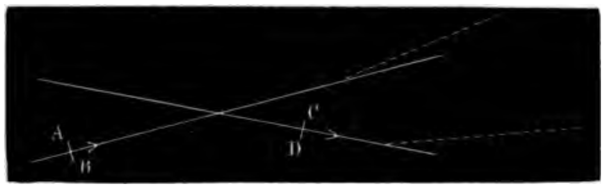
$$\frac{2\pi\rho mm'a'^2 a^2 v \cos\alpha \cos^2\beta}{\kappa c^3} = \frac{2\pi\rho mm'a'^2 a^2 \sin^3 \epsilon . v^2 w^2}{\kappa^4 c^3}$$

The kinetic energy of A B is diminished by the same amount.

With the help of these results we may find the way in which two vortices affect each other in all cases.

Case 1.—Vortices moving in the same direction.

Fig 4.



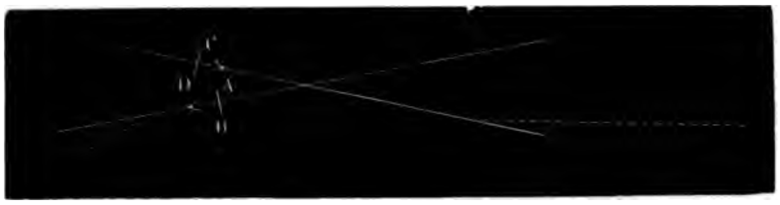
1. When the positions of the vortices when nearest together are as represented in fig. 4.

The way in which their paths are deflected is indicated by the dotted lines.

The vortex C D increases in radius and energy and its velocity is decreased.

Case II.—Vortices moving in opposite directions.

Fig 5.



The position of the vortices when nearest together is represented in fig. 5.

The way their paths are deflected is indicated by the dotted lines in the figure.

The vortex C D decreases in radius and energy and its velocity is increased.

The vortex A B increases in radius and energy and its velocity is decreased.

These results may be summed up in the following rule. The vortex which first passes through the point of intersection of the directions of motion of the vortices is deflected towards the direction of motion of the other: it increases in radius and energy and its velocity is decreased; the other vortex is deflected in the same direction: it decreases in radius and energy and its velocity is increased.

(Note added March 21st, 1882.)

Sir WILLIAM THOMSON has pointed out to me that when n is very great the time of vibration of the single vortex which for this case is (equation 23)

$$2\pi\Big/\sqrt{\frac{\varpi 2\pi^2 e^2}{l^2}\log\frac{nl}{\pi e}}$$

when l is the wave-length $2\pi a/n$, does not agree infinitely nearly as it ought with the value obtained by him for the rapidity of the transverse vibrations of a straight columnar vortex, which by formula 61 of his paper "On thè Vibrations of a Columnar Vortex," is

$$2\pi\Big/\frac{\omega 2\pi^2 e^2}{l^2}\left\{\log\frac{l}{2\pi e}+\cdot 1159\right\}$$

The results would agree approximately if $l/2e$ were indefinitely great compared with n, but it obliges us to neglect the factor $\log n$, and thus the agreement instead of getting better as it ought gets worse as n increases. The way I determined A_n is only suitable when $n\kappa$, is small, as it is only allowable to expand $\cos n\phi$ as $1-\frac{n^2\phi^2}{2}$ when $n\phi$ remains small within the limits of integration. I have therefore endeavoured to determine A_n in a way which shall not be open to these objections.

With the notation of the paper since

$$A_n=\frac{1}{\pi}\int_0^{2\pi}\frac{\cos n\phi . d\phi}{(a^2+\rho^2+\zeta^2-2a\rho\cos\phi)^{\frac{3}{2}}}$$

we may easily prove that if $a-\rho$ be small compared with a, that

$$a\left(\frac{dA_{n-1}}{da}-\frac{dA_n}{da}\right)=(n-1)A_{n-1}-(n-3)A_n$$

and

$$a\left(\frac{dA_{n+1}}{da}-\frac{dA_n}{da}\right)=(n+3)A_n-(n+1)A_{n+1}$$

If we make these substitutions we find from equations (10) and (12)

$$\frac{d\alpha_n}{dt}=\tfrac{1}{2}ma\beta_n\{\mathfrak{A}_1+\tfrac{1}{2}((n-1)\mathfrak{A}_{n+1}-(n+1)\mathfrak{A}_{n-1})\}$$

$$\frac{d\beta_n}{dt}=\tfrac{1}{2}ma\alpha_n\{\mathfrak{A}_1-\mathfrak{A}_n\}$$

Hence we have only to find an expression for $\mathfrak{A}_n$

$$\mathfrak{A}_n=\frac{1}{\pi((a+\rho)^2+\zeta^2)^{\frac{3}{2}}}\int_0^{2\pi}\frac{\cos n\phi}{\left(\kappa_1{}^2+\kappa^2\sin^2\frac{\phi}{2}\right)^{\frac{3}{2}}}$$

Now as $\kappa_1{}^2$ is very small this integral will be very large, and as the large part arises when ϕ is small or nearly 2π, we may write as the approximate value of $\mathfrak{A}_n$

$$\frac{2}{\pi\{(a+\rho)^2+\zeta^2\}^{\frac{3}{2}}}\int_0^{\infty}\frac{\cos n\phi d\phi}{\left(\kappa_1{}^2+\frac{\phi^2}{4}\right)^{\frac{3}{2}}}$$

$$=\frac{4}{\pi\{(a+\rho)^2+\zeta^2\}^{\frac{3}{2}}}\int_0^{\infty}\frac{\cos 2n\phi . d\phi}{(\kappa_1{}^2+\phi^2)^{\frac{3}{2}}}$$

Let us consider the more general integral

$$u=\int_0^\infty \frac{\cos s\phi d\phi}{(a^2+\phi^2)^{\frac{2p+1}{2}}}$$

Considered as a function of *s* it is easy to prove that the integral satisfies the differential equation

$$\frac{d^2u}{ds^2}-\frac{2p-1}{2s}\frac{du}{ds}-a^2u=0$$

or if $u=s^pv$

$$\frac{d^2v}{ds^2}+\frac{1}{s}\frac{dv}{ds}-v\left(\frac{p^2}{s^2}+a^2\right)=0$$

The solution of this differential equation may be expressed in terms of Bessel's functions of the first and second kinds ; we shall, however, only solve here the special case which we require.

It is easily proved that

$$\int_0^\infty \frac{\cos s\phi}{(a^2+\phi^2)^{\frac{3}{2}}}d\phi=-\frac{s}{a^2}\frac{d}{ds}\int_0^\infty \frac{\cos s\phi}{(a^2+\phi^2)^{\frac{1}{2}}}d\phi$$

Since $p=0$ for $\int_0^\infty \frac{\cos s\phi}{(a^2+\phi^2)^{\frac{1}{2}}}d\phi$ it satisfies the differential equation

$$\frac{d^2u}{ds^2}+\frac{1}{s}\frac{du}{ds}-a^2u=0$$

This is the equation solved by Professor Stokes in his paper "On the effect of Internal Friction on the Motion of Pendulums," Camb. Phil. Trans., 1850, and quoted by Sir William Thomson, Phil. Mag., September, 1880.

The solution is there shown to be

$$u=\left(\mathrm{E}+\mathrm{D}\log\frac{1}{as}\right)\left(1+\frac{a^2s^2}{2^2}+\frac{a^4s^4}{2^24^2}+\ldots\right)$$

$$+\mathrm{D}\left(\frac{a^2s^2}{2^2}\mathrm{S}_1+\frac{a^4s^4}{2^24^2}\mathrm{S}_2+\ldots\right)$$

where

$$\mathrm{S}_n=1^{-1}+2^{-1}+\ldots n^{-1}$$

and if as for $\int_0^\infty \frac{\cos s\phi}{(a^2+\phi^2)^{\frac{1}{2}}}d\phi$, $u=0$ when *s* is infinite E/D is shown to be

$$\log 8+\pi^{-\frac{1}{2}}\Gamma'\tfrac{1}{2}=\cdot 11593$$

so that

$$\int_0^\infty \frac{\cos s\phi}{(a^2+\phi^2)^{\frac{3}{2}}}d\phi=\mathrm{D}\Big[\Big(\cdot 11593+\log\frac{1}{as}\Big)\Big(1+\frac{a^2s^2}{2^2}+\frac{a^4s^4}{2^24^2}+\dots$$
$$+\Big(\frac{a^2s^2}{2^2}\mathrm{S}_1+\frac{a^4s^4}{2^24^2}\mathrm{S}_2+\dots\Big)$$

Therefore

$$\int_0^\infty \frac{\cos s\phi}{(a^2+\phi^2)^{\frac{3}{2}}}d\phi=-\frac{s}{a^2}\mathrm{D}\Big[\Big(\cdot 11593+\log\frac{1}{as}\Big)\Big(\frac{a^2s}{2}+\frac{a^4s^3}{2^24}+\dots\Big)$$
$$+\frac{1}{s}\Big(1+\frac{a^2s^2}{2^2}+\frac{a^4s^4}{2^24^2}+\dots\Big)$$
$$+\Big(\frac{a^2s^2}{2}\mathrm{S}_1+\frac{a^4s^3}{2^24}\mathrm{S}_2+\dots\Big)$$

Now since

$$\int_0^\infty \frac{d\phi}{(a^2+\phi^2)^{\frac{3}{2}}}=\frac{1}{a^2}$$

putting $s=0$ we find $\mathrm{D}=-1$.

Therefore

$$\int_0^\infty \frac{\cos s\phi.d\phi}{(a^2+\phi^2)^{\frac{3}{2}}}=\frac{1}{a^2}\Big\{1+\frac{a^2s^2}{2^2}+\frac{a^4s^4}{2^24^2}+\dots\Big\}$$
$$+\Big(\cdot 11593+\log\frac{1}{as}\Big)\Big(\frac{s^2}{2}+\frac{a^2s^4}{2^24}+\dots\Big)$$
$$+\Big(\frac{s^2}{2}\mathrm{S}_1+\frac{a^2s^4}{2^24}\mathrm{S}_2+\dots\Big)$$

Now approximately

$$\mathfrak{A}_n=\frac{1}{2\pi a^3}\int_0^\infty\frac{\cos 2n\phi d\phi}{(\kappa_1{}^2+\phi^2)^{\frac{3}{2}}}$$

hence A_n is found, and if we substitute these values in the expressions for $\frac{d\alpha_n}{dt}$, $\frac{d\beta_n}{dt}$ we shall be able to find the time of vibration in any particular case.

If we suppose $n\kappa_1$ is small, then approximately

$$\mathfrak{A}_n=\frac{1}{2\pi a^3}\Big\{\frac{1}{\kappa_1{}^2}+n^2\Big\{2\Big(\log\frac{1}{2n\kappa_1{}^2}+\cdot 11593\Big)+1\Big\}\Big\}$$

or if we neglect 1.23 in comparison with $2\log\frac{1}{2n\kappa_1}$

$$\mathfrak{A}_n=\frac{1}{2\pi a^3}\Big\{\frac{1}{\kappa^1}+2n^2\log\frac{1}{2n\kappa_1}\Big\}$$

Now

$$\frac{d\alpha_n}{dt}=\tfrac{1}{2}ma\beta_n\{\mathfrak{A}_1+\tfrac{1}{2}\}(n-1)\mathfrak{A}_{n+1}-(n+1)\mathfrak{A}_{n-1})$$
$$\frac{d\beta_n}{dt}=\tfrac{1}{2}ma\alpha_n\{\mathfrak{A}_1-\mathfrak{A}_n\}$$

Substituting we find

$$\frac{d\alpha_n}{dt}=-\frac{m\beta_n}{2\pi a^2}(n^2\log 2\kappa_1+(n^2-1)(n+1)\log n+1-(n-1)\log n-1)$$

$$=-\frac{m}{2\pi a^2}f\beta_n \quad \text{say}$$

$$\frac{d\beta_n}{dt}=\frac{m\alpha_n}{2\pi a^2}(n^2\log 2n\kappa_1-\log 2\kappa_1)$$

$$=\frac{m}{2\pi a^2}g\alpha_n \quad \text{say}$$

Thus

$$\frac{d^2\beta_n}{dt}+\left(\frac{m}{2\pi a^2}\right)^2 fg\beta_n=0$$

or the time of vibration

$$=2\pi\Big/\frac{\omega e^2}{2a^2}\sqrt{fg}$$

Now if n be not very large

$$f=n^2\log 2\kappa_1=-n^2\log\frac{a}{e}$$

$$g=(n^2-1)\log 2\kappa_1=-(n^2-1)\log\frac{a}{e}$$

and the time of vibration

$$=2\pi\Big/\frac{\omega e^2}{2a^2}n\sqrt{n^2-1}\log\frac{a}{e}$$

If n be large

$$f=n^2\log 2\kappa_1+n^2\log n=-n^2\log\frac{a}{ne}$$

$$g=n^2\log 2n\kappa_1=-n^2\log\frac{a}{ne}$$

and the time of vibration

$$=2\pi\Big/\frac{\omega e^2}{a^2}n^2\log\frac{a}{ne}$$

or if l be the wave length $=2\pi a/n$

$$=2\pi\Big/\frac{2\pi^2\omega e^2}{l^2}\log\frac{l}{2\pi e}$$

which agrees approximately with Sir WILLIAM THOMSON'S result.

X. THE BAKERIAN LECTURE.—*Chemical Theory of Gunpowder.*

By H. DEBUS, *Ph.D., F.R.S.*

Received February 8,—Read February 23, 1882.

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ACCORDING to BELLANI, the English army used cannon at the Battle of Crecy in the year 1346. The correctness of this report has been doubted since English and French writers in their description of the battle do not mention the use of cannon. However this may be, it is certain, that from the middle of the 14th century the application of powder to the purposes of the art of war became more and more general, until towards the close of the Middle Ages heavy ordnance was used by all European armies. The effect of this new application of gunpowder upon the civilisation of our race is usually considered to have been of the same importance as the invention of the art of printing or the discovery of America. And, although 536 years have passed away since the Battle of Crecy was fought, we have to this day no satisfactory account of the chemical reactions which occur during the combustion of gunpowder, no theory to enable us to determine the quantitative relations of the products of combustion *à priori* from the composition of the powder. The attempts, which have been made from time to time by eminent men to supply solutions of the problems indicated, have been, as is well known, unsuccessful.

In the following pages I propose to describe a theory which explains in a satisfactory manner the chemical reactions which occur during and after the explosion, not only of a powder of normal composition, but, generally, of a mixture of x molecules of saltpetre, y atoms of carbon, and z atoms of sulphur.

Until about the year 1856 the metamorphosis of gunpowder was represented in chemical text-books to take place according to the equation

$$2KNO_3+3C+S=K_2S+N_2+3CO_2$$

This equation is obviously not correct, because besides sulphide, also sulphate and carbonate of potassium are formed.

After the publication of BUNSEN's and SCHISCHKOFF's[*] classical investigation in 1857, the incorrectness of the equation was generally recognised, and the view expressed that the explosion of gunpowder could not be represented by a chemical equation on account of its complex nature. Passing by for the present the papers published between the years 1858 and 1874, I propose to take at once into consideration the most recent and important investigation by NOBLE and ABEL;[†] I do so because we receive from the pages of their papers a very complete account of our present knowledge of the combustion of gunpowder.

Five different descriptions of powder were used in their experiments.

1. Pebble powder (P.); 2. Rifle large grain (R. L. G.); 3. Rifle fine grain (R. F. G.); 4. Fine grain (F. G.); and 5. Spanish pebble powder.

The first four descriptions were manufactured at Waltham Abbey.

It will be convenient for the purposes of reference to give in the following table the composition of these powders.

TABLE I.—Showing the composition of the powders used by NOBLE and ABEL.

Constituents of powder.	Powders from Waltham Abbey.								Spanish.	
	P.		R. L. G.		R. F. G.		F. G.			
Saltpetre		74·67		74·95		75·04		73·55		75·30
Potassic sulphate		0·09		0·15		0·14		0·36		0·27
Potassic chloride		..		..		..		..		0·02
Sulphur		10·07		10·27		9·93		10·02		12·42
Charcoal Carbon	12·12	14·22	10·86	13·52	10·67	14·09	11·36	14·59	8·65	11·34
Charcoal Hydrogen	0·42		0·42		0·52		0·49		0·38	
Charcoal Oxygen	1·45		1·99		2·66		2·57		1·68	
Charcoal Ash	0·23		0·25		0·24		0·17		0·63	
Water		0·95		1·11		0·80		1·48		0·65

NOBLE and ABEL burnt from 100 to 700 grms. of powder in hermetically closed steel cylinders.[‡] The analyses of the products of combustion obtained in 31 experiments have been published, and of these it will be desirable to reproduce a few representative cases in Table II.

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[*] Pogg. Ann., Bd. cii., p. 321.

[†] Phil. Trans., Vol. 165 (1875), p. 49, Vol. 171 (1880), p. 203.

[‡] Ibid., Vol. 165, p. 61.

TABLE II.—Containing the results of nine experiments calculated for 1 grm. of powder.

Products of combustion.	F. G.			R. L. G.			P.		
K_2CO_3	0·2429	·2615	·3255	·3007	·3017	·3635	·2879	·3098	·3680
$K_2S_2O_3$	·1851	·1666	·0780	·1166	·0740	·0369	·1845	·0338	·0761
K_2SO_4	·1288	·1268	·1204	·1171	·1395	·0625	·0733	·0658	·0523
K_2S	·0000	·0196	·0252	·0230	·0337	·0565	·0128	·1055	·0220
KCNS	·0009	·0004	·0004	·0000	·0003	·0015	·0022	·0013	·0033
KNO_3	·0010	·0005	·0005	·0032	·0002	·0000	·0014	·0011	·0025
$(NH_4)_4H_2(CO_3)_3$.	·0186	·0002	·0005	·0003	·0002	·0006	·0003	·0004	·0007
S	·0026	·0068	·0306	·0041	·0262	·0480	·0111	·0340	·0484
H_2S	·0090	·0080	·0088	·0041	·0127	·0067	·0129	·0084	·0086
CO	·0316	·0339	·0343	·0303	·0390	·0472	·0419	·0473	·0362
CO_2	·2689	·2678	·2650	·2597	·2610	·2677	·2630	·2770	·2710
CH_4	·0003	·0000	·0005	·0006	·0000	·0007	·0007	·0012	·0013
H	·0006	·0008	·0007	·0005	·0007	·0005	·0005	·0005	·0005
N	·1096	·1071	·1096	·1201	·1108	·1077	·1075	·1139	·1090
No. of experiment	XL.	XIX.	LXXV.	I.	IV.	XXXIX.	XXXVIII.	XIV.	LXXVII.

From this table it is clear that not only powders of different description, but also mixtures of the same nature, will yield during combustion the products in variable quantities. The salts of potassium, especially the hyposulphite and sulphide, vary considerably in different experiments.

NOBLE and ABEL draw the following conclusions from the results of their investigations.

1. "The variations in the composition of the products of explosion furnished in close chambers by one and the same powder, under different conditions as regards pressure, and by two powders of similar composition under the same conditions as regards pressure, are so considerable, that no value whatever can be attached to any attempt to give a general chemical expression to the metamorphosis of a gunpowder of normal composition (p. 137).

"Any attempt to express, even in a comparatively complicated chemical equation, the nature of the metamorphosis which a gunpowder of average composition may be considered to undergo when exploded in a confined space, would therefore only be calculated to convey an erroneous impression as to the simplicity or the definite nature of the chemical results and their uniformity under different conditions, while it would, in reality, possess no important bearing upon an elucidation of the theory of explosion of gunpowder (p. 85).

2. "The proportions in which the several constituents of solid powder-residue are formed, are quite as much affected by slight accidental variations in the conditions which attend the explosion of one and the same powder in different experiments, as

by decided differences in the composition as well as in the size of grain of different powders (p. 137).

3. "Very small grain powder, such as F. G. and R. F. G. furnish decidedly smaller proportions of gaseous products than a large grain powder (R. L. G.), while the latter again furnishes somewhat smaller proportions than a still larger powder (pebble), though the difference between the gaseous products of these two powders is comparatively inconsiderable.

4. "In all but very exceptional results, the solid residue furnished by the explosion of gunpowder contains as important constituents, potassium carbonate, sulphate, hyposulphite and sulphide, the proportion of carbonate being very much higher, and that of sulphate very much lower than stated by recent investigators."

The view of NOBLE and ABEL may be briefly stated as follows:—

One and the same description of powder, exploded several times in succession, will yield the products of combustion, in the different experiments, in variable proportions; hence: the metamorphosis of gunpowder cannot be represented by a chemical equation.

One might suppose that, perhaps, the pressure developed during explosion had an influence on the quantities of the products of combustion. From a comparison of the analytical results and the corresponding pressures, published by NOBLE and ABEL, this, however, appears not to be the case.[*] (See p. 85 of their first memoir.)

According to NOBLE and ABEL, the chemical metamorphosis of gunpowder during explosion is a very complicated process, which cannot be explained with the data at their disposal. BERTHELOT[†] arrived at a different conclusion.

The composition of the powders of Waltham Abbey can, according to him, be represented by the symbols

$$2KNO_3+3C+S$$

which require for 100 parts of powder:

Saltpetre	74·8
Carbon	13·3
Sulphur	11·8

The analyses gave:

Saltpetre	73·55	to	75·04
Carbon	10·67		12·12
Sulphur	9·93		10·27

---

[*] An increase of pressure appears to diminish the amount of carbonic oxide. But this is not always the case, and when it does occur, it is not sufficient to explain the variations in the other products of combustion.

[†] Comptes Rendus, tom. lxxxii., p. 487.

The combined weights of potassic sulphocyanate, ammonic carbonate, hydrogen, marsh gas, and sulphuretted hydrogen amount, according to Table II., to about 1·5 per cent. They evidently originate from secondary reactions, and may, accordingly, be neglected in the following considerations.

A theory of the explosion of gunpowder ought to explain the formation of potassic carbonate, potassic sulphate, potassic sulphide, carbonic acid and carbonic oxide. Potassic hyposulphite is not a primary product, but is formed during the analysis of the powder residue.

If we select two from several experiments published by NOBLE and ABEL, viz.:—one in which the maximum amount of potassic carbonate and the minimum of sulphate were produced, and another which yielded the largest quantity of potassic sulphate and the smallest of carbonate, then, according to M. BERTHELOT, the explosion which produced the results of the first case, may be represented by three equations—

$\frac{1}{3}$ of the powder was transformed according to equation

$$2KNO_3+3C+S=K_2S+3CO_2+N_2, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad \text{I.}$$

$\frac{1}{2}$ according to

$$2KNO_3+3C+S=K_2CO_3+CO_2+CO+N_2+S, \quad . \quad . \quad . \quad . \quad . \quad \text{II.}$$

$\frac{1}{6}$ according to

$$2KNO_3+3C+S=K_2CO_3+1{\cdot}5CO_2+0{\cdot}5C+S+N_2 \quad . \quad . \quad . \quad . \quad \text{III.}$$

and in the second case, with a maximum of potassic sulphate,

$\frac{1}{3}$ of the powder was transformed according to equation I,
about $\frac{1}{2}$ according to III,
$\frac{1}{8}$ according to

$$2KNO_3+3C+S=K_2SO_4+2CO+C+N_2, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad \text{IV.}$$

and $\frac{1}{12}$ according to

$$2KNO_3+3C+S=K_2SO_4+CO_2+C_2+N_2 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad \text{V.}$$

Between the limits, marked by these two cases, are contained all the experimental results of NOBLE and ABEL. If, therefore, we assume that in a given experiment one portion of the powder used burnt according to the equations of the first, and the rest according to those of the second case, the calculated results will agree with the observations. And if the proportions of powder, which are transformed according to the one or other system of equations, be changed from experiment to experiment, the quantities of the products of combustion obtained in each experiment can be calculated in a satisfactory manner.

The assumption that during explosion one portion of the powder is transformed according to one and another portion according to another equation or system of equations is justified in the opinion of M. BERTHELOT by the further assumption, that the local conditions in a mass of burning powder are not the same in all parts, and that the cooling is too rapid to allow the products to assume a state of chemical equilibrium.

If the products were left in contact at a high temperature for a longer time, they would react upon each other, and the final result would be the same as that represented by equation V., to which corresponds the greatest evolution of heat.

This theory of M. Berthelot is very ingenious, but does not agree with experience. Considerable amounts of carbon ought to be left free at the end of each explosion. In 28 experiments of Noble and Abel no free carbon was left, and only in three cases small insignificant quantities had escaped combustion. The equations III., IV. and V. cannot be applied to the combustion of the powders of Waltham Abbey. But even if the theory were correct, it would possess no practical value, because the quantities of the powder which would burn according to each of the two systems of equations, could not be known *à priori*, but would have to be found by experiment.

Berthelot invented his theory in order to explain the remarkable result of Noble and Abel's experiments, that the same description of powder, or powders of similar composition, yield the products of explosion, in different experiments, in variable proportions.

We will now proceed to show that this result can be explained, without hypothesis or theory, in a very simple manner.

For this purpose it is desirable to express the analytical results of Noble and Abel in a manner different from the one adopted by these investigators.* If we divide the numbers of Table II. by the corresponding molecular weights, we obtain another table expressing the number of molecules of the products obtained in the different experiments by the explosion of 1 grm. of powder. For Experiment XIX. we have :—

K_2CO_3	0·2615	gramme or	0·00189	mol.
K_2SO_4	0·1268	„	0·00072	„
$K_2S_2O_3$	0·1666	„	0·00087	„
K_2S_2	0·0252	„	0·00017	„
S	0·0012	„	0·00004	atom.
CO_2	0·2678	„	0·00608	mol.
CO	0·0339	„	0·00121	„
N	0·1071	„	0·00765	atom.
H_2S	0·0080	„	0·00023	mol.
CH_4	0·0000	„	0·00000	„
H	0·0008	„	0·00080	atom.
KCNS	0·0004	„		
KNO_3	0·0005	„		
$Am_4H_2(CO_3)_3$ . .	0·0002	„		

Potassic carbonate, sulphate, sulphide, and hyposulphite, carbonic acid and carbonic oxide, nitrogen and sulphuretted hydrogen together form more than 98 per cent. of the exploded powder; accordingly, hydrogen, marsh gas, ammonic carbonate, potassic sulphocyanate and undecomposed saltpetre may, as non-essential products, be left out of consideration.

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* Noble and Abel discuss only the percentage quantities of the products of explosion.

Potassic carbonate, sulphate, hyposulphite, and sulphide contain very nearly the entire amount of potassium of the exploded powder. If, therefore, we add the number of molecules of these bodies and multiply the sum by two, we obtain the number of molecules of saltpetre in 1 grm. of powder. In order to compare the results of experiments made with the same or with different descriptions of gunpowder it is desirable to calculate these results, not, as is usually done, for the same quantity of powder, but for such quantities as contain equal amounts of saltpetre or oxygen.

The products of Experiment XIX. contain 0·00730 atom of potassium and are derived from a weight of powder containing 0·0073 mol. of saltpetre. The same products contain 0·00189 mol. of K_2CO_3, hence:

$$0{\cdot}00730 : 0{\cdot}00189 :: 16 : x \text{ and } x=4{\cdot}14.$$

And if the same mode of calculation is extended to the other products, we obtain

$$\begin{array}{l} 4{\cdot}14K_2CO_3+1{\cdot}58K_2SO_4+13{\cdot}3CO_2+8{\cdot}38N_2 \\ \qquad +1{\cdot}90K_2S_2O_3+2{\cdot}65CO+0{\cdot}08S \\ \qquad +0{\cdot}37K_2S_2 \qquad\qquad +0{\cdot}5H_2S \end{array}$$

and, from these numbers, the following composition of the F. G. powder,

$$\begin{array}{l} 16KNO_3+20{\cdot}09C+6{\cdot}70S \\ +0{\cdot}76N+1{\cdot}0H \\ \qquad +5{\cdot}69O. \end{array}$$

By a similar calculation we obtain:

Experiment LXXV.

$$\left.\begin{array}{l} 5{\cdot}10K_2CO_3+13{\cdot}08CO_2+0{\cdot}56H_2S \\ 1{\cdot}50K_2SO_4+2{\cdot}65CO \\ 0{\cdot}89K_2S_2O_3+8{\cdot}5N_2 \\ 0{\cdot}48K_2S_2+1{\cdot}58S \end{array}\right\} = \left\{\begin{array}{l} 16KNO_3+20{\cdot}83C \\ 0{\cdot}5N_2+6{\cdot}38S \\ 4{\cdot}78O \\ 1{\cdot}12H \end{array}\right.$$

Experiment IV.

$$\left.\begin{array}{l} 4{\cdot}75K_2CO_3+12{\cdot}92CO_2+0{\cdot}80H_2S \\ 1{\cdot}74K_2SO_4+3{\cdot}03CO \\ 0{\cdot}85K_2S_2O_3+8{\cdot}6N_2+1{\cdot}11S \\ 0{\cdot}65K_2S_2 \end{array}\right\} = \left\{\begin{array}{l} 16KNO_3+20{\cdot}7C \\ 6{\cdot}65S+0{\cdot}6N_2 \\ +1{\cdot}60H \\ +4{\cdot}63O \end{array}\right.$$

Experiment XXXIX.

$$\left.\begin{array}{l} 5{\cdot}70K_2CO_3 + 13{\cdot}18CO_2 + 0{\cdot}41H_2S \\ +0{\cdot}78K_2SO_4 + 3{\cdot}64CO \\ +0{\cdot}41K_2S_2O_3 + 8{\cdot}3N_2 \quad +2{\cdot}14S \\ +1{\cdot}10K_2S_2 \end{array}\right\} = \left\{\begin{array}{l} 16KNO_3 + 22{\cdot}52C \\ +6{\cdot}35S + 0{\cdot}3N_2 \\ +0{\cdot}82H \\ +3{\cdot}45O \end{array}\right.$$

Experiment XIV.

$$\left.\begin{array}{l} 4{\cdot}77K_2CO_3 + 13{\cdot}44CO_2 + 0{\cdot}51H_2S \\ +0{\cdot}79K_2SO_4 + 3{\cdot}62CO \\ +0{\cdot}36K_2S_2O_3 + 8{\cdot}67N_2 \quad +0{\cdot}21S \\ +2{\cdot}04K_2S_2 \end{array}\right\} = \left\{\begin{array}{l} 16KNO_3 + 21{\cdot}83C \\ +6{\cdot}31S + 0{\cdot}67N_2 \\ +1{\cdot}02H \\ +1{\cdot}05O \end{array}\right.$$

Experiment LXXVII.

$$\left.\begin{array}{l} 5{\cdot}95K_2CO_3 + 13{\cdot}78CO_2 + 0{\cdot}55H_2S \\ +0{\cdot}67K_2SO_4 + 2{\cdot}88CO \\ +0{\cdot}89K_2S_2O_3 + 8{\cdot}7N_2 \quad +2{\cdot}93S \\ +0{\cdot}44K_2S_2 \end{array}\right\} = \left\{\begin{array}{l} 16KNO_3 + 22{\cdot}61C \\ +6{\cdot}81S + 0{\cdot}7N_2 \\ +1{\cdot}10H \\ +5{\cdot}64O \end{array}\right.$$

These equations represent only the quantitative relations between the constituents of the powders and the products of explosion, and, accordingly, fractions of atoms and molecules are admissible.

Experiments XIX. and LXXV. were made with F. G., Experiments IV. and XXXIX. with R. L. G., and Experiments XIV. and LXXVII. with P. powder of Waltham Abbey.

It will be noticed by comparing two experiments, made with the same description of powder, that the composition of the powder deduced from one of the experiments exhibits considerable differences from the composition derived from the other experiment, and neither of them agrees with the composition found by direct analysis.

The composition of the R. L. G. powder may serve as an example.

According to Experiment IV. . . $16KNO_3 + 20{\cdot}7C + 6{\cdot}65S + 4{\cdot}63O$
„ „ Experiment XXXIX. $16KNO_3 + 22{\cdot}52C + 6{\cdot}35S + 3{\cdot}45O$
„ „ Analysis of powder . $16KNO_3 + 19{\cdot}51C + 6{\cdot}92S$

The products of combustion of Experiment IV. contain 1·2, and of Experiment XXXIX. 3 atoms more carbon than the powder used in these experiments, or in other words: the products of explosion in Experiment XXXIX. were found to contain 1·67 per cent. more carbon than the R. L. G. powder which was exploded.

The results of other experiments, made by NOBLE and ABEL, differ from each other in a similar manner.

One is forced to conclude either that the methods of analysis adopted by NOBLE and ABEL do not yield exact results, or that the powders exploded did not possess the composition which was attributed to them.

Gunpowder is a mechanical mixture of saltpetre, charcoal, and sulphur. It can hardly be expected that such a mixture should, even if the greatest care has been taken by the manufacturer, be perfectly homogeneous. Moreover, the burning of wood into charcoal will not always yield a product containing the same percentage amount of carbon, and as gunpowder is a mixture of 75 parts of saltpetre, 10 of sulphur, and 15 of charcoal, it appears *à priori* probable that the same description of powder from the same manufacturer will not always possess the same percentage composition. The amount of carbon, more particularly, may be expected to vary more or less.

In order to test this conclusion by experiment, I requested the late Mr. WILLS to analyse a sample of R. L. G. and one of pebble (P.) powder, both obtained from the Royal Arsenal, Woolwich. His results, placed side by side with those of NOBLE and ABEL, are given below.

	R. L. G.		P.	
	NOBLE and ABEL.	WILLS.	NOBLE and ABEL.	WILLS.
Saltpetre	74·95	75·10	74·67	74·26
Sulphur	10·27	8·96	10·07	9·51
Charcoal—				
Carbon . . .	10·86	12·09	12·12	11·58
Hydrogen. . .	0·42	0·54	0·42	0·51
Oxygen . . .	1·99	2·12	1·45	2·55
Ash.	0·25	0·20	0·23	0·33
Water	1·11	0·85	0·95	0·76
	99·85	99·86	99·91	99·50

It will be noticed that the amounts of carbon and sulphur found by WILLS differ considerably from those found by NOBLE and ABEL. But the best proof that the same description of powder from the same works may vary much in composition has been furnished by NOBLE and ABEL themselves. In their first memoir they assign to R. L. G. powder the above composition; in their second paper "On Fired Gunpowder" they publish the following analysis:—

Saltpetre	74·43
Sulphur	10·09
Charcoal—	
Carbon	12·40
Hydrogen	0·40
Oxygen	1·27
Ash	0·21
Water	1·05
	99·85

The sample used in the first analysis was taken from the top, the one employed in the second from the bottom of the same barrel. Two analyses of powder out of the same barrel, executed by the same chemists, gave amounts of carbon which differ from each other by no less than 1·54 per cent.! We will now consider the effect of such a difference in the percentage of carbon on the relative quantities of the products of combustion.

If in one experiment 100 grms. of powder containing 75 parts of saltpetre, 10 parts of sulphur, and 10·86 parts of carbon, and in a second experiment 100 grms. of powder with 74·43 parts of saltpetre, 9 parts of sulphur, and 12·40 parts of carbon had been exploded, then, *cæteris paribus*, in the second experiment more potassic carbonate, more sulphide and less sulphate must have formed than in the first, and the quantitative differences of the same products furnished by the two experiments would be almost as great as the greatest differences actually observed by NOBLE and ABEL for the same description of powder in the whole series of their experiments. We arrive in this manner at a very simple explanation of the experimental results upon which NOBLE and ABEL have based the conclusions mentioned under No. 1 and 2 on p. 525, without the necessity of having recourse to a theory like the one advanced by M. BERTHELOT, or renouncing all explanation like NOBLE and ABEL.

It follows from Table I. that the differences of composition between P., R. L. G., R. F. G., and F. G. powders, one compared with the other, are not greater than the differences between two samples of R. L. G. powder taken out of the same barrel. From this it appears probable that only one mixture of saltpetre, charcoal, and sulphur is prepared at Waltham Abbey, and that from this one mixture the powders P., R. L. G., R. F. G., and F. G., differing only in size of grain and, perhaps, density, are manufactured. We may take then for the composition of the said powders the mean of the numbers of all published analyses. Taking into consideration only the saltpetre, sulphur, and carbon, we obtain—

R. L. G. First analysis by Noble and Abel . .	$16KNO_3+19{\cdot}51C+6{\cdot}92S$
„ Second „ „ . .	$16KNO_3+22{\cdot}40C+6{\cdot}83S$
F. G.	$16KNO_3+20{\cdot}8C+6{\cdot}8S$
P..	$16KNO_3+21{\cdot}86C+6{\cdot}79S$

and for the mean $16KNO_3+21{\cdot}14C+6{\cdot}83S$.

If also the analyses of the late Mr. Wills are taken into consideration, then the mean composition of the powders of Waltham Abbey would be represented by the symbols

$$16KNO_3+21{\cdot}18C+6{\cdot}63S$$

The differences of composition found by analysis for the same description of powder are also of great importance in the calculations of the analytical results of the products of explosion. Noble and Abel determine the potassium in x parts of the solid powder residue, and by means of the number so obtained calculate the weight of the total residue. The amount of potassium in the quantity of powder exploded is known from the analysis of the powder, and as the whole of this potassium must reappear in the solid residue, it is easy to find the total residue if the weight of potassium in x parts of it are known.

The total weight of the powder residue subtracted from the weight of the powder used in the experiment gives as difference the total weight of the gases produced by the explosion.

This mode of calculation requires that the exact composition of the powder used in each experiment should be known. Messrs. Noble and Abel assume the composition of the powders of Waltham Abbey, P., R. F. G., and F. G., to be constant; they also consider in their first memoir that of R. L. G. to be so, but in their second memoir they base the calculations of the later experiments on the second analysis of this description of powder. This assumption is, as a matter of fact, not correct; on the contrary, we may take it as highly probable, that in any two experiments made by Noble and Abel with the same class of powder, the powder used in the one experiment was not exactly of the same composition as the powder used in the other. Accordingly the weight of the solid residue as calculated by Noble and Abel will have been found in some experiments too high, in others too low, and as a consequence of this the total weight of the gaseous products cannot have been exact. The correctness of this is proved by the differences between the composition of the powders calculated from the products of combustion, and the composition deduced from direct analysis (see pp. 530, 531). These errors will, however, compensate each other if we take the mean of the analytical results of all the experiments.

Before we proceed to do so it will be desirable to consider another circumstance which is not without influence on the final results of Messrs. Noble and Abel.

They burnt their powders in steel cylinders in quantities from 100 to 750 grms., so that the solid products of combustion after explosion remained from 60 to 120 seconds in contact in a fluid condition at a high temperature. Amongst these products we have not, as is usually assumed, potassium monosulphide, but disulphide, free sulphur, and potassium carbonate. But a mixture of polysulphides of potassium and potassium carbonate at a bright red or a white heat has a most powerful corroding effect upon metals. It is well known that the celebrated STAHL was of opinion that MOSES had dissolved the golden calf of the Israelites by means of a polysulphide of potassium.

"And he took the calf which they had made, and burnt it in the fire, and ground it to powder and strewed it upon the water, and made the children of Israel drink of it." —(Exod. xxxii. 20.)

Sulphur and ash of wood were known to the Jews, and these, at a high temperature, form liver of sulphur, capable of dissolving gold. The affinity of iron for sulphur is very strong. We may expect that, according to temperature, pressure, time of cooling, and last but not least, the condition of the surface of the cylinder, more or less of the sulphur of the powder would unite with the iron of the apparatus. This would have the same effect upon the products as if a powder with less sulphur had been burnt. If 400 grms. of powder of the average composition had been exploded, and 10 grms. of its sulphur were to unite with the iron of the apparatus, the potassium carbonate produced would be about 0·20 time greater than the amount obtained by the combustion of the same quantity of powder without this removal of sulphur by the iron of the apparatus.

According to the description given by NOBLE and ABEL of the solid powder residue, considerable quantities of ferrous sulphide were contained in it. Hence the variations in the quantities of the products of combustion of powders exploded in NOBLE and ABEL's apparatus will partly be due to the chemical action of iron and sulphur at high temperatures.

It will be observed by an inspection of the equations on pages 529 and 530, that considerably more oxygen was found in the products of combustion than was contained in the saltpetre of the exploded powder. This excess of oxygen cannot have been derived from the charcoal, or the moisture of the powder, because if it had, equivalent quantities of hydrogen ought to have been liberated. Charcoal contains more hydrogen than is necessary to form water with its oxygen. It is this excess of hydrogen which is set free, or which unites with sulphur, carbon or nitrogen forming sulphuretted hydrogen, marsh gas, or ammonia.

Hence, the excess of oxygen in the products of explosion must originate from some other source. NOBLE and ABEL, like LINCK, KAROLYI and others, adopted the method proposed by BUNSEN and SCHISCHKOFF for the analysis of the solid powder residue. This method requires that the aqueous solution of the substance should be digested with cupric oxide in order to convert the potassic sulphide into hydrate. The question suggested itself whether or not oxygen from the cupric oxide had formed with potassic sulphide, sulphate or hyposulphite.

27·3 grms. pure potassic hydrate were dissolved in water, which previously had been boiled in order to expell the air, and the solution divided into two equal parts. One of these parts was saturated with sulphuretted hydrogen in an atmosphere of hydrogen and then mixed with the other, and the solution of potassic sulphide, so obtained, digested with powder of sulphur in sufficient quantity to produce pentasulphide. The analysis of this liquid gave the following results:—

10 cub. centims. diluted with previously boiled water were acidulated with hydric chloride and heated to the boiling point. A precipitate of 1·094 grms. of sulphur was obtained. The filtrate evaporated to dryness and ignited, the dry residue moistened with hydric chloride, and then raised to a red heat gave 1·387 grms. of potassic chloride, corresponding to 0·726 grm. of potassium. On the assumption that 1 atom of sulphur had escaped as sulphuretted hydrogen, the liquid, according to these determinations, contains sulphur and potassium in the proportion of 4·67 atoms of the former to 2 atoms of the latter.

10 cub. centims. of the sulphide mixed with a solution of zinc sulphate, and the filtrate tested with an iodine solution in presence of some starch required 0·2 cub. centim. of the iodine liquid in order to produce a blue colour. 1 cub. centim. of the iodine solution corresponded to 1 cub. centim. of a solution of sodium hyposulphite containing 24·8 grms. of the salt in 1 litre.

Therefore, 10 cub. centims. of the polysulphide of potassium contain:

0·0038 grm. potassic hyposulphite
2·119 grms. potassic polysulphide.

To check these numbers, Mr. Cowper at my request dissolved the zinc sulphide in hydric nitrate of 1·5 sp. grav., and determined the zinc and the sulphur in the solution according to the usual methods.

He obtained:

Sulphur	1·285 grms.
Zinc.	0·556 grm.

or for every atom of zinc 4·69 atoms of sulphur.

180 cub. centims. of the solution of potassic polysulphide were digested in a hermetically closed flask with pure and previously ignited cupric oxide at common temperatures. The liquid assumed a brown colour, which still could be observed after two days' digestion. The flask was now placed in water of 35° C., whereupon the colour rapidly disappeared. The contents of the flask were now placed upon a filter, the cupric oxide and sulphide well washed, and filtrate and wash water united and kept in a closed bottle.

I. 25 cub. centims. of the liquid so prepared, neutralised with acetic acid, mixed with some starch solution, required 24·6 cub. centims. of iodine solution for the production of the blue colour.

25 cub. centims. in another experiment required 24 cub. centims. iodine solution.

25 cub. centims. in a third experiment required 24·2 cub. centims. iodine solution.

The mean of these experiments is 24·26 cub. centims. of iodine solution for 25 cub. centims. of the filtrate, hence, 100 cub. centims. of the united filtrate and wash water contain 1·844 grms. of potassic hyposulphite.

II. 100 cub. centims. of the same filtrate evaporated with pure hydric sulphate gave 3·653 grms. of potassic sulphate, which dissolved in water to a clear and neutral liquid. A similar quantity of hydric sulphate to that which had been used in this experiment, and out of the same bottle, left no residue after evaporation. 3·653 grms. of K_2SO_4 contain 1·6375 grms. of potassium, 1·844 grms. of $K_2S_2O_3$ contain 0·757 grm. of potassium; therefore, more than one-third, nearly one-half, of the potassium of the $K_2S_{4\cdot67}$ in the original solution appears after treatment with cupric oxide as potassic hyposulphite.

The presence of potassic hyposulphite is assumed on account of the behaviour of the liquid with iodine solution. A direct proof of its presence appeared to be desirable.

Reactions of the filtrate of the cupric oxide.

a. Hydric chloride caused turbidity after some time, probably from sulphur.

b. Barium chloride, a white precipitate, only partly soluble in hydric chloride.

c. Cupric sulphate gave after neutralisation with acetic acid a blue precipitate, which turned dark brown at 70–80° C.

A mixture of sodic hyposulphite, potassic acetate and cupric sulphate behaved in a similar manner.

d. Lead acetate and silver nitrate, respectively, gave the same reactions as they do with a solution of sodic hyposulphite.

122 cub. centims. of the strongly alkaline filtrate were neutralised with acetic acid and allowed to evaporate over hydric sulphate under the receiver of an air-pump. After a few days a great number of prismatic crystals were observed. These crystals warmed with alcohol fused into an oily liquid, which recrystallised on cooling and did not dissolve in alcohol.

Alcohol added to the mother liquor of the crystals produced a crystalline precipitate.

The original crystals, and the crystalline precipitate united weighed 2·634 grms.

According to the dètermination with iodine solution 122 cub. centims. of the filtrate from the cupric oxide, contain 2·249 grms. anhydrous, or 2·604 grms. hydrated salt of the formula $3K_2S_2O_3, 5H_2O$. The crystals dissolved easily in 3 cub. centims. of water with absorption of heat; from this solution 2·457 grms. of salt were reobtained.

0·497 grm. of the same, dissolved in water and mixed with a solution of strontium chloride, gave, after two days' standing, a very small precipitate.

0·903 grm. of the salt, dissolved in 50 cub. centims. of water, gave, on addition of 1 grm. of barium chloride a white precipitate, which left, after treatment with boiling water, 0·017 grm. of barium sulphate. The filtrates of the barium sulphate yielded a fine crop of barium hyposulphite.

As this salt is, according to my experience, easily obtained in a pure form, the whole of the potassium hyposulphite was converted into the barium compound for the following analytical determinations.

0·882 grm. baric hyposulphite boiled with hydric nitrate gave 0·763 grm. of baric sulphate; the filtrate of this gave, on addition of barium chloride, another quantity of the same salt, which, after ignition and treatment with hydric chloride, weighed 0·753 grm.

Therefore in 100 parts:

	Experiment.	Theory.	
		BaS_2O_3	BaS_2O_3, H_2O.
Barium	50·86	51·70	48·41
Sulphur	23·60	24·15	22·61

The potassium salt from which the barium hyposulphite had been prepared gave the following reactions:—

a. Hydric chloride produced, a few moments after its addition, a slightly yellow turbidity.

b. Cupric sulphate gave a slight turbidity at common temperatures, on boiling, a copious brown precipitate.

c. Lead acetate, a white precipitate, which did not change its colour at 100° C.

d. Ferric chloride gave the usual violet colour.

e. Silver nitrate, a white precipitate, which turned black at higher temperatures.

According to these experiments there can be no doubt that, by the treatment of a solution of a mixture of potassic penta- and tetra-sulphides with cupric oxide, large quantities of potassic hyposulphite are formed.

The mother liquor of the potassic hyposulphite, from which this salt had been removed by means of alcohol, was left to evaporate in vacuo. The crystalline residue, heated in a platinum crucible, produced an oily liquid, which crystallised on cooling, like potassic acetate, and weighed 2·616 grms.

It was converted by means of hydric sulphate into 2·404 grms. of potassic sulphate.

2·616 grms. of potassic acetate should give, according to theory, 2·320 grms. of potassic sulphate. If we assume that, in the solution obtained by the treatment of potassic polysulphides with cupric oxide, only potassic hyposulphite and potassic hydrate are present, then, according to the numbers given on page 536, 122 cub. centims. of the filtrate from the cupric oxide should have given 2·697 grms. potassic acetate, instead of 2·616 as found by experiment.

Hence, we may conclude that 100 cub. centims. of the filtrate from the cupric oxide contain:

1·844 grms. potassic hyposulphite,
1·263 „ „ hydrate,

and a very small amount of potassic sulphate.

At my request, Mr. COWPER digested solutions of the other sulphides of potassium with cupric oxide and examined the products for potassium hyposulphite. He found that in a solution of potassic tersulphide (K_2S_3) nearly $\frac{1}{2}$, in one of potassic disulphide about $\frac{1}{3}$, and in one of potassic sulph-hydrate about $\frac{1}{12}$ of the potassium appears after treatment with cupric oxide as hyposulphite. All these experiments lead to the conclusion that the potassic hyposulphite found in solid powder residues by BUNSEN and SCHISCHKOFF'S method had been formed during the analysis of the said residues, and was not one of the original products of explosion. This conclusion is supported by the observation of PAPE,* according to which potassic hyposulphite is decomposed at 225° C. into sulphate and pentasulphide of potassium.

At the conclusion of my experiments in July 1879, I communicated the results to Mr ABEL, and he has since then confirmed my observations.

NOBLE and ABEL say at the end of their second memoir (Phil. Trans., 1880, p. 277), "that although it would seem that in certain cases and under certain exceptional circumstances potassium hyposulphite does exist as a secondary, it exists in no case as a primary product, and should not, therefore, be reckoned among the normal constituents of powder residues."

Wishing to obtain a clear conception of the mode of action of cupric oxide in the analysis of a powder residue, I instituted the following experiments :—

6·157 grms. potassic sulphate, and 8·541 grms. of potassic carbonate were dissolved in 100 cub. centims. of water; the solution filled the space of 103·5 cub. centims.

10 cub. centims. of this liquid were mixed with 5 cub. centims. of a solution containing:

Potassium	0·389 grm.
Sulphur	0·498 „

or 3·12 atoms of sulphur for every 2 atoms of potassium.

The solution so prepared was then digested for two days with previously ignited cupric oxide in a well-stoppered bottle, at ordinary temperatures.

The mixture appeared brown, but became decolorised at 35° C.

The contents of the bottle were placed upon a filter; the black oxide and sulphide were well washed with boiling water, and both, filtrate and wash water united, were kept in a stoppered bottle. They filled the space of 578 cub. centims.

111 cub. centims. of the filtrate required 3·7 cub. centims. of the iodine solution.

192 cub. centims. of the filtrate required 6·3 cub. centims. iodine liquid. Hence, the entire filtrate contained 0·3623 grm. of potassic hyposulphite.

91 cub. centims. of the filtrate acidulated with hydric chloride, boiled for some minutes, separated from precipitated sulphur by filtration, and mixed with baric

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* Pogg. Ann., Bd. cxxii., p. 408.

chloride, gave a precipitate of baric sulphate weighing 0·158 grm. The whole filtrate therefore contains 0·749 grm. of potassic sulphate.

91 cub. centims. of the filtrate, acidulated with hydric chloride and evaporated to dryness, gave, after treatment of the residue with hydric sulphate, 0·395 grm. potassic sulphate. From this number it follows that the entire filtrate contained 1·124 grms. of potassium.

91 cub. centims. of the filtrate gave, with manganous sulphate, a precipitate, which generated with dilute hydric sulphate 0·042 grm. of carbonic acid. 578 cub. centims. of the filtrate contain 0·266 grm. carbonic acid, corresponding to 0·834 grm. potassic carbonate.

From the solution of the precipitate caused by manganous sulphate, 0·094 grm. of Mn_3O_4 was obtained, corresponding to 0·597 grm. for the entire filtrate; 0·461 grm. of this is derived from manganous carbonate, the remainder, 0·136 grm. of Mn_3O_4, from the manganous hydrate precipitated by potassic hydrate. 0·136 grm. of Mn_3O_4 corresponds to 0·139 grm. of potassium or 0·310 grm. of K_2S_3. The black cupric oxide and sulphide was dissolved in concentrated hydric nitrate and the solution precipitated by baric chloride; the weight of the baric sulphate was found to be 2·754 grms.

The following table contains the results of these experiments:—

	Composition of the original solution.	Found by analysis.
Potassic carbonate	0·829	0·834
„ sulphate	0·597	0·749
„ tersulphide	0·887	0·310
„ hyposulphite . . .	0·000	0·362
Potassium	1·124	1·124
Sulphur in CuO	0·000	0·378
„ in $K_2S_2O_3$	0·000	0·122

A considerable error attaches to the determination of the sulphur. The cupric oxide had been ignited in a Hessian crucible. From this, it appears, it became contaminated with silica and alumina. The baric sulphate, precipitated from the solution of the cupric oxide in hydric nitrate by baric chloride, contained a copper compound which I regard as a silicate, since it could not be got rid of even after long continued boiling with hydric chloride.

The analysis gives quite a wrong idea of the composition of the original solution. Not only is a portion of the potassic tersulphide converted into hyposulphite, it even appears that some has been oxidised into sulphate.

The following experiments prove the absence of sulphates in the reagents used for the above determinations.

15 cub. centims. of the solution of potassic tersulphide, acidulated with hydric

chloride, boiled, filtered and precipitated with baric chloride gave only 0·0005 grm. baric sulphate. 80 grms. of cupric oxide were dissolved in hydric chloride and the solution mixed with 1 grm. of baric chloride; even after two days' standing no precipitate could be observed.

0·854 grm. potassic carbonate, examined in a similar manner, was found to contain only 0·0007 grm. of sulphate.

1 grm. of sodic hyposulphite in 100 cub. centims. of water gave with barium chloride a crystalline precipitate which proved to be completely soluble in boiling water. Baric chloride, added to the water used in the experiments, and hydric chloride caused no precipitate.

1 grm. of sodium hyposulphite was dissolved in 200 cub. centims. of water, hydric chloride added, and the solution boiled during three-quarters of an hour.

Baric chloride precipitated 0·022 grm. of baric sulphate. Another similar experiment yielded 0·028 grm. baric sulphate.

From these experiments it is clear that in a solution containing hyposulphites and sulphates, the latter cannot accurately be determined by the ordinary method. The error from this source is, however, not sufficiently great to account for the discrepancy between the sulphate taken and found in the experiment described on page 539, and exhibited in the table given on that page.

Accordingly, it appears probable that by the action of cupric oxide upon potassic tersulphide, in *presence of potassic carbonate*, not only potassic hyposulphite but also potassic sulphate is formed.

15 cub. centims. of a solution of potassic sulphide were acidulated by addition of hydric chloride, and by boiling and filtration were separated from the precipitated sulphur. The potassic chloride left, after evaporation of the filtrate from the sulphur, was converted, by means of hydric sulphate, into potassic sulphate. The weight of this salt was found to be 2·554 grms.

5 cub. centims. of potassic sulphide solution, mixed with potassic hydrate and oxydised by chlorine, yielded 3·58 grms. of baric sulphate. 10 cub. centims. of potassic sulphide solution, precipitated by zinc sulphate, and the filtrate from the zinc sulphide examined by means of iodine solution for hyposulphites, gave numbers which indicated in 15 cub. centims. of the sulphide solution 0·114 grm. of potassic hyposulphite. Sulphuric acid was not found in the sulphide solution.

15 cub. centims. of potassic sulphide solution, out of the same bottle from which the quantities used in the above experiments had been taken, were mixed with solutions of 1·188 grms. of potassic sulphate, and of 1·650 grms. of potassic carbonate.

The mixture so prepared had the following composition :—

	Grms.		Grms.
Potassic carbonate . . .	1·650		
„ sulphate. . . .	1·188		
„ sulphide. . . .	2·534	Sulphur . .	1·4366
		Potassium .	1·0977
„ hyposulphite . .	0·114	Sulphur . .	0·0384
		Potassium .	0·0468

It was digested for a few days in a closed flask with 40 grms. of cupric oxide at common temperatures, and finally decolorised by the raising of its temperature for a short time to 35° C.

The contents of the flask were placed upon a filter and the oxide and sulphide washed with hot water. The volume of filtrate and wash water measured 1081 cub. centims.

The analysis of the liquid gave the following results:—

200 cub. centims. gave 0·394 grm. of baric sulphate; according to these numbers 1081 cub. centims. contain 1·589 grms. of potassic sulphate.

312 cub. centims. were precipitated with manganous sulphate. From the precipitate so obtained, 0·140 grm. of carbonic acid was developed by means of sulphuric acid, and 0·443 grm. of Mn_3O_4, prepared by the usual method. Hence, 1081 cub. centims. of filtrate contain 1·521 grms. of potassic carbonate and 0·708 grm. of potassium in the form of hydrate.

139 cub. centims. of the liquid required 6·4 cub. centims. of iodine solution. Therefore, 1081 cub. centims. contain 0·945 grm. of potassic hyposulphite.

200 cub. centims., evaporated with concentrated sulphuric acid, gave 1·069 grms. of potassic sulphate, containing 0·4793 grm. of potassium, or in 1081 cub. centims. 2·590 grms. of potassium.

From these data we find the following composition for the solution:—

	Composition of original solution.		Composition according to analysis.	
Potassic carbonate	1·650		1·521	
„ sulphate	1·188		1·589	
„ sulphide	2·534	S =1·4366	1·633	S =0·925
		K=1·0977		K=0·708
„ hyposulphite. . .	0·114	S =0·0384	0·945	S =0·318
		K=0·0468		K=0·387
Potassium	2·609		2·590	

The mixture of cupric oxide and sulphide, dissolved in concentrated nitric acid, yielded 8·16 grms. of baric sulphate, corresponding to 1·120 grms. of sulphur.

	Sulphur.	
	Contained in original solution.	Found by analysis.
Potassic sulphate	0·2180	0·291
„ sulphide	1·4366	. .
„ hyposulphite . . .	0·0384	0·318
Cupric oxide	0·0000	1·120
	1·693	1·729

From these experiments we conclude that by treatment of a solution containing potassic tersulphide, carbonate, and sulphate with cupric oxide, not only potassic hyposulphite but also potassic sulphate is formed.

Experiments with potassic monosulphide.

A solution of 5 grms. of pure potassic hydrate, in 100 cub. centims. of water, was divided into two equal parts, one of these was saturated with sulphuretted hydrogen and then mixed with the other.

20 cub. centims. of this solution gave, after evaporation with hydric chloride, 1·1015 grms. of neutral potassic chloride, corresponding to 0·812 grm. of potassic monosulphide (K_2S).

20 cub. centims. of the solution of the monosulphide were mixed with 2·219 grms. of potassic carbonate and 0·950 grm. of sulphate, and the mixture digested with pure cupric oxide at common temperatures, in a stoppered bottle, until all the potassic sulphide had been decomposed.

The contents of the bottle were treated in the same manner as in former experiments.

The filtrate of the cupric oxide measured 415 cub. centims.

160 cub. centims. required 2·1 cub. centims. of the iodine solution, therefore we have, in 415 cub. centims of the filtrate, 0·103 grm. of potassic hyposulphite.

It was ascertained, by special experiment, that the original potassic monosulphide did not contain hyposulphite.

60 cub. centims. of the liquid gave 0·185 grm. of baric sulphate, corresponding to 0·953 grm. of potassic sulphate in 415 cub. centims. of the filtrate.

100 cub. centims. gave with manganous sulphate a precipitate from which 0·174 grm. of carbonic acid and 0·428 grm. of Mn_3O_4 were obtained. It follows from these numbers that 415 cub. centims. of filtrate contain 0·758 grm. of monosulphide and 2·261 grms. of carbonate of potassium.

40 cub. centims. of the filtrate evaporated with hydric chloride, and the remaining potassic chloride converted by means of hydric sulphate into potassic sulphate, gave

carbonic acid and oxide, and these, together with the potassic sulphide, nitrogen and free sulphur amount to about 96 per cent. of the exploded powder. The following calculation leads to the conclusion that the oxygen of the charcoal and of the moisture in the powder does not enter into the composition of the principal products of explosion, but is eliminated in union with hydrogen in the form of water.

According to analysis, the composition of the F. G. powder is represented by the symbols

$$16KNO_3+0{\cdot}044K_2SO_4+6{\cdot}88S+20{\cdot}81C+10{\cdot}78H+3{\cdot}52O$$

if ash and moisture of the charcoal are left out of consideration. In Experiment 17 of NOBLE and ABEL the hydrogen in ammonic carbonate, sulphuretted hydrogen, and in a free state, added together is equal to 4·1 atoms. This number deducted from 10·78 atoms, the hydrogen of the charcoal, leaves as difference 6·68 atoms. These 6·68 atoms of hydrogen must have formed water, for which operation 3·34 atoms of oxygen, or almost the exact quantity contained in the charcoal, is required.

Also the moisture of the powder cannot have contributed any oxygen, because if it had, an equivalent quantity of hydrogen ought to have been set free, or entered into combination with nitrogen, carbon, or sulphur. And as in all other experiments of NOBLE and ABEL, executed with F. G. powder from Waltham Abbey, the number of atoms of hydrogen, free or combined, which occur amongst the products of combustion, is less than 4·1, it follows, that in these other experiments also the oxygen of the charcoal or of the moisture of the powder has taken no part in the explosion. And finally, since several hundred grammes of powder were exploded in a *hermetically* closed steel cylinder, oxygen from the atmosphere cannot have entered into the composition of products of combustion of the powder.

If, then, we find in one of the experiments, after the potassic hyposulphite has been replaced by its equivalent of potassic sulphide, that the sum of the quantities of oxygen contained in the potassic carbonate and sulphate, carbonic acid, and oxide exceeds the oxygen derived from the decomposed saltpetre, we may assume that this excess of oxygen is owing to some sulphate which had been formed during the process of analysis, and, accordingly, we shall be justified in deducting this excess of sulphate from the total quantity found.

The errors which have been explained on p. 533, due to the mode of calculation, will compensate each other, if the *mean of all the experimental results* be taken.

If all these corrections are carried out we obtain in the form of a chemical equation an approximately correct expression of the metamorphosis of the powders of Waltham Abbey. This equation, deduced from the 31 experiments published by NOBLE and ABEL, is:

$$\left.\begin{array}{l} 4\cdot98K_2CO_3+13\cdot13CO_2+0\cdot84S \\ +0\cdot90K_2SO_4+3\cdot23CO \quad +0\cdot67H_2S \\ +2\cdot10K_2S_2 \quad +17\cdot34N \end{array}\right\}=\left\{\begin{array}{l} 16KNO_3 \ +1\cdot34N \\ +21\cdot35C+1\cdot34H \\ +6\cdot63S \end{array}\right\} \quad . \ . \quad \text{(I.)}$$

The powder constituents on the right-hand side of the sign of equality have been calculated from the composition of the products of explosion.

The same constituents, as found by the direct analysis of the powders, are represented by the symbols:

$$16KNO_3+21\cdot18C+6\cdot63S$$

which are in close agreement with those deduced from the products of explosion. Powder of this composition, burnt according to the method of NOBLE and ABEL, will form the products of explosion in quantities as represented by equation (I.), if the small quantities of secondary products arising from the presence of hydrogen in charcoal, such as marsh gas, ammonia, and free hydrogen, are neglected.

The sulphuretted hydrogen is either the product of the direct union of hydrogen and sulphur at comparatively low temperatures, or of the action of carbonic acid and water upon potassic sulphide. In either case its formation has no direct connexion with the explosion, and it ought to be likewise omitted from an equation representing the metamorphosis of gunpowder.

0·84 atom of sulphur is represented as free, because there are no data to show how much sulphur has united with the iron of the apparatus.

It is usual to represent the potassic sulphide as monosulphide. Further on it will be shown that this is not correct, but that disulphide is really produced.

We may then replace equation (I.) by the more simple one

$$\left.\begin{array}{r} 16KNO_3+21C+5S=5K_2CO_3+13CO_2 \\ 1K_2SO_4+3CO \\ 2K_2S_2 \quad +8N_2 \end{array}\right\} \quad . \ . \ . \ . \ . \quad \text{(II.)}$$

A portion of the sulphur of the powder has united with hydrogen and iron, hence the difference of sulphur in equations (I.) and (II.).

A powder consisting of

$$16KNO_3+21C+5S$$

exploded according to the method of NOBLE and ABEL in a vessel the substance of which is not attacked by the products of combustion, ought, *cæteris paribus*, to yield the products of explosion always in the proportions represented in equation (II.).

Under very great pressures the amount of carbonic oxide appears to diminish to a small extent; this variation of the carbonic oxide has, however, only a slight influence on the other products, amounting in the case of potassic carbonate, potassic sulphate, and potassic disulphide generally to less than 0·1 of a molecule. (See note, p. 526.)

Equation (II.) expresses only the quantitative relations between the powder constituents and the products of explosion; the reactions which occur during explosion, which of them are simultaneous, and the order in which they succeed each other have still to be determined.

The solid products of explosion possess the composition and the properties of liver of sulphur prepared with an insufficient quantity of sulphur. We can conceive that, at first, all the potassium of the saltpetre forms with carbon and oxygen potassic carbonate, and that, in another stage, sulphur acts on the potassic carbonate and produces the mixture known as the solid powder residue. Or we may conceive that potassic sulphate is the first product, and that this is afterwards reduced by carbon to potassic disulphide and carbonate. Both conceptions would lead to the same results.

The experiments of KAROLYI,* executed more than 17 years ago, contain the key to a chemical theory of gunpowder, and allow us to form an idea of the nature of the reactions, and the order in which they follow each other during the combustion of powder. He proposed to himself to decide by experiment whether or not the nature of the products and their relative quantities are dependent on the pressure which obtains during explosion. CRAIG† had asserted that under great pressure, such as would exist during the explosion in a piece of ordnance, much more potassic sulphide was formed than had been obtained by BUNSEN and SCHISCHKOFF under ordinary atmospheric pressure. KAROLYI took 36·836 grms. of Austrian cannon powder, which had, according to his opinion, a composition similar to that of the powder employed by BUNSEN and SCHISCHKOFF, and enclosed the same in a small metallic cylinder, suspended in the centre of a 60-pounder hermetically closed shell. The air was then pumped out of the shell by means of an air pump, and the powder exploded by an electric current. As soon as the pressure of the gases in the cylinder had attained a certain magnitude the cylinder burst, and its contents were scattered about the space of the exhausted shell.

The capacity of the shell and the amount of powder had been so adjusted that after explosion the gases in the shell should possess a tension of about 1·5 atmospheres; they were allowed to escape into tubes and sealed up for analysis. The solid products, which were removed from the shell by means of water, as well as the gases, were examined by the methods of BUNSEN.

A similar experiment was made with 34·153 grms. of rifle powder.

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* Pogg. Ann., Bd. cxviii., p. 544.

† SILLIMAN'S Am. J., [2], vol. xxxi., p. 429; Chem. News, vol. iv., p. 18.

COMPOSITION of the powders.

	Cannon powder.	Rifle powder.
Saltpetre.	73·78	77·15
Sulphur	12·80	8·63
Charcoal—		
Carbon	10·88	11·78
Hydrogen	0·38	0·42
Oxygen	1·82	1·79
Ash	0·31	0·28

COMPOSITION of the products of explosion.

	Cannon powder. Grms.	Rifle powder. Grms.
Potassic carbonate	7·14	7·096
„ sulphate	13·61	12·354
„ hyposulphite	1·04	0·605
„ sulphide	0·04	0·000
Carbonic acid	6·40	7·442
„ oxide	0·97	0·504
Nitrogen	3·60	3·432
Hydrogen	0·04	0·047
Marsh gas	0·15	0·167
Sulphuretted hydrogen	0·10	0·079
Carbon	0·94	0·887
Sulphur	1·73	0·397
Ammonic carbonate	0·99	0·908
Loss	0·08	0·235

Cannon powder gave 30·77 per cent. of gas and 69·25 per cent. solid residue. Rifle powder gave 34·86 per cent. of gas, and 65·14 per cent. solid residue.

KAROLYI, comparing his results with those of BUNSEN and SCHISCHKOFF, arrives at the conclusion that the nature and quantities of the products of explosion are not much influenced by the conditions under which the combustion takes place, but that the composition of the powder determines in a great measure the proportions in which the products of explosion are formed. Besides this, he deduces no other conclusions from his experiments. KAROLYI'S observations do not support CRAIG'S assertion ; the pressure in the metallic cylinder before explosion must have been great, yet very little or no potassic sulphide has been formed.

A few years later (1869) FEDOROW* published the results of some experiments executed by him on the explosion of gunpowder. He, like CRAIG, concludes that under high pressure more potassic sulphide is produced than would be formed by the same powder under lower pressures.

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* Zeitschrift für Annalytische Chemie, Bd. ix., p. 127 ; STRECKER, Jahresbericht, 1869, p. 1059.

The experiments of FEDOROW, however, do not prove anything of the kind. He fired powder from a pistol and from a 9-pounder cannon, and it appears that the solid products of the powder fired from the pistol were collected in a glass tube. A charge of 0·75 grm. of powder gave a residue with proportionally more potassic sulphate and less of potassic sulphide than one of 1·5 grms., and relatively still smaller was the potassic sulphate and greater the sulphide in the residue obtained by the firing of 3 lbs. of powder from the cannon.

M. FEDOROW concludes: "A smaller amount of powder remains unconsumed, less potassic sulphate, but more sulphide and carbonate are formed under higher than under lower pressures. Time acts like pressure. If the combustion of the powder is retarded the same effects follow as if the pressure had been increased. A charge of 1·5 grms. of a mixture consisting of 100 parts of meal powder, and 0·5 part of stearic acid, gave a residue with less potassic sulphate, but with more carbonate and hyposulphite than a similar charge of ordinary powder would have done."

Potassic sulphide is, as is well known, a substance endowed with great attraction for oxygen, not only at high, but also at ordinary temperatures. It appears that the air was not excluded from the glass tube into which M. FEDOROW fired his charges for the collection of the solid residues. A greater percentage of the potassium sulphide formed by a small charge, than of the sulphide of the products of a larger charge, will in this manner be oxidised.

It thus appears that M. FEDOROW's experiments can be explained without reference to pressure. But as far as the retardation of the combustion by stearic acid is concerned, we cannot ascribe to retardation an effect which is due to the stearic acid itself. Stearic acid at a red heat reduces potassic sulphate to sulphide and probably carbonate.

M. FEDOROW could not collect and examine the gases.

The experiments of FEDOROW do not establish a relation between pressure and the nature and quantities of the products of explosion, and, consequently, do not invalidate the conclusions of KAROLYI, as stated on page 547.

We will now proceed to explain, by means of the analytical results of KAROLYI, some of the reactions which take place during the combustion of gunpowder. For this purpose it will be desirable to express the composition of the Austrian powders, and the products of explosion, by chemical formulæ.

no constituents of the saltpetre occur in them, they may be regarded as merely accessory products not directly concerned in the explosion.

As the composition of the Austrian service powders does not differ much from that of the powders of Waltham Abbey, it will be interesting to contrast the results of Karolyi with those obtained by Noble and Abel.

Remarkable differences will be observed if equation (I.), page 545 is compared with the composition of the products of combustion observed by Karolyi and represented by means of chemical symbols on page 549. Equation (I.), as well as Karolyi's results, are calculated for quantities of powder containing 16 mols. KNO_3.

Noble and Abel, according to equation (I.), found about a quarter of the potassium of the decomposed saltpetre as potassic sulphide, whereas in Karolyi's experiments the sum of the potassium in the hyposulphite, and in the sulphide of the products of combustion of cannon powder amounted to only $\frac{1}{26}$th, and in that of the rifle powder only to $\frac{1}{40}$th of the potassium of the saltpetre of the exploded powder. It seems to follow that the production of such small quantities of potassic hyposulphite and sulphide cannot be the direct result of the chief reactions of the explosion. Karolyi obtained much more potassic sulphate and less potassic carbonate than the English chemists; a considerable quantity of the carbon of his powders remained unconsumed, whilst in Noble and Abel's experiments, as a rule, every trace of this element was burnt, although the English powders contain somewhat more carbon than the Austrian. Similar differences occur between the results obtained with the gaseous products. The gases obtained by Karolyi were combustible, those of Noble and Abel were not. The gases from the Austrian powders contained more hydrogen and marsh gas and less sulphuretted hydrogen than those from the mixtures of Waltham Abbey.

	Mean percentage by volume.	
	Austrian.	Waltham Abbey.
Hydrogen	6·41	2·50
Marsh gas	2·86	0·31
Sulphuretted hydrogen . . .	0·76	2·56

Karolyi inclosed his powders in a thin brass cylinder hermetically closed, and ignited the charge by means of a galvanic current. As soon as the tension of the gases developed by the combustion had reached a certain magnitude the metal cylinder exploded, and its contents were scattered against the cold sides of the exhausted 60-pounder shell. Thus the combustion of the powder was interrupted before its completion. The correctness of this view is rendered apparent if the effect is considered which would have followed if the products of Karolyi had remained in contact for some time at a red heat. The free carbon and hydrogen, and the constituents of the marsh gas would have been burnt at the expense of the oxygen of the potassic sulphate; the quantity of the latter would have been diminished and that of the sulphide increased, the free sulphur would have decomposed potassic carbonate with production of potassic sulphate and sulphide; in short, the products of combustion would have formed in similar proportions as we find them in NOBLE and ABEL'S experiments. The products of several hundred grammes of powder remained in a fluid condition at a white heat in NOBLE and ABEL'S steel cylinder from 60 to 100 seconds in contact, whereas the 36 grms. of burning powder were scattered after the explosion of KAROLYI'S brass cylinders over the cold sides of a large iron shell, and their combustion occupied only a very small fraction of a second. The reactions between the powder constituents had time to complete themselves in NOBLE and ABEL'S steel cylinder, but they had not in KAROLYI'S small brass vessel. And although the time of the combustion in the experiments of the last-named chemist was very short, less than one second, we find all the saltpetre of the powder decomposed, and its constituents incorporated in the potassium salts, carbonic oxide and carbonic acid, or set free as nitrogen.

At the same time two of the potassium salts, the hyposulphite and sulphide, occur in such small quantities that we may regard them as secondary products, not connected with the chief reactions of the explosion, and accordingly neglect them.

Hence we have, as chief products formed in KAROLYI'S experiments: potassic sulphate and potassic carbonate, carbonic acid and nitrogen, and perhaps carbonic oxide.

The combustion of gunpowder accordingly consists of two distinct stages; a process of oxidation, which is finished in a very short time, occupying only a very small fraction of a second, and causing the explosion, and during which potassium carbonate and sulphate, carbonic acid, some carbonic oxide and nitrogen are produced, and a process of reduction, which succeeds the process of oxidation and requires a comparatively long time for its completion. As the oxygen of the saltpetre is not sufficient to oxidise all the carbon to carbonic, and all the sulphur to sulphuric acid, a portion of the carbon and a portion of the sulphur are left free at the end of the process of oxidation. The carbon so left free reduces, during the second stage of the combustion, potassic sulphate, and the free sulphur decomposes potassic carbonate. Hydrogen and marsh gas, which are formed by the action of heat upon charcoal, likewise reduce potassic sulphate, and some hydrogen combines with sulphur, forming sulphuretted hydrogen.

Great variations of pressure appear to affect the proportions of the different products in a very slight degree, so that it may be regarded as doubtful whether pressure has any influence on them.

KAROLYI'S experiments happen to be arranged in such a manner that the combustion of his powders could only proceed to the end of the first and the commencement of the second stage; in NOBLE and ABEL'S explosions, the reactions of the second stage were also completed.

This view of the combustion of gunpowder explains not only the experiments of

Noble, Abel, and Karolyi, but is also in perfect accordance with the thermochemical relations of the products of explosion.

The heat of formation of a molecule of potassic sulphate is much greater than that of one of potassic sulphide, hence, the production of the former is to be expected during explosion. In short, the formation of the molecules of potassic carbonate, potassic sulphate, and carbonic acid is accompanied by the greatest evolution of heat.

Karolyi examined the products of explosion according to Bunsen and Schischkoff's method, which does not yield exact values for potassic sulphide and sulphate. The errors arising from this source will, however, be very small, if the potassic sulphide in the original powder residue is small. The small amount of potassic hyposulphite found by Karolyi proves that the products of combustion contained, in his experiments, very little potassic sulphide. As the potassic hyposulphite must be regarded as a product of oxidation of the potassic sulphide, it has been replaced in the following calculations by its equivalent of potassic sulphide. Nevertheless, the quantity of the latter does not exceed 0·32 of a molecule, hence the error caused by the method of analysis may be neglected.

We will now take the chief products of explosion observed by Karolyi, and calculate from their composition the quantities of powder constituents which took part in their formation, and arrange the results in form of equations.

Cannon powder.

$$\left.\begin{array}{l} 3{\cdot}05\ K_2CO_3 \\ 4{\cdot}62\ K_2SO_4 \\ 0{\cdot}33\ K_2S_2 \\ 9{\cdot}23\ CO_2 \\ 2{\cdot}04\ CO \\ 7{\cdot}55\ N_2 \end{array}\right\} = \left\{\begin{array}{l} 16\ \ KNO_3 \\ 14{\cdot}32\ C \\ 5{\cdot}28\ S \\ -0{\cdot}9\ N \\ 0{\cdot}13\ O \end{array}\right.$$

Rifle powder.

$$\left.\begin{array}{l} 3{\cdot}30\ K_2CO_3 \\ 4{\cdot}49\ K_2SO_4 \\ 0{\cdot}20\ K_2S_2 \\ 11{\cdot}49\ CO_2 \\ 1{\cdot}15\ CO \\ 8{\cdot}00\ N_2 \end{array}\right\} = \left\{\begin{array}{l} 16\ \ KNO_3 \\ 15{\cdot}94\ C \\ 4{\cdot}89\ S \\ 3{\cdot}99\ O \end{array}\right.$$

The entire quantity of saltpetre contained in the powders taken for these experiments was decomposed, and its constituents, with the exception of very small quantities of potassium and nitrogen, in potassic sulphocyanate and ammonia, reappear in the above chief products. The cannon powder contains for every 16 mols. of KNO_3 about 3 atoms of sulphur more than the rifle powder (p. 549). Nevertheless, during the first stage of the combustion the quantities of sulphur consumed in the formation of potassic sulphate are nearly the same in the experiments with both powders, and the potassic carbonate and disulphide are also almost identical.

The ratios of the oxygen in the potassic carbonate, sulphate, and carbonic acid are as follows:—

	Oxygen.		
	K_2CO_3	K_2SO_4	CO_2
Cannon powder	9·15	18·44	18·46
	1	2	2
Rifle powder	9·90	17·96	22·98
	1	1·81	2·33

The amount of oxygen in the products of explosion of the rifle powder has been found about 1·2 per cent. too high, consequently an error attaches to one or more of the analytical determinations.

The metamorphosis of the cannon powder during the first stage of the combustion can almost exactly, that of the rifle powder approximately, be represented by the equation

$$10(KNO_3)+8C+3S=2(K_2CO_3)+3(K_2SO_4)+6(CO_2)+5(N_2) \quad . \quad . \quad \text{(III.)}$$

It is worthy of notice that the ratios of the oxygen in the three principal products, potassic carbonate, potassic sulphate, and carbonic acid, are, according to equation (III.), of all possible ratios the most simple, if these products are to be formed by the combustion of a mixture of saltpetre, carbon, and sulphur.

From these considerations it appears to follow that during the explosion of gunpowder, or the first stage of its combustion, the constituents of powders which differ in their composition will act on each other in certain fixed stöchiometrical proportions.

It may be assumed as highly probable that of the infinite number of mixtures, which can be prepared from saltpetre, carbon, and sulphur, some will be more combustible than others, and, among the more combustible mixtures, one will be found containing the constituents in proportions most favourable for their transformation into the chief products of explosion. In this most combustible mixture the number of the molecules of saltpetre and of the atoms of carbon and sulphur will probably stand in simple arithmetical relations to each other, and if a mixture containing the constituents in other proportions be ignited, they will tend to react on each other in the stöchiometrical proportions of the most combustible mixture.

Equation (III.) can be transformed into:

$$16KNO_3+12{\cdot}8C+4{\cdot}8S=3{\cdot}2K_2CO_3+4{\cdot}8K_2SO_4+9{\cdot}6CO_2+8N_2$$

and as during the first stage of the combustion some carbonic oxide is probably also formed, we may write the equation instead

$$16KNO_3+13C+5S=3K_2CO_3+5K_2SO_4+9CO_2+CO+8N_2 \quad . \quad . \quad \text{(IV.)}$$

This equation explains the experiments of KAROLYI in a very satisfactory manner.

If the reactions went a little beyond the first stage of the combustion, and we add to the left of the sign of equality 1·32 atoms of carbon, which would reduce some potassic sulphate with formation of potassic sulphide and carbonate, carbonic acid and oxide, according to equations which will be given afterwards, numbers are obtained which are almost identical with those calculated from KARÓLYI'S observations on the products of explosion of cannon powder (p. 552). The same remark applies to the products of the rifle powder, except the carbonic acid.

The excess of oxygen found in the products of rifle powder indicates that some error has occurred in the determinations of these products, and it seems to have influenced, almost exclusively, the carbonic acid. If the quantity of this substance is calculated according to the available oxygen of the decomposed saltpetre, a number is obtained which agrees very well with equation (IV.).

From the foregoing observations it follows that powders of the composition of the Austrian service powders—

$$16KNO_3+19{\cdot}8C+8{\cdot}7S$$
$$\text{and}\quad 16KNO_3+20{\cdot}57C+5{\cdot}66S$$

burn during the first stage of the metamorphosis according to the equation:

$$16KNO_3+13C+5S=3K_2CO_3+5K_2SO_4+9CO_2+CO+8N_2$$

and that the carbon and sulphur which the powders contain, beyond the quantities required by this equation, remain free.

The very combustible sporting powder of BUNSEN and SCHISCHKOFF contained

$$16KNO_3+13{\cdot}3C+6{\cdot}3S$$

Therefore, it will be seen that the constituents of the service powders react upon each other during the first stage of the explosion nearly in the same stöchiometrical quantities in which they are contained in the more highly combustible sporting powder.

If it be correct that equation (IV.) represents proportions of saltpetre, carbon, and sulphur, in which these substances will burn with greater facility than they do in the proportions of the service powders, then we can by means of equation (IV.) calculate the composition of a powder which shall be distinguished by its great combustibility.

Besides saltpetre, carbon and sulphur, hydrogen, oxygen, ash, and moisture are contained in gunpowder. The weight of these latter constituents is about 4 per cent. of the mixture. If we add 4 per cent. to the quantities represented by the symbols

$$16KNO_3+13C+5S$$

and consider hydrogen, oxygen, ash, and moisture united with carbon to charcoal, we obtain for the percentage composition of the most combustible mixture asked: in what proportions must the constituents of a given mixture of saltpetre, carbon, and sulphur react during the process of explosion, and what must be the ratios of the chief products of explosion, so that on one hand the total quantity of heat developed is as great as possible, and, on the other hand, the amounts of heat produced by the formation of the chief products shall stand to each other in a simple relation? the answer would be: the combustion must take place according to equation (IV.).

But not only does this equation correspond to the most simple relations of the heat of formation of the principal products, it likewise requires the most simple distribution of the oxygen of the decomposed saltpetre. If the combustion of a mixture of saltpetre, carbon, and sulphur is to produce potassic carbonate and sulphate, carbonic acid and nitrogen, and if the oxygen of each of the first three products is to stand to the oxygen of the others in the most simple ratios possible, then the mixture must burn according to equation (III.), p. 553, and as the proportions expressed by equation (IV.) closely approach to those of equation (III.), it follows that equation (IV.) fulfils all the conditions and consequences explained in the foregoing lines. And, perhaps, these relations are the cause why mixtures of saltpetre, carbon, and sulphur of different composition burn during the first stage of the explosion according to equation (IV.), and if they contain more carbon and sulphur than is required by this equation, the excess of the two elements will remain free.

These interesting conclusions I deduce from the analytical data of KAROLYI and the corrected results of NOBLE and ABEL's experiments. Their investigations, however, do not give any information about the reactions of the second stage of the combustion of gunpowder, the reduction of potassic sulphate by carbon, and the decomposition of potassic carbonate by sulphur. Hitherto it has been assumed that potassic monosulphide is formed; this is, however, a mistake.

According to BERZELIUS and MITSCHERLICH[*] the products of the decomposition of potassic carbonate by sulphur at a white heat are potassic sulphate and disulphide:

$$4K_2CO_3+7S=K_2SO_4+3K_2S_2+4CO_2$$

The question we have now to solve is: which of the sulphides of potassium is formed by the action of carbon upon potassic sulphate at a white heat?

BAUER[†] and WITTSTOCK[‡] obtained potassic carbonate and a polysulphide, but the amount of sulphur in the polysulphide was not determined.

A mixture of

26·5	grms. of	potassic sulphate
10·54	„	„ carbonate
2·5	„	charcoal

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[*] GMELIN-KRAUT, 'Handbuch der Chemie,' Bd. ii., p. 39.

[†] Ibid., p. 33.

[‡] Ibid., p. 33.

was exposed in a porcelain crucible for half an hour to a temperature approaching white heat. The contents of the crucible dissolved completely in water, forming a deep yellow solution.

20 cub. centims. of this solution acidulated with hydric chloride, gave a copious precipitate of sulphur, and the filtrate of this precipitate left, after evaporation and treatment with hydric sulphate, 2·943 grms. of potassic sulphate. The potassium of

1·960 grms. potassic sulphate
0·779 „ „ carbonate

of the salts originally taken, is, accordingly, present in 20 cub. centims. of the solution.

20 cub. centims. of the same solution gave by the usual method 1·038 grms. of baric sulphate, corresponding to 0·775 grm. of potassic sulphate.

19·28 cub. centims. of the same solution placed for some days in contact with 5 grms. of cupric oxide and the carbonic acid in the liquid determined according to Bunsen's method gave 0·37 grm. of this substance, which corresponds to 1·203 grms. of potassic carbonate in 20 cub. centims. of the solution.

1·96 grms. of potassic sulphate originally taken contain 0·36 grm. of sulphur.
0·775 „ „ „ found in 20 cub. centims. contains 0·142 grm. of sulphur.
Sulphur in potassic sulphide=0·218 grm.

20 cub. centims. of the solution contain 1·318 grms. of potassium.

1·203 grms. of potassic carbonate contain 0·680 grm. of potassium, and 0·775 grm. of potassic sulphate 0·347 grm. of potassium; the difference of 0·291 grm. represents the potassium in potassic sulphide.

Hence the potassic sulphide produced by the action of the carbon of the charcoal upon the mixture of potassic sulphate and carbonate contains:

	Grm.
Potassium.	0·291
Sulphur	0·218
	0·509

and the composition of the salts in 20 cub. centims. of the solution of the fused mass is:

	Found.	Taken.
Potassic sulphate.	0·775	1·960
„ carbonate	1·203	0·779
„ sulphide.	0·509	0·000

The composition of the potassic sulphide can be represented by the symbols

$$K_2S_{1\cdot82}$$

Therefore, potassic sulphate and the carbon of charcoal, react, under the conditions of the experiment, principally according to the equation

$$4K_2SO_4+7C=2K_2CO_3+2K_2S_2+5CO_2$$

NOBLE and ABEL calculate their potassic sulphide as monosulphide, and in a special column give, as free sulphur, the sulphur not contained in the monosulphide, potassic sulphate and potassic hyposulphite. This so-called free sulphur was in reality contained in the residues partly in union with potassium as disulphide, partly with iron as ferrous sulphide.

If we imagine their free sulphur all combined with their potassic monosulphide, we obtain:

In 15 experiments united to 2 atoms of potassium, from 1·7 to 2·44 atoms of sulphur.

In seven experiments from 2·44 to 3·00 atoms of sulphur.

And in three experiments from 3·0 to 3·7 atoms of sulphur. The mean of 25 experiments would give us for 2 atoms of potassium 2·42 atoms of sulphur. But as a portion of the so-called free sulphur was in union with iron, it follows that in the powder residues 2 atoms of potassium were on an average combined with less than 2·42 atoms of sulphur. These numbers, however, apply only to their corrected results.

According to BERZELIUS and MITSCHERLICH the action of sulphur upon potassic carbonate at a white heat produces K_2S_2; the reduction of potassic sulphate in presence of carbonate, according to my own experiments, gives $K_2S_{1\cdot 82}$; the mean of NOBLE and ABEL's experiments for the composition of the potassic sulphide formed by the explosion of powder in their apparatus is less than 2·42 atoms of sulphur for every 2 atoms of potassium, hence we may conclude that the potassium sulphide formed during the second stage of the combustion of gunpowder is the disulphide, or at least contains the two elements in a proportion closely approaching the proportion in the disulphide. The following considerations confirm this conclusion.

The mean composition of the English service powder is:

$$16KNO_3+21{\cdot}18C+6{\cdot}63S$$

The Austrian rifle powder contains:

$$16KNO_3+20{\cdot}57C+5{\cdot}66S$$

The products of combustion of the former are, according to NOBLE and ABEL,

$$4{\cdot}98K_2CO_3+0{\cdot}90K_2SO_4+2{\cdot}10K_2S+13{\cdot}13CO_2+3{\cdot}23CO+0{\cdot}84S$$

Those of the Austrian rifle powder, according to KAROLYI, are

$$3{\cdot}27K_2CO_3+4{\cdot}52K_2SO_4+0{\cdot}2K_2S_3+11{\cdot}46CO_2+1{\cdot}15CO+4{\cdot}74C+0{\cdot}79S$$

The possibility of dissociation requires the additional equation

$$K_2CO_3+K_2S_2+O_7=2K_2SO_4+CO_2 \quad . \; . \; . \; . \; . \; . \; . \quad \text{(VII.)}$$

The final results of the reactions represented by equations (III.), (IV.), (V.), (VI.), and (VII.) can be expressed by one equation.

For this purpose let x, y, and z be positive numbers, and a denote how many molecules of carbonic oxide are formed by the combustion of a quantity of powder containing x molecules of saltpetre, y atoms of carbon, and z atoms of sulphur. The general equation representing the qualitative and quantitative relations between the constituents of a mixture of saltpetre, carbon, and sulphur on the one hand, and the products of *complete* combustion on the other, will then be :

$$\left.\begin{aligned} xKNO_3+yC+zS= &\ \tfrac{1}{28}[4x+8y-16z-4a](K_2CO_3) \\ &+\tfrac{1}{28}[20x-16y+4z+8a](K_2SO_4) \\ &+\tfrac{1}{28}[-10x+8y+12z-4a](K_2S_2) \\ &+\tfrac{1}{28}[-4x+20y+16z-24a](CO_2) \\ &+\tfrac{1}{2}xN_2 \\ &+aCO \end{aligned}\right\} \; . \; . \quad \text{(VIII.)}$$

As far as the application of this equation is concerned, the following remarks are, perhaps, not unnecessary.

The charcoal of gunpowder contains, besides carbon, also oxygen and hydrogen, ash and moisture. The oxygen ofthe charcoal is, as has already been proved before (p. 544), eliminated with some of the hydrogen as water. The rest of the hydrogen of the charcoal with nitrogen, carbon, and sulphur respectively, forms by-products, the total weight of which, as a rule, does not exceed 2 per cent. of the powder burnt.

The products of combustion, with the exception of those prepared by NOBLE and ABEL, contain always some unburnt carbon and sulphur, and frequently undecomposed saltpetre.

Hence we have:

a. Chief products: K_2CO_3, K_2SO_4, K_2S_2, CO_2, CO, and N_2.
b. By-products: H_2, H_2S, CH_4, NH_3, H_2O, and KCNS.
c. Constituents of powder, not burnt: KNO_3, C, and S.

Equation (VIII.) enables us to calculate, from that portion of the powder which produces the chief products (mentioned under *a*), the quantities of these products formed during complete combustion.

We will now proceed to prove, by examples, the correctness of this equation.

BUNSEN and SCHISCHKOFF found in 1 grm. of sporting powder, and in the products of its combustion, the following quantities:—

1 grm. of powder—

Saltpetre	0·7899
Sulphur	0·0984
Charcoal—	
Carbon	0·0769
Hydrogen	0·0041
Oxygen	0·0307

Chief products of combustion—

Potassic sulphate	0·4227
„ carbonate	0·1264
„ hyposulphite	0·0327
„ sulphide	0·0213
Carbonic acid*	0·2159
„ oxide	0·0094
Nitrogen	0·0998
	0·9282

By-products—

Potassic sulphocyanate	0·0030
Hydrogen	0·0002
Sulphuretted hydrogen	0·0018
Oxygen	0·0014
Ammonia and water	0·0139
	0·0203

Powder constituents not decomposed or burnt—

Saltpetre	0·0372
Carbon	0·0073
Sulphur	0·0014
	0·0459

Hence—

Chief products	0·9282
By-products	0·0203
Unburnt powder	0·0459
	0·9944

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* Inclusive of the carbonic acid of the ammonic carbonate.

If the potassic hyposulphite is replaced by its equivalent of potassic disulphide, the quantities of the chief products expressed in molecular weights, and calculated for 16 mols. of decomposed saltpetre, we obtain:

$$1{\cdot}97K_2CO_3+5{\cdot}25K_2SO_4+0{\cdot}45K_2S_2+10{\cdot}59CO_2+7{\cdot}7N_2+0{\cdot}33K_2S+0{\cdot}71CO.$$

From these symbols we calculate the following composition of the powder:

$$16KNO_3+13{\cdot}2C+6{\cdot}48S+0{\cdot}66O.$$

If now we substitute, in equation (VIII.), for x the number 16, for y 13·2, for z 6·5, and for a 0·71, we find for the chief products:

$$2{\cdot}25K_2CO_3+5{\cdot}01K_2SO_4+0{\cdot}74K_2S_2+10{\cdot}25CO_2+0{\cdot}70CO+8N_2$$

numbers which closely agree with those found by experiment.

The powder used by **Bunsen** and **Schischkoff** contains, according to analysis:

$$16KNO_3+13{\cdot}3C+6{\cdot}3S$$

a composition which is nearly the same as that deduced from the chief products of combustion.

The analytical method in this case causes no appreciable error in the determination of the potassic sulphate. Neither, according to my own analysis, does potassic monosulphide, nor, according to **Noble** and **Abel**'s analysis of the residues of mining powder, does potassic disulphide produce potassic sulphate by treatment with cupric oxide. **Bunsen** and **Schischkoff**'s residues contained a mixture of mono- and disulphide, and of both only a comparatively small quantity.

Linck[*] examined the products of combustion of the Würtemberg service powder according to the method employed by **Bunsen** and **Schischkoff**. He obtained the following results:—

Composition of the powder—

Saltpetre	0·7470
Sulphur	0·1245
Charcoal—	
Carbon	0·0905
Hydrogen	0·0041
Oxygen	0·0278
Water	0·0060
	0·9999

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[*] Ann. der Chemie und Pharm., Bd. cix., p. 53.

Chief products of combustion—

Potassic sulphate	0·2891
„ carbonate	0·1537
„ hyposulphite . . .	0·0374
„ disulphide	0·0959
Carbonic acid*	0·2345
„ oxide	0·0118
Nitrogen	0·0952
	0·9176

By-products—

Potassic sulphocyanate . .	0·0116
Sulphuretted hydrogen . .	0·0238
Hydrogen	0·0003
Oxygen	0·0001
Ammonia and water . . .	0·0098
	0·0456

Powder constituents not decomposed or burnt—

Saltpetre	0·0120
Carbon	0·0183
Sulphur	0·0031
	0·0334

Hence—

Chief products	0·9176
By-products	0·0456
Not burnt	0·0334
	0·9966

If we express the quantities of the chief products by means of molecular weights, and calculate how much of each would be formed by the combustion of a quantity of powder containing 16 mols. of saltpetre, we obtain :

$$2{\cdot}43K_2CO_3+3{\cdot}64K_2SO_4+0{\cdot}43K_2S_2O_3+1{\cdot}47K_2S_2+11{\cdot}71CO_2+0{\cdot}92CO+14{\cdot}94N$$

and expressed in powder constituents :

$$16KNO_3+15{\cdot}06C+7{\cdot}44S-1{\cdot}06N-0{\cdot}52O.$$

The powder contained, according to analysis :

$$16KNO_3+16{\cdot}3C+8{\cdot}4S.$$

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* Inclusive of the carbonic acid in the ammonic carbonate.

The by-products contain no oxygen of the saltpetre, and with the exception of the potassium and nitrogen in the potassic sulphocyanate, and the nitrogen of the ammonia, their elements are derived from the charcoal and sulphur. The chief products should, accordingly, contain all the oxygen of the decomposed saltpetre. This, however, is not the case; 0·52 of an atom is wanting, and if the potassic hyposulphite, as we must assume, has been formed by the oxidising action of the cupric oxide upon the potassic disulphide, then no less than 1·81 atoms or nearly $\frac{1}{26}$th of the oxygen of the decomposed saltpetre have disappeared. LINCK himself finds in the products of combustion 1·75 per cent. or about $\frac{1}{22}$nd less oxygen than in the original powder, and this loss of oxygen would have appeared still greater if he had not assumed that the oxygen of the hyposulphite had been derived from the saltpetre. On the other hand, LINCK finds in the products of explosion 0·71 per cent. or $\frac{1}{12}$th to $\frac{1}{13}$th more of carbon, and 0·9 or $\frac{1}{13}$th to $\frac{1}{14}$th more of sulphur than in the original powder. Accordingly, it follows, either that LINCK's powder has not the composition which he ascribes to it, or that some considerable errors attach to the analytical data of the products of explosion. Hence, no near agreement can here be expected between theory and experiment.

If we replace the potassic hyposulphite found, by its equivalent of potassic disulphide, express the quantity of the chief products by molecular weights, and calculate the composition of that portion of the powder which was transformed into the chief products, we obtain

$$16KNO_3+15·06C+7·44S-1·06N-1·81O.$$

and if in equation (VIII.) we place,

$$x=16 \qquad y=15$$
$$z=7·5 \qquad a=1·0$$

we find the following theoretical values, which are placed by the side of the experimental numbers

		Theory.	Experiment.
$16KNO_3$ $15C$ $7·5S$ $=$	K_2CO_3 . . .	2·14	2·43
	K_2SO_4 . . .	4·21	3·64
	K_2S_2	1·64	1·90
	CO_2	11·85	11·71
	CO	1·0	0·92

Some mistake appears to have occurred in the determination of the potassic sulphate and carbonate. The calculation could likewise be carried out in the following manner: The unburnt portion of the powder, and the quantities of the elements contained in the by-products, could be subtracted from the composition of the quantity of powder taken for experiment, and from the rest, the quantities of the products might be

The mean composition of the powders of Waltham Abbey is expressed by the symbols

$$16KNO_3+21\cdot18C+6\cdot63S$$

and because, during their combustion, 0·67 mol. of sulphuretted hydrogen is formed, there remain

$$16KNO_3+21\cdot18C+5\cdot96S$$

for the formation of the principal products.

But as the portion of the sulphur which has united with the iron of the apparatus has not been determined by direct experiment, we are obliged to form an estimate of its amount from other considerations.

According to the remarks on pages 557–559, potassium disulphide is produced by the metamorphosis of gunpowder. The combustion of a quantity of powder containing 16 mols. of saltpetre produces, if we take the mean of all the experiments of NOBLE and ABEL, 0·90 mol. of K_2SO_4 and 2·1 mols. of K_2S_2, for which quantities 1 mol. of K_2SO_4 and 2 mols. of K_2S_2 have been placed in equation (II.). From this it follows that 5 atoms of sulphur have taken part in the metamorphosis, and that 0·96 atom of sulphur has remained free or has united with the iron of the explosion apparatus.

The experiments of KAROLYI support this conclusion. The Austrian cannon powder containing 8·7 atoms, and the rifle powder 5·66 atoms of sulphur for every 16 mols. of saltpetre, and in the principal products of combustion of the former 5·25 atoms, and in those of the latter 4·89 atoms of this element were found, the rest of the sulphur having remained free. In spite of the great difference of the amounts of sulphur in the two descriptions of powders, we find in their products of combustion, for 16 mols. of decomposed saltpetre, almost the same quantity of sulphur, 5 atoms, in both cases. In NOBLE and ABEL'S experiments the sum of the sulphur in the sulphate and disulphide is likewise equal to 5 atoms. From this equality we may conclude that in NOBLE and ABEL'S, as in KAROLYI'S experiments, during the first stage of combustion, or the stage of explosion, the powders were transformed according to equation (IV.), page 553, and that 5 of the 5·96 atoms of sulphur entered into combination. The remaining 0·96S should, during the second stage, have reacted with potassic carbonate according to equation (V.), page 559, but as this reaction appears not to have occurred, we may conclude, with great probability, that the 0·96 atom of sulphur united with the metal of the apparatus.

Since in NOBLE and ABEL'S experiments all the powder introduced into their apparatus was completely burnt, and as the sum of the weights of the secondary products, after deducting the sulphuretted hydrogen, is very small, and finally, because potassium, oxygen, and carbon in the principal products of combustion occur almost in the same proportions as in the saltpetre and charcoal of the original powder, we can substitute for x and y, in equation (VIII.), the values derived directly from the composition of the powder.

COMPOSITION of the powder

	Saltpetre	61·66
	Sulphur	15·05
Charcoal	Carbon	17·93
	Hydrogen	0·66
	Oxygen	2·23
	Water	1·66
		99·19

which may be represented by the symbols:

$$16KNO_3+12{\cdot}3S+\underbrace{39{\cdot}18C+16{\cdot}16H+3{\cdot}65O}_{\text{charcoal.}}$$

This mixture contains sulphur and carbon nearly in the same proportions as they occur in the service powder of Waltham Abbey [S : C=1 : 3·21], and therefore differs by containing much less saltpetre.

The products of combustion, calculated for 16 mols. of decomposed saltpetre, yielded the following results :—

$$\begin{array}{lll} 3{\cdot}66K_2CO_3+13{\cdot}47CO_2 & +2{\cdot}96H_2S+2C & \\ +0{\cdot}04K_2SO_4+14{\cdot}10CO & +1{\cdot}11CH_4+1{\cdot}27S & \\ +4{\cdot}11K_2S_2 \ + \ 7{\cdot}96N_2 & +2{\cdot}2H_2 & \\ +0{\cdot}36KCNS & +0{\cdot}08[NH_4]_4H_2(CO_3)_3 & \end{array}$$

from which we calculate the composition of the powder as follows :—

$$16KNO_3+12{\cdot}85S+\underbrace{34{\cdot}94C+16{\cdot}2H+4{\cdot}42O}_{\text{charcoal.}}$$

If we contrast the above results with those obtained by the explosion of the service powders of Waltham Abbey, (I.), page 545, it is seen that the large excess of carbon and sulphur in the mining powder has not diminished the amount of carbonic acid, but greatly increased the quantity of carbonic oxide. In the service powders, the oxygen of the charcoal is eliminated, with hydrogen, as water; in the mining powder it is found at the end of the combustion in union with carbon as carbonic oxide. The hydrogen of the charcoal thus set free partly remains so, partly unites with carbon and nitrogen respectively, forming marsh gas and ammonia, and during the cooling of the products, at a lower temperature, gives rise to the generation of much sulphuretted hydrogen.

In consequence of the great excess of charcoal and sulphur, carbonic oxide, marsh gas and sulphuretted hydrogen are, calculated for 16 mols. of decomposed saltpetre, from four to five times greater in the products of the mining powder than in those of the service powder. The gases of the former are combustible, those of the latter are not.

The potassic sulphocyanate has been formed by the metamorphosis of CURTIS and HARVEY's powder in quantities ten times as large as were observed amongst the products of the Waltham Abbey mixtures. It is well known that potassic carbonate, sulphur and charcoal, at a white heat, in an atmosphere containing nitrogen, will produce potassic sulphocyanate. The amount of potassic sulphate, as might be expected, present among the products of the mining powder is almost *nil*, and the source of the by-products of the combustion of gunpowder is laid bare.

If we discard the by-products, we obtain for the combustion of a powder with an excess of carbon the equation:

$$16KNO_3+28C+8S=4K_2CO_3+4K_2S_2+12CO_2+12CO+8N_2$$

or more simply:

$$4KNO_3+7C+2S=K_2CO_3+K_2S_2+3CO_2+3CO+2N_2$$

which represents with sufficient exactness the reactions between saltpetre, carbon, and sulphur, when an excess of carbon is present and is introduced in the form of charcoal. Carbonic acid and oxide have been found by experiment to be a little higher, in consequence of the action of the oxygen of the charcoal, potassic carbonate a little lower, in consequence of the formation of some potassic sulphocyanate, than is required by the foregoing equation.

It is also worthy of notice that in spite of the presence of free carbon, more than 13 mols. of CO_2 have remained undecomposed.

From the remarks of the preceding pages it follows that during the complete metamorphosis of powder, the reactions between the constituents of saltpetre, the carbon of the charcoal, and sulphur, take place according to equations (III.), (IV.), (V.), (VI.), and (VII.), and that by means of equation (VIII.) the products, namely, potassic carbonate, potassic sulphate, potassic disulphide, carbonic acid and nitrogen, which are formed during the combustion of a weight of powder containing x mols. of saltpetre, y atoms of carbon, and z atoms of sulphur, can be calculated in a satisfactory manner. It now remains to calculate, by means of the same equation, the quantities of heat, gas, and energy which powders of various composition are able to produce.

For this purpose we assume that we have to deal with mixtures of saltpetre, sulphur, and pure *carbon*, and that the combustion is complete, viz.: that it runs to the end of the second stage. If we conceive that during the transformation of the powder no carbonic oxide is formed, we should, as a consequence, have a considerable simplification of equation (VIII.) without influencing much the calculated amounts of gas and heat.

The conversion of the carbonic oxide to carbonic acid could only take place at the expense of the oxygen in the potassic sulphate; if it occurred according to the equation

$$K_2SO_4+4CO=K_2S+4CO_2$$

the volume of the gas would not be changed. But since potassic disulphide is formed, we have to base our calculations on equation

$$2K_2SO_4+7CO=K_2CO_3+K_2S_2+6CO_2$$

from which it follows that, if no carbonic oxide but only carbonic acid is produced, the volume of the entire gas will be diminished by $\frac{1}{7}$th of the volume of the carbonic oxide which in reality is formed.

The greatest amount of carbon in gunpowders generally, as far as I know, is contained in the mixtures of Waltham Abbey, and these also produce the largest quantity of carbonic oxide, 3 mols. or 6 vols. for every 16 mols. of decomposed saltpetre. In addition to 3 mols. of carbonic oxide, 13 mols. of carbonic acid and 8 mols. of nitrogen are generated, which together amount to 24 mols. or 48 vols. of gas.

Now, if in place of carbonic oxide, carbonic acid had been formed, the volume of the entire gas would have been 47·14 instead of 48 vols. In other words, if we frame our calculation on the assumption that only carbonic acid and no oxide has resulted from the combustion, we shall find for the English service powders 1·8 per cent. less gas than was actually obtained by experiment. And as other descriptions of powder contain less carbon than those of Waltham Abbey, in their case the error will be smaller than 1·8 per cent. If then we calculate the volumes of gas which mixtures of saltpetre, carbon, and sulphur in various proportions will produce, on the assumption that no carbonic oxide, but only carbonic acid is formed, we shall obtain numbers that will not differ much from the sum of the volumes of carbonic acid, carbonic oxide, and nitrogen produced by gunpowders containing corresponding quantities of saltpetre, carbon, and sulphur.

By adding the coefficients of carbonic acid and nitrogen of equation (VIII.), and putting $x=16$, and $a=0$, we obtain for the sum, G, of the molecules of carbonic acid and nitrogen, which a mixture of 16 mols. of saltpetre, y atoms of carbon, and z atoms of sulphur, by its complete combustion, can produce, the equation

$$G=\frac{160+20y+16z}{28}$$

and for the volume, V

$$V=\frac{160+20y+16z}{14} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad \text{(IX.)}$$

In BUNSEN and SCHISCHKOFF'S experiment 16 mols. of saltpetre, 13·3 atoms of carbon, and 6·3 atoms of sulphur were consumed in the formation of the chief products of combustion (page 562). The values $y=13·3$, $z=6·3$, placed in equation (IX.), give $V=37·62$. Now 16 mols. of saltpetre, 13·3 atoms of carbon, and 6·3 atoms of sulphur correspond to 1977·2 parts by weight, and if these parts are expressed in grammes, then

1 vol. of gas will be equal to 11·19 litres, and 37·62 vols.=420967·8 cub. centims. Hence, 1 grm. of the powder would produce 212·9 cub. centims. of gas.

But only 92·8 per cent. of the powder was transformed, according to equation (VIII.), therefore

$$\frac{212·9\times 92·8}{100}=197·5 \text{ cub. centims.}$$

BUNSEN and SCHISCHKOFF found 193·1 cub. centims. If we deduct from this number 4·5 cub. centims., the volume of the gaseous by-products, hydrogen, sulphuretted hydrogen, and oxygen, and add 7·4 cub. centims. for the carbonic acid of the ammonic carbonate, we obtain 196 cub. centims. for the gas found by experiment.

Hence:

Experiment.	Theory.
196 cub. centims.	197·5 cub. centims.

The chief products of the combustion of LINCK's powder contain, according to his analysis, the powder constituents in the proportion

$$16KNO_3+15·06C+7·44S.$$

If we substitute in equation (IX.) for y the number 15, and for z 7·5, we obtain $V=41·42$. $16KNO_3+15C+7·5S$ corresponds to 2036 parts by weight. Expressed in grammes 1 vol. of the gas will be equal to 11190 cub. centims., therefore 41·42 vols. =463489·8 cub. centims., and 1 grm. of the powder would yield 227·6 cub. centims.

As only 91·7 per cent. of the Würtemberg service powder was transformed, according to equation (VIII.), we have

$$\frac{227·6\times 91·7}{100}=208·7 \text{ cub. centims. for the theoretical volume.}$$

LINCK found 218·35 cub. centims.; but from this must be subtracted 15·67 cub. centims. for sulphuretted hydrogen, 3·56 cub. centims. for hydrogen, and 0·09 for oxygen, leaving 199·3 cub. centims. of gas as the product of combustion of 1 grm. of powder according to equation (VIII.). Adding 5·8 cub. centims. for carbonic acid in ammonic carbonate, we obtain

Experiment.	Theory.
205·1 cub. centims.	208·7 cub. centims.

If the difficulties which have to be overcome in order to obtain exact results in the determination of the products of explosion of gunpowder are considered, the differences between the theoretical and experimental numbers appear to come within the errors of observation.

In NOBLE and ABEL's experiments 16 mols. of saltpetre, 21·35 atoms of carbon, and 5·1 atoms of sulphur were transformed according to equation (VIII.). If in equation (IX.) for y the number 21·35, and for z the number 5·1 is substituted, the volume of gas is found to be 47·73.

$16KNO_3+21{\cdot}35C+5{\cdot}1S$ are equal to 2035·4 parts by weight, and 47·73 vols.= 534098·7 cub. centims., if the weight is expressed in grammes, therefore: 1 grm. of the powder yields 262·4 cub. centims. of gas.

On an average, 93·75 per cent. of the weight of the powder was transformed, according to equation (VIII.), hence we have:

$$\frac{262{\cdot}4\times 93{\cdot}75}{100}=246{\cdot}0 \text{ cub. centims.}$$

for the theoretical volume of gas formed by the combustion of 1 grm. of powder. The mean of Noble and Abel's observations is 268·7 cub. centims. From this number 14·5 cub. centims. must be subtracted for the volumes of sulphuretted hydrogen, marsh gas, and hydrogen, leaving 254·2 cub. centims. for the carbonic acid, carbonic oxide, and nitrogen. Therefore we obtain, for the volume of the gas produced by 1 grm. of service powder from Waltham Abbey,

Experiment.	Theory.
254·2 cub. centims.	246 cub. centims.

the difference between the two numbers is only 1·9 cub. centim. greater than the greatest difference between two observations made with R. L. G. powder.

The three descriptions of powder, Bunsen and Schischkoff's, Linck's, and Noble and Abel's, contain from 13 to 21 atoms of carbon, and from 6·3 to 8·4 atoms of sulphur for every 16 mols. of saltpetre, and are good representatives of gunpowder in general.

The values calculated for the volumes of the gases furnished by these powders are near enough to those found by experiment to show the correctness of the theoretical considerations on which equation (IX.) has been framed, and to justify the use of this equation for the determination of the volume of gas produced by mixtures which contain saltpetre, carbon, and sulphur in proportions different from those of the Waltham Abbey, the Würtemberg, or Bunsen and Schischkoff's powder.

The amount of heat generated by the combustion of a mixture of saltpetre, *pure* carbon, and sulphur can be found in the following manner:

If we assume, as in the case of the calculation of the gas, that no carbonic oxide is formed, that is to say put $a=0$, multiply the heat of formation of each product with its coefficient in equation (VIII.), add the products thus formed and subtract from the sum the heat of formation of saltpetre: the difference will be equal to the heat generated by the combustion of the mixture.

quantity, when compared with the amount given off by the formation of potassic carbonate, potassic sulphate, potassic disulphide, and carbonic acid. The following condition has a greater influence on the heat of combustion of ordinary gunpowder.

The combustion ought to be complete; but in BUNSEN and SCHISCHKOFF'S as well as in LINCK'S experiments, a not inconsiderable portion of the powder remained unburnt. In every calorimetric determination all the products ought to be carefully examined, and this it seems was not done by those who have determined the heat of combustion of gunpowder. From the foregoing remarks we conclude that no close agreement can be expected between the heat of combustion calculated by means of equation (X.) for a mixture of 16 mols. of KNO_3, y atoms of C, and z atoms of S, and that generated by an ordinary gunpowder containing saltpetre, carbon, and sulphur in the same proportions.

In BUNSEN and SCHISCHKOFF'S powder we have for every 16 mols. of saltpetre, 13·3 atoms of C, and 6·3 atoms of S. If we substitute for y the number 13·3 and for z the number 6·3 in equation (X.), we obtain:

$$W=1546688 \text{ cal.}$$

$16KNO_3+13{\cdot}3C+6{\cdot}3S=1977{\cdot}2$ parts by weight, or one part of their powder would furnish 782 units of heat. BUNSEN and SCHISCHKOFF found 619·5. This number is, I believe, the result of one experiment made with 0·71 grm. of powder; it is evidently much too small. NOBLE and ABEL found, for the heat of combustion of the powders of Waltham Abbey, values which vary between 696 and 727 units. Their numerous calorimetric determinations were made by the combustion of the powders in the explosion apparatus, and several hundred grammes were used in each experiment. It is known that the combustion under such conditions is complete. But as the English powders contain much more carbon than BUNSEN and SCHISCHKOFF'S, or 4 per cent. less of saltpetre, they ought to have produced less heat. From these considerations it seems to follow that in BUNSEN and SCHISCHKOFF'S experiment a portion of the powder taken escaped combustion.

It has been shown that 16 mols. of saltpetre, 21 atoms of C, and 5 atoms of S take part in the metamorphosis of the powders of Waltham Abbey. If for y the number 21, for z the number 5, are substituted in equation (X.), the value of W is found to be equal to 1427789 cal.; but $16KNO_3+21C+5S=2028$ parts by weight, hence 1 grm. of powder would generate 704 units of heat.

NOBLE and ABEL found in the first series, comprising five experiments, numbers which give a mean of 702·34 units for 1 grm. of powder.

In another series of 19 experiments greater numbers were obtained than in the first. The mean of all 24 experiments is equal to 719·9 cal.

The theoretical number of 704 cal., however, corresponds to a mixture of saltpetre, sulphur, and pure carbon.

93·75 per cent. of the English service powders are transformed, according to equation sulphur, which shall, of all possible mixtures of this nature, possess the greatest energy; the results so obtained will be nearly correct for ordinary gunpowders. This would be the most general form of the problem of the explosion of gunpowder which could be proposed for solution to a chemist.

Equations (IX.) and (X.), viz. :—

$$V=\tfrac{160}{14}+\tfrac{20}{14}y+\tfrac{16}{14}z \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad \text{(IX.)}$$

$$W=1000[1827{\cdot}154-16{\cdot}925y-8{\cdot}788z] \quad . \; . \; . \; . \; . \; . \quad \text{(X.)}$$

at once show that, if for a given weight of saltpetre, 16 mols., the carbon and sulphur of the mixture were allowed to increase, the volume of gas generated by combustion would likewise increase, but the quantity of heat will grow smaller, and if the carbon and sulphur be diminished the gas will also become less, but the heat of combustion will increase.

NOBLE and ABEL have called attention to the fact* that the products of heat and gas obtained by them in their various experiments with different descriptions of powder do not differ much from each other. The explanation of this interesting observation is to be found in equations (IX.) and (X.).

The work which can be performed by a given weight of powder will, *cæteris paribus*, be proportional to the volume of gas and amount of heat, respectively, which the powder can produce by its combustion, and hence, will be proportional to the product of both. This is, however, only approximately correct, because, if we have to compare the work which two powders of different composition can produce, the gases given off by the one will not contain the elements in the same proportion as those produced by the other; a portion of the energy developed will be consumed in the performance of interior work during the expansion of the carbonic acid. This portion is, however, very small.

The product of (IX.) and (X.) is:

$$10440{\cdot}88-12{\cdot}09y^2+1208{\cdot}39y-15{\cdot}95yz+993{\cdot}867z-5{\cdot}022z^2=\frac{W.V}{1000\times 2}=E \;. \qquad \text{(XI.)}$$

the factor 1000 in (X.) has been omitted, or W divided by 1000, and V by division by 2 has been converted from volumes into molecules.

The equations (IX.) and (X.) are based on the assumption that no carbonic oxide is formed during the combustion of a mixture of saltpetre, carbon, and sulphur. In consequence, the volume of gas calculated by formula (IX.) for a mixture of the composition like those of Waltham Abbey is 1·8 per cent. smaller, and the amount of heat according to equation (X.) 2·6 per cent. larger than it would have been if the carbonic oxide had been taken into consideration. These errors nearly compensate each other in the product E in the equation (XI.), so that the values of E are but *little* affected by putting a, the carbonic oxide, $=0$ in equation (VIII.).

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* Phil. Trans., 1880, p. 230.

Equation (XI.) can be used for the calculation of the relative energies of weights of powder containing 16 mols. of saltpetre, y atoms of carbon, and z atoms of sulphur.

The question now arises for what values of y and z will E in equation (XI.) assume a maximum value, provided that y and z render the coefficients of equation (VIII.) positive,—the condition which must be fulfilled in a chemical equation.

If we put $a=0$ in equation (VIII.), we obtain:

$$\left.\begin{array}{r} x\mathrm{KNO_3} \\ +y\mathrm{C} \\ +z\mathrm{S} \end{array}\right\} = \left\{\begin{array}{l} \frac{1}{28}[4x+8y-16z](\mathrm{K_2CO_3}) \\ +\frac{1}{28}[20x-16y+4z](\mathrm{K_2SO_4}) \\ +\frac{1}{28}[-10x+8y+12z](\mathrm{K_2S_2}) \\ +\frac{1}{28}[-4x+20y+16z](\mathrm{CO_2}) \\ +\frac{1}{2}x\mathrm{N_2} \end{array}\right. \quad \ldots\ldots \quad \text{(XII.)}$$

in which, as in (VIII.), x, y, and z denote positive numbers. Let a rectangular coordinate system be given with its origin in point A, and the coordinates of a point P be represented by x, y, and z. The coefficients of potassic carbonate, sulphate, and disulphide in (XII.) will for certain values of x, y, and z be equal to 0. The equations:

$$\begin{aligned} 4x+8y-16z&=0 \\ 20x-16y+4z&=0 \\ -10x+8y+12z&=0 \end{aligned}$$

satisfied by these values represent three planes which form a trihedral angle with its vertex in the origin and one edge in the $x\,\mathrm{A}\,y$ plane. The points within the trihedral angle have coordinates which will render all the coefficients of (XII.) positive, those situated outside give values for x, y, and z, which will make at least one of the three coefficients of the potassium salts negative. Hence, coordinates of the points within the trihedral angle denote quantities of saltpetre, carbon, and sulphur, which can transform themselves completely into potassic carbonate, sulphate, disulphide, carbonic acid, and nitrogen, whereas the points outside represent, by their coordinates, quantities of the powder constituents which cannot do so entirely, because one or the other of these constituents is in excess or defect.

The points on the faces of the trihedral angle correspond to mixtures which will burn with the production of two, those on the edges with only one of the three potassium salts.

But to show the connexion between the quantities of the constituents of a given powder and those of its products of combustion, we need only consider relative, and not absolute quantities.

If a straight line be drawn through the origin within the trihedral angle the ratios of the coordinates of every point upon it will be the same.

A plane at right angles to the x axis will cut the faces of the trihedral angle so as to form a triangle B D C (see fig. 1), and the coordinates of the points inside this triangle will represent all possible proportions of carbon and sulphur which can with a given weight of saltpetre transform themselves into the products of combustion indicated in equation (XII.). If then in (XII.) we attribute to x the constant value 16, we obtain:

$$\left.\begin{array}{l} 16KNO_3 \\ +yC \\ +zS \end{array}\right\} = \left\{\begin{array}{l} \frac{1}{28}[\quad 64+\ 8y-16z](K_2CO_3) \\ +\frac{1}{28}[\ 320-16y+\ 4z](K_2SO_4) \\ +\frac{1}{28}[-160+\ 8y+12z](K_2S_2) \\ +\frac{1}{28}[-\ 64+20y+16z](CO_2) \\ +8N_2 \end{array}\right. \quad . \quad . \quad . \quad . \quad \text{(XIII.)}$$

and from it the equations:

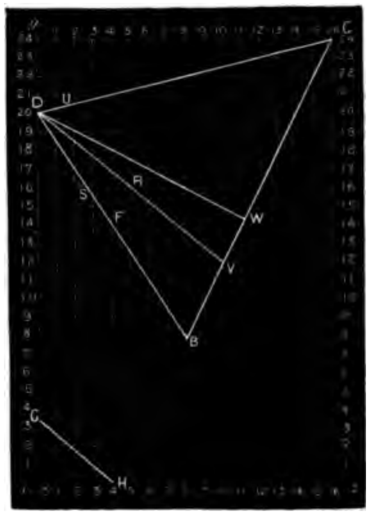
$$64+\ 8y-16z=0 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad \text{(XIV.)}$$

$$320-16y+\ 4z=0 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad \text{(XV.)}$$

$$-160+\ 8y+12z=0 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad \text{(XVI.)}$$

of the lines of intersection of the plane at right angles to the x axis with the faces of the trihedral angle, in other words, the sides of the triangle B D C (fig. 1).

Fig. 1.



It will be observed that

(XIV.) is the equation of B C
(XV.) „ „ D C
(XVI.) „ „ B D.

The points of the side B C represent, by their coordinates, mixtures of carbon and sulphur with 16 mols. of saltpetre, which will by complete combustion produce no potassic carbonate; those of side D C such as will not form potassic sulphate, and finally, the coordinates of the points of side B D denote quantities of carbon and sulphur which will burn with 16 mols. of saltpetre without the formation of potassic disulphide. The coefficient of carbonic acid will never vanish, but be always positive, because if it is equated to zero it will represent the line G H in figure, which does not intersect the triangle B D C.

All points outside the triangle B D C have coordinates which render at least one of the three coefficients of the potassium salts in equation (XIII.) negative, and consequently have reference to mixtures of carbon and sulphur with 16 mols. of saltpetre, which contain either too much or too little of one or both of the two elements named.

The coordinates of the line B D represent mixtures of carbon and sulphur with 16 mols. of saltpetre, which will burn without the formation of potassic disulphide, and those of the line B C, such as will not produce potassic carbonate; hence, it may be concluded that the coordinates of B, the point of intersection of the two lines will correspond to a powder which will be transformed without formation of potassic carbonate and disulphide, and will only yield, as products of its combustion, potassic sulphate, carbonic acid, and nitrogen. The coordinates of point B are:

$$y=8$$
$$z=8$$

and these values substituted in equation (XIII.).

$$16KNO_3+8C+8S=8K_2SO_4+8CO_2+8N_2$$

or simplified:

$$2KNO_3+C+S=K_2SO_4+CO_2+N_2$$

Accordingly a mixture of 82·1 parts of saltpetre, 4·8 parts pure carbon, and 13 parts of sulphur may be expected to produce during complete combustion only potassic sulphate, carbonic acid, and nitrogen, and this conclusion is in perfect accord with the thermochemical relations of the reacting substances and with experimental results. And by a similar method of reasoning we arrive at the conclusion that the coordinates of the point D represent a mixture which will burn according to the equation:

$$16KNO_3+20C=8K_2CO_3+12CO_2+8N_2$$

or simplified:

$$4KNO_3+5C=2K_2CO_3+3CO_2+2N_2$$

and those of point C according to

$$16KNO_3+24C+16S=8K_2S_2+24CO_2+8N_2$$

or simplified:

$$2KNO_3+3C+2S=K_2S_2+3CO_2+N_2$$

if in this last equation the potassic disulphide be changed into monosulphide, then the equation would become identical with the old one, which for many years was supposed to represent the metamorphosis of all sorts of gunpowders.

As already observed, all points within the triangle represent, by their coordinates, mixtures of carbon and sulphur with 16 mols. of KNO_3, which besides carbonic acid and nitrogen, will yield during their combustion three potassium salts.

The geometrical construction of the coefficients of equation (XIII.), does not only offer the advantage of representing by the coordinates of the points within the triangle B D C all possible proportions of saltpetre, carbon, and sulphur which can transform themselves into potassic carbonate, potassic sulphate, potassic disulphide, carbonic acid and nitrogen, but it also enables us to deduce at once, geometrically, the quantities of these products of combustion.

If we desire to know the composition of all those mixtures which shall contain variable quantities of carbon and sulphur, but shall all produce by their combustion the same amount of potassic carbonate, we can deduce the answer from the following considerations:

For such mixtures the coefficient of the potassic carbonate in equation (XIII.) must assume a constant value. Hence,

$$64+8y-16z=c.$$

and

$$y=2z+\frac{c-64}{8}$$

the equation of a line parallel to the side B C of the triangle. The coordinates of the points of such a line indicate the composition of mixtures which will burn with production of the same amount of potassic carbonate. The amount of potassic carbonate is constant for each parallel line, but changes from one line to another. Now as they intersect the lines B D and D C, it is only necessary to ascertain the amounts of potassic carbonate corresponding to the points of one of these sides in order to know the amount of potassic carbonate formed by the combustion of a mixture represented by the coordinates of any point within the triangle.

Similar considerations lead to the equation:

$$y=\tfrac{1}{4}z+\frac{320-c}{16}$$

for mixtures which will burn with production of equal quantities of potassic sulphate.

This is the equation of a line parallel to the side D C. All these parallel lines intersect the line D B. If, then, we know the quantity of potassic sulphate corresponding to each point of D B, we shall likewise know the amount of this salt which any mixture, the composition of which is represented by the coordinates of one of the points of the triangle, can produce by its combustion. Now the powders, the composition of which is given by the coordinates of the points of B D produce by their combustion an amount of potassic sulphate which in molecules is directly expressed by the length of the corresponding abscissæ of the points.

Since x has been taken constant $=16$, the sum of the molecules of the potassium salts must always be $=8$, and as the points of the line B D represent only mixtures which burn with the production of two of these salts, potassic carbonate and sulphate, it is only necessary in order to know the respective quantities of each of these salts for a point F on B D, to subtract the value of the abscissa of F from 8 to obtain the molecules of potassic carbonate which would be produced by the combustion of a mixture the composition of which is given by the coordinates of F.

The coefficient of potassic disulphide in equation (XIII.) is $=-160+8y+12z$, from which we deduce the equation :

$$y=-\tfrac{3}{2}z+\frac{c+160}{8}$$

which is the equation of a line parallel to side B D. The points of such a line represent by their coordinates mixtures which will burn with the production of the same amount of potassic disulphide, which amount is constant for the same line, but changes from one to another. This amount is found for a mixture represented by the coordinates of a point P, if through P a line is drawn parallel to B D, and the abscissa of the point of intersection with the side D C is ascertained; half the length of this abscissa represents the number of molecules of potassic disulphide formed by the combustion of the mixture represented by point P.

For mixtures which are to burn with the evolution of the same quantity of carbonic acid we have :

$$-64+20y+16z=c$$

or

$$y=-\tfrac{4}{5}z+\frac{c+64}{20}$$

an equation which represents a line parallel to G H, on which for two points y, z and y', z', five times the difference of the ordinates is equal to four times the difference of the abscissæ.

The line D V in our figure is parallel to G H. In order to find the amount of carbonic acid which is developed by the combustion of a mixture the composition of which is represented by the coordinates of a point P, we have to draw through P a line parallel to D V or G H, and determine the length of the ordinate of the point of intersection with the side B C; this length is equal to the number of molecules of carbonic acid, because for all mixtures represented by the points of the side B C, the number of molecules of carbonic acid produced is equal to the number of atoms of carbon the mixtures contain.

We will now proceed to determine, by aid of the method just explained, the quantities of the products of combustion of a mixture the composition of which is represented by the coordinates of the point R on D V, $y=16$, $z=5$. A line drawn through R parallel to D C intersects D B in the point S; the abscissa of $S=3$, hence 3 mols. of potassic sulphate are produced.

A line drawn through R parallel to B C, cuts the side D B, in F; the abscissa of $F=4$; $8-4=4$; hence we obtain 4 mols. of potassic carbonate.

A line through R parallel to D B, intersects D C, in point U; the abscissa of $U=2$; $\frac{2}{2}=1$; hence 1 mol. of potassic disulphide is formed.

R is a point of D V, the ordinate of V, y, $=12$, hence we have 12 mols. of carbonic acid.

Nitrogen is for all mixtures a constant $=8N_2$, therefore the equation for the metamorphosis of a mixture, the composition of which is expressed by the coordinates of the point R is:

$$16KNO_3+16C+5S=4K_2CO_3+3K_2SO_4+K_2S_2+12CO_2+8N_2$$

The great advantage of the geometrical construction of the coefficients of equation (XIII.) consists in this, that we can at once ascertain by an inspection of figure B C D, the influence of all possible variations of the quantities of carbon and sulphur in given mixtures, upon the proportions of the corresponding products of combustion.

Similar considerations enable us to find the quantities of gas and heat.

If we add the constant 8 for nitrogen to the number of molecules of carbonic acid determined as previously explained, we obtain the total number of gas molecules produced by the combustion of a mixture represented by the coordinates of a given point.

The heat generated is found by equation (X.).

For powders which shall produce by their combustion the same amount of heat, we have:

$$1{\cdot}92y=-z+\frac{1827154-c}{8788}$$

for which we may adopt without serious error

$$2y=-z+\frac{1827154-c}{8788}\quad.\ .\ .\ .\ .\ .\ .\ .\ .\quad\text{(XVII.)}$$

This is the equation of a line perpendicular to the side B C. For an appropriate value of C it becomes:

$$2y=-z+40$$

and then represents the line D W in the figure.

Mixtures, the composition of which can be represented by the points of such a line, will generate by their combustion very nearly the same amount of heat. A powder composed of $16KNO_3+20C$ corresponds to the point D, and one consisting of $16KNO_3+14{\cdot}4C+11{\cdot}25S$ to the point W. The first generates according to equation (X.) 1,488,654, and the second 1,484,569 units of heat, two numbers which differ only by 0·27 per cent., and may therefore be considered identical for practical purposes.

A line drawn through the point R, perpendicular to the side B C, intersects the latter in the point $y=13{\cdot}25$, $z=10{\cdot}6$; hence two powders composed of

$$16KNO_3+16C+5S$$
$$\text{and } 16KNO_3+13{\cdot}25C+10{\cdot}6S$$

will generate by their combustion the same, or more correctly, nearly the same amount of heat.

Consequently, if we know the heat of combustion of all the mixtures represented by the coordinates of the points of the line B C, then we know likewise the heat of combustion of all the mixtures the composition of which is represented by *any* point within the triangle. And we arrive at the same conclusion with regard to the amount of gas which a mixture can produce, the composition of which is represented by any point inside the triangle B C D. According to equation (X.) the heat of combustion reaches its maximum when y and z assume their smallest values, and, on the other hand, when y and z are greatest the heat of combustion will be a minimum. Therefore, an inspection of the triangle B D C teaches that of all the infinite number of mixtures of saltpetre, carbon, and sulphur which can be transformed according to equation (XIII.), the one which is composed of

$$16KNO_3+8C+8S$$

will produce by its combustion the greatest, and the one composed of

$$16KNO_3+24C+16S$$

the smallest quantity of heat: the first is represented by point B, the second by point C of the figure.

Further, it follows from equation (IX.) that the first of the above mixtures will form the smallest, and the second the largest quantity of gas.

If then we place ourselves at the point B of line B C, to which corresponds the generation of the greatest quantity of heat and that of the smallest quantity of gas, and move from B towards C, the amounts of heat produced by the mixtures represented by the coordinates of the several points will constantly decrease, and the volumes of gas increase, until the former reach in C their minimum, and the latter their maximum value.

We calculate for B and C

	Volume of gas.	Units of heat.
B . . .	32	1,621,450
C . . .	64	1,280,346

and between these numbers, 32 and 64 for the volume of the gas, and 1,280,346 and 1,621,450 for the units of heat, fluctuate the quantities of heat and gas which any possible mixture of 16 mols. of saltpetre with carbon and sulphur can produce, provided that these constituents, during combustion, transform themselves according to equation (XIII.). We will now show that the product, E, of the units of heat and the molecules of gas as given by equation (XI.), is greater for mixtures represented by points of line B C than for such as are represented by any other point within the triangle.

If we take, on the line D W, perpendicular to B C, the point $y=17$, $z=6$, then the mixture corresponding to this point and the one corresponding to point W will produce the same quantity of heat. The amount of gas generated by the mixture represented by point $y=17$, $z=6$, must be less than the quantity produced by the mixture corresponding to point W. Because, if we draw a line through $y=17$, $z=6$, parallel to D V, the point of intersection with B C will lie between W and V, but the further the point of intersection is away from W in the direction towards B the smaller the volumes of gas will be. Hence, the product of gas and heat for $y=17$, $z=6$ must be smaller than the one for point W, and the further we proceed from W towards D the smaller this product must be. But what holds good for the line D W also applies to every other perpendicular which can be drawn to B C.

Therefore the maximum value of the product of gas and heat must be produced by a mixture the composition of which is expressed by the coordinates of one of the points of the line B C.

If we represent the function on the right of the equation (XI.) by $F(y, z)$, and the equation of the line B C (XIV.) by $\phi(y, z)=0$, then the differential equation

$$\frac{dF}{dz}\cdot\frac{d\phi}{dy}-\frac{dF}{dy}\cdot\frac{d\phi}{dz}=0$$

together with $\phi(y, z)=0$ give the values of y and z, for which E in equation (XI.) becomes a maximum. We find $y=38{\cdot}02$ and $z=23{\cdot}0$. Hence, a powder composed of

$$16KNO_3+38C+23S$$

will by its complete combustion produce amounts of heat and gas the product of which will be the required maximum. But a mixture which shall transform itself according to equation (XIII.), can only contain per 16 mols. of saltpetre from 8 to 24 atoms of carbon, and from 8 to 16 atoms of sulphur. Hence, the product of the quantities of heat and gas will be a maximum for a powder composed of

$$16KNO_3+24C+16S$$

because according to the coefficients of equation (XI.) E will become greater and greater when y increases from 8 to 24, and z from 8 to 16, until it reaches its maximum value at a point $y=38$ and $z=23$, outside the triangle B C D.

For the purpose of calculating the values of E for different mixtures, we may simplify the coefficients and constant of equation (XI.), and write accordingly

$$10441-12{\cdot}1y^2+1208{\cdot}4y-16yz+994z-5z^2=E \quad . \quad . \quad . \quad \text{(XVIII.)}$$

E has been calculated by means of this equation for different values of y and z with the following results:—

$y=8$, $z=8$, $E=25941{\cdot}8$
$y=10$, $z=9$, $E=28416$
$y=12$, $z=10$, $E=30719{\cdot}4$
$y=14$, $z=11$, $E=32852$
$y=16$, $z=12$, $E=34813{\cdot}8$
$y=18$, $z=13$, $E=36604{\cdot}8$
$y=20$, $z=14$, $E=38225$
$y=22$, $z=15$, $E=39674{\cdot}4$
$y=24$, $z=16$, $E=40953{\cdot}0$

If, therefore, the carbon and the sulphur increase in different mixtures, the carbon by 2 atoms and the sulphur by 1 atom from

$$16KNO_3+8C+8S$$

to

$$16KNO_3+24C+16S$$

then parallel with this change of the carbon and sulphur, a regular increase of the product of heat and gas takes place, until for 24C and 16S it becomes a little more than one and-a-half times as great as for 8C and 8S.

If saltpetre and sulphur remain constant, and the carbon alone changes, then also an increase of the carbon is followed by one of E.

$y=8$, $z=8$, $E=25941{\cdot}8$
$y=16$, $z=8$, $E=32261{\cdot}8$
$y=18$, $z=8$, $E=33599{\cdot}8$
$y=22$, $z=8$, $E=35985{\cdot}8$

The energy of a powder of the composition:

$$16KNO_3+22C+8S$$

in about $\frac{2}{5}$ths greater than that of one containing:

$$16KNO_3+8C+8S.$$

If saltpetre and carbon are constant, but the sulphur changes, we obtain the following values for E:—

$y=14,\quad z=4,\quad E=27987{\cdot}0$
$y=14,\quad z=11,\quad E=32852{\cdot}0$

$y=16,\quad z=6,\quad E=30925{\cdot}8$
$y=16,\quad z=8,\quad E=32261{\cdot}8$
$y=16,\quad z=12,\quad E=34813{\cdot}8$

$y=18,\quad z=8,\quad E=33599{\cdot}8$
$y=18,\quad z=13,\quad E=36604{\cdot}8$

$y=20,\quad z=0,\quad E=29769$
$y=20,\quad z=5,\quad E=33014$
$y=20,\quad z=9,\quad E=35430$
$y=20,\quad z=14,\quad E=38225$

$y=22,\quad z=8,\quad E=35985{\cdot}4$
$y=22,\quad z=15,\quad E=39674{\cdot}4$

It follows from these examples that for a constant quantity of saltpetre, in varying mixtures of saltpetre, carbon, and sulphur, the relative energy of the mixtures increases with both the carbon and the sulphur, and reaches its maximum for 24 atoms of carbon and 16 atoms of sulphur, the highest amounts of these constituents which can exist in a powder according to equation (XIII.).

The difference of E for two mixtures of the same amount of saltpetre, but varying quantities of carbon and sulphur, becomes much smaller with equal weights of such mixtures. If, then, we multiply x, y, and z with their respective molecular or atomic weights, and divide E by the sum of the numbers so obtained, we find the relative energy, say E′, of equal weights of various mixtures.

The following table gives the value of E′ for mixtures which contain 16 mols. of KNO_3, y atoms of C, and z atoms of S.

$y=8,\quad z=8,\quad E'=13{\cdot}18$
$y=16,\quad z=8,\quad E'=15{\cdot}63$
$y=18,\quad z=8,\quad E'=16{\cdot}09$
$y=22,\quad z=8\quad E'=16{\cdot}84$

$y=11$, $z=6$, $E'=13{\cdot}91$
$y=13$, $z=6$, $E'=14{\cdot}58$
$y=16$, $z=6$, $E'=15{\cdot}46$
$y=21$, $z=6$, $E'=16{\cdot}62$

$y=11$, $z=6$, $E'=13{\cdot}91$
$y=11$, $z=9{\cdot}5$, $E'=14{\cdot}41$

$y=14$, $z=4$, $E'=14{\cdot}63$
$y=14$, $z=8$, $E'=15{\cdot}11$
$y=14$, $z=11$, $E'=15{\cdot}38$

$y=16$, $z=2{\cdot}66$, $E'=15{\cdot}11$
$y=16$, $z=6$, $E'=15{\cdot}46$
$y=16$, $z=8$, $E'=15{\cdot}63$
$y=16$, $z=12$, $E'=15{\cdot}88$

$y=18$, $z=8$, $E'=16{\cdot}09$
$y=18$, $z=13$, $E'=16{\cdot}28$

$y=20$, $z=0$, $E'=16{\cdot}03$
$y=20$, $z=5$, $E'=16{\cdot}37$
$y=20$, $z=9$, $E'=16{\cdot}52$
$y=20$, $z=14$, $E'=16{\cdot}59$

$y=21$, $z=4$, $E'=16{\cdot}54$
$y=21$, $z=6$, $E'=16{\cdot}62$

$y=22$, $z=8$, $E'=16{\cdot}84$
$y=22$, $z=15$, $E'=16{\cdot}81$

$y=24$, $z=16$, $E'=16{\cdot}95$

It follows from these numbers that E' becomes greater when y or z, or both simultaneously, increase, but proportionately less so than is the case with weights of mixtures which contain equal weights of saltpetre, viz., 16 mols.

The smallest value of E' is 13·18, the highest 16·95; hence the latter is about 28 per cent. greater.

The highest value of E, on the other hand, is more than 50 per cent. greater than the lowest. Further, it is apparent that for mixtures for which y and z assume high values the differences of E' become very small.

The powder

$$16KNO_3+14C+4S$$

differs from

$$16KNO_3+14C+11S$$

by 7 atoms of sulphur.

The two mixtures

$$16KNO_3+22C+8S$$
$$\text{and } 16KNO_3+22C+15S$$

differ by the same amount of sulphur.

The two former show, for E′, the difference 0·75, the latter only 0·03; indeed, many of the various mixtures for which y and z have high values, give, when equal weights are considered, almost the same number for E′.

If we draw a line through the triangle B C D (see fig. 1), from the point $y=22$, $z=8$ to the point $y=8$, $z=8$, it will be observed that for mixtures represented by the coordinates of the points on the right-hand side of this line, the value of E′ only increases very little if the sulphur is increased beyond 8 atoms and the carbon kept constant. This circumstance is of great practical importance. The analyses of military and sporting powders known to me, all give for 16 mols. of saltpetre an amount of sulphur which varies between 5·5 and 8·7 atoms. There would be very little, if any, gain in energy if, for 16 mols. of saltpetre, more than about 8 atoms of sulphur were introduced into the powder; especially would this be the case with mixtures in which for 16 mols. of saltpetre more than 16 atoms of carbon are present.

E′ obtains its maximum value, 16·95, when the powder contains:

$$16KNO_3+24C+16S.$$

Such a large amount of sulphur does not, according to the foregoing remarks, contribute much to the value of E′, whereas, on the other hand, it must be very detrimental to the metal of the ordnance. For the mixture

$$16KNO_3+22C+8S$$

we have $E'=16{\cdot}84$, hence, only 0·67 per cent. less than for $16KNO_3+24C+16S$.

If carbon and sulphur undergo a further diminution, the decrease of E′ becomes more rapid; for

$$16KNO_3+21C+4S$$

$E'=16{\cdot}54$. If, therefore, we had to choose between the two mixtures

$$16KNO_3+24C+16S$$
$$\text{and } 16KNO_3+22C+8S$$

for the composition of a service powder the second would recommend itself as the more suitable.

We will now compare the composition and energy of the ordinary gunpowders with the results of the foregoing theoretical considerations.

The composition of the powders of Waltham Abbey can be represented by the symbols:

$$16KNO_3+21{\cdot}18C+6{\cdot}63S$$

which corresponds nearly to 75 parts of saltpetre, 10 parts of sulphur, and 15 parts of charcoal.

About these numbers fluctuate the compositions of the service powders of most nations.

	Composition of gunpowders.		
	Saltpetre.	Charcoal.	Sulphur.
England	75	15	10
Sweden	75	15	10
Russia	75	15	10
Prussia	74	16	10
Saxony	74	16	10
United States . . .	76	14	10
Austria	75·5	14·5	10

If, therefore, the composition of a gunpowder is required which shall possess nearly the greatest energy, and at the same time contain the smallest amount of sulphur compatible with this condition, an experience extending over 500 years has selected a mixture which contains saltpetre, carbon, and sulphur nearly in the theoretical proportions.

Composition of powders of Waltham Abbey. . $16KNO_3+21{\cdot}18C+6{\cdot}63S$
Theoretical composition. . . . . $16KNO_3+22C+8S$.

We concluded from KAROLYI's experiments that the most inflammable and combustible mixture is represented by

$$16KNO_3+13C+5S$$

BUNSEN and SCHISCHKOFF found in their sporting powder

$$16KNO_3+13{\cdot}3C+6{\cdot}3S$$

The value of E' for the proportions of saltpetre, carbon, and sulphur exhibited in the powders of Waltham Abbey is very nearly 16·62; for BUNSEN and SCHISCHKOFF's sporting powder 14·58. Consequently 12·2 per cent. of the energy of the English service powder has been sacrificed in order to obtain the greater combustibility of the sporting powder.

According to composition, the service powders of France, Spain, Belgium, and Würtemberg are intermediate between the two powders just considered. They fluctuate about the proportions required by the symbols

$$16KNO_3+16C+8S$$

E' for these powders equals 15·63, or about 6 per cent. less than for the English; but they will, probably, be more inflammable and combustible than the latter.

It is worthy of notice that the points which represent, by their coordinates, the proportions of saltpetre, carbon, and sulphur in the gunpowders considered in this paper, are situated between two ordinates on our triangle B C D, for which respectively z assumes the values 5·5 and 8·7. The powders of Waltham Abbey, and BUNSEN and SCHISCHKOFF's sporting powder contain per 16 mols. of saltpetre, nearly the same amount of sulphur; the former are represented by a point near the line D C, the latter by one near the line B D, of our figure B C D.

Summary of the main results.

1. The mean composition of the powders of Waltham Abbey can be represented by the symbols:

$$16KNO_3+21·18C+6·63S$$

A powder of this composition is transformed in NOBLE and ABEL's apparatus according to the equation:

$$16KNO_3+21C+5S=5K_2CO_3+K_2SO_4+2K_2S_2+13CO_2+3CO+8N_2 \quad . \quad \text{(II.)}$$

The residue of the sulphur, 1·63 atoms, unites partly with hydrogen, partly with the iron of the apparatus.

2. The ordinary service and sporting powders contain for every 16 mols. of saltpetre from 13 to 22 atoms of carbon, and from 5·5 to 8·7 atoms of sulphur.

3. A powder composed of *pure* carbon, saltpetre, and sulphur furnishes by its *complete* combustion potassic carbonate, potassic sulphate, potassic *disulphide*, carbonic acid, carbonic oxide, and nitrogen, as chief products.

4. An increase of pressure appears, *cæteris paribus*, to diminish the amount of carbonic oxide, and, in consequence, according to equation (VIII.), to increase the quantities of potassic carbonate, potassic disulphide, and carbonic acid, and diminish that of potassic sulphate. These fluctuations depending on pressure are, however, *very small*. In NOBLE and ABEL's Experiment No. 38, the pressure amounted to 18·6 tons, in Experiment No. 77 to 31·4 tons on the square inch, Experiment No. 38 gave for every 16 mols. of decomposed saltpetre 3·36 mols. of carbonic oxide, and Experiment No. 77, 2·9 mols. of this gas, or, for a difference of 12·8 tons in pressure, one of 0·46 mol. of carbonic oxide. A diminution of 0·5 mol. of carbonic oxide corresponds to one of 0·143 mol. in the amount of potassic sulphate, and an increase of 0·071 mol. in that of the potassic carbonate and disulphide, and 0·428 mol. in the quantity of carbonic acid. These fluctuations are probably not caused directly by the pressure, but by the differences in the rate of cooling after explosion.

5. The combustion of gunpowder takes place in two stages, one succeeding the other.

(*a*.) A process of oxidation during which potassic sulphate, carbonate, carbonic acid and nitrogen, and, *perhaps*, some carbonic oxide, but no potassic disulphide, are produced.

(*b*.) A process of reduction during which carbon and sulphur left free at the end of the first stage react with some of the products formed during that stage; the free carbon reducing potassic sulphate, with formation of potassic disulphide, potassic carbonate, and carbonic acid; the free sulphur decomposing potassic carbonate with the production of potassic disulphide, potassic sulphate, and carbonic acid [equations (V.) and (VI.)].

6. The first stage of the combustion, the explosion proper, takes place with powders of various composition according to equation:

$$10KNO_3+8C+3S=2K_2CO_3+3K_2SO_4+6CO_2+5N_2 \quad . \quad . \quad . \quad \text{(III.)}$$

But as some carbonic oxide is probably produced at the same time, the following will more correctly represent the reactions.

$$16KNO_3+13C+5S=3K_2CO_3+5K_2SO_4+9CO_2+CO+8N_2 \quad . \quad . \quad \text{(IV.)}$$

The constituents of the powder and the products of combustion are, according to (IV.), nearly in the same ratios as according to (III.).

7. The oxygen in the potassic carbonate stands to the oxygen in the potassic sulphate and carbonic acid, respectively, in equation (III.), in the most simple ratios which can exist, if these substances are to be produced by the combustion of a mixture of saltpetre, carbon, and sulphur. In other words, equation (III.) represents the most simple distribution of the oxygen of the decomposed saltpetre amongst the products of the first stage of the combustion. And because the products are, according to equation (IV.), nearly in the same proportions they assume to (III.), it follows that the distribution of the oxygen between potassic sulphate, carbonate, and carbonic acid, according to (IV.), nearly corresponds to the most simple possible distribution.

8. If the greatest possible amount of heat is to be evolved by the combustion of a mixture of saltpetre, carbon, and sulphur, and if at the same time potassic sulphate, carbonate, and carbonic acid are to be formed in such proportions that the heat of formation of one of them shall stand to the heat of formation of each of the others in the most simple ratio, then the combustion must take place according to equation (IV.).

The heat produced by the formation of 3 mols. of potassic carbonate stands to that produced by the formation of 5 mols. of potassic sulphate and 9 mols. of carbonic acid respectively, as

$$1 : 2{\cdot}05 : 1{\cdot}04$$

9. The ordinary gunpowders contain more carbon and sulphur than is required by equation (IV.).

This excess of carbon and sulphur is left free at the end of the first stage of the combustion.

The free carbon now acts according to equation:

$$4K_2SO_4+7C=2K_2CO_3+2K_2S_2+5CO_2 \quad . \quad . \quad . \quad . \quad . \quad . \quad \text{(VI.)}$$

the free sulphur upon the potassic carbonate as follows:

$$4K_2CO_3+7S=K_2SO_4+3K_2S_2+4CO_2 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad \text{(V.)}$$

and both united form the second stage of the combustion. These reactions are endothermic; heat is not evolved but consumed; they are not of an explosive nature, and in practice are probably seldom complete.

The reactions of this second stage increase the volume of the gas formed during the first stage of the combustion and diminish the temperature of the products. A portion of the carbonic oxide is formed during the second stage by the action of free carbon or potassic disulphide upon carbonic acid.

10. The reactions represented by equations (III.), (IV.), (V.), and (VI.) can be expressed by one equation. If x, y, and z are positive numbers, and a indicates how many molecules of carbonic oxide are formed by the combustion of a weight of powder, containing x molecules of saltpetre, y atoms of carbon, and z atoms of sulphur, the following will be the general equation representing the complete chemical metamorphosis of powder.

$$\left.\begin{array}{r} xKNO_3 \\ +yC \\ +zS \end{array}\right\}=\left\{\begin{array}{l} \tfrac{1}{28}[4x+8y-16z-4a](K_2CO_3) \\ +\tfrac{1}{28}[20x-16y+4z+8a](K_2SO_4) \\ +\tfrac{1}{28}[-10x+8y+12z-4a](K_2S_2) \\ +\tfrac{1}{28}[-4x+20y+16z-24a](CO_2) \\ +aCO \\ +\tfrac{1}{2}xN_2 \end{array}\right.$$

11. If $x=16$, and $a=0$, the volume of the gas, (V.), generated by complete combustion is nearly

$$=\frac{160+20y+16z}{14} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad \text{(IX.)}$$

and the units of heat, W,

$$=1000[1827{\cdot}154-16{\cdot}925y-8{\cdot}788z] \quad . \quad . \quad . \quad . \quad . \quad . \quad \text{(X.)}$$

y signifies in these equations the number of carbon and z that of the sulphur atoms in a weight of powder containing 16 mols. of saltpetre.

The volume of gas becomes greater and the heat of combustion diminishes with an increase of y and z, and *vice versâ*.

The mixture containing :

$$16KNO_3+8C+8S$$

produces the greatest amount of heat and the smallest quantity of gas, and the mixture represented by the symbols :

$$16KNO_3+24C+16S$$

the largest volume of gas and the smallest quantity of heat.

12. The product E obtained by the multiplication of V and W (equations IX. and X.) will approximately represent the relative energies of mixtures of various composition.

The mixture represented by the symbols

$$16KNO_3+24C+16S$$

is of all the infinite number of mixtures which can transform themselves according to equation (XIII.) the one for which E assumes the greatest value. Hence, a powder of this composition possesses the greatest energy.

13. If a mixture of saltpetre, carbon, and sulphur were required which shall possess nearly the greatest energy, and at the same time contain the smallest amounts of carbon and sulphur compatible with this condition, theory would point to the mixture :

$$16KNO_3+22C+8S$$

The service powders of most nations fluctuate about :

$$16KNO_3+21{\cdot}2C+6{\cdot}6S$$

14. Gunpowder, however, does not contain pure carbon, but besides this element hydrogen and oxygen as constituents of the charcoal.

The oxygen is eliminated with a portion of the hydrogen in the form of water, the remainder of the hydrogen remains either free or unites with carbon, sulphur, and nitrogen respectively, producing sulphuretted hydrogen, ammonia, and marsh gas.

These secondary products only amount from 1 to 2 per cent. of the powders.

15. Mining powders contain much more carbon than is required according to equation (XIII.). In consequence, the oxygen of the charcoal is not eliminated during the combustion of these powders with hydrogen, as water, but in combination with carbon as carbonic oxide and carbonic acid.

The hydrogen thus left free causes the formation of a comparatively large proportion of sulphuretted hydrogen and marsh gas. The potassic sulphocyanate is also produced in quantities much larger than those formed by the service powders on account of the carbon left free at the end of the combustion. If we neglect these secondary products, then the combustion of mining powder may be represented by the simple equation :

$$16KNO_3+28C+8S=4K_2CO_3+4K_2S_2+12CO_2+12CO+8N_2$$

The reactions can also be represented as follows:

$$16KNO_3+28C+8S=16KNO_3+22C+8S+6C$$

16 mols. of saltpetre, 22 atoms of carbon, and 8 atoms of sulphur transform themselves according to (XIII.) as follows:

$$16KNO_3+22C+8S=4K_2CO_3+4K_2S_2+18CO_2+8N_2$$

If, now, 6 atoms of carbon act on 6 mols. of CO_2 we obtain

$$6CO_2+6C=12CO$$

In this manner we can conceive the combustion of a mining powder to take place according to the same equations which apply to ordinary gunpowder, and that the excess of carbon in the mining powder causes, subsequently, the reduction of a portion of the carbonic acid to carbonic oxide.

XI. *On the Refraction of Plane Polarized Light at the Surface of a Uniaxal Crystal.*

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THE laws of the reflexion and refraction of polarized light at the surface of a crystal in accordance with the electro-magnetic theory of light have been discussed by LORENTZ (Schlömilch Zeitschrift, vol. xxii.), FITZGERALD (Phil. Trans., Vol. 171, 1880), and myself (Proc. Camb. Phil. Society, 1881). When a plane wave of electro-magnetic disturbance falls on the surface of separation between two different dielectric media six equations of condition are obtained. Three of these express the conditions that the electric displacement perpendicular to the surface and the electromotive force along the surface should be the same in the two media, while the other three do the same for the magnetic force and displacement. In all cases the six equations reduce to only four.

Let us suppose we know the amount and direction of the electric displacement in the incident wave. If both media are isotropic, these four equations give us the amounts and directions of the electric displacements in the reflected and refracted waves.

If the second medium is crystalline the possible directions of vibration in a wave travelling in it are known when the position of the wave is known; two of the equations as before give the amount and direction of the electric displacement in the reflected wave, the other two give the amounts of the displacements in the two refracted waves; the directions of these displacements being known from the position of the waves with reference to the axes of the crystal.

In general we have two refracted waves, the ordinary and extraordinary. Now, according to the electro-magnetic theory, light obeys the same laws as to propagation, reflexion, and refraction as this electro-magnetic disturbance, and the direction of the light vibrations coincides with that of the electric displacement, while the intensity of the light is measured by the energy of the disturbance. Our equations then give us the intensities of the two refracted rays which arise in general when a wave of polarized light falls on a crystal.

Consider now such an incident wave. Experiment tells us that there are two positions for its plane of polarization, in either of which one or other of the refracted waves disappears. The same result follows from the theory, and if we know the position of the face of incidence with reference to the axes of the crystal and also the directions of the two refracted wave normals, the theory enables us to calculate the angle between these two positions of the plane of polarization of the incident light.

But this angle is capable of direct measurement by experiment, and so the truth of the theoretical formulæ can be tested.

The experiment in its simplest form is as follows: a plane polarized beam of sodium light falls on a crystal of Iceland spar cut in the form of a prism, the position of whose faces relatively to the optic axis can be determined. The angle of incidence is observed and also the deviations of the two emergent wave normals.

From these data we can calculate the positions of the refracted wave normals in the crystal, and also of course the positions of the planes of polarization of the light travelling in these two directions respectively.

Let us suppose the polarizer to be a NICOL'S prism, mounted in such a manner that the position of the plane of polarization of light emerging from it can be determined by means of a graduated circle attached to it; turn the NICOL until the extraordinary refracted wave disappears, then observe the position of the plane of polarization. But, there being only one ray, the ordinary, traversing the crystal prism, we can obtain from theory the azimuth of the plane of polarization of the incident light. Let us call this measured from some fixed plane θ_0. If the position of this fixed plane with reference to the NICOL can be found with accuracy we have here a means of comparing theory and experiment. We can eliminate the uncertainty in our knowledge of the relative position of the two planes of reference by turning the NICOL until the ordinary ray disappears, and reading the circle again; the difference between the two circle readings is the angle through which the plane of polarization has been turned; but the theory gives us again in this case, when there is only one refracted ray, the extraordinary, the value θ_E of the azimuth, measured from the same plane as before, of the plane of polarization of the incident light; the difference $\theta_0 \backsim \theta_E$ should be equal to the difference between the circle readings. Or again, having obtained as above a value for θ_0, alter the position of the spar prism so as to change the angle of incidence, and proceed as before; we can thus get a series of values of θ_0, the position of the plane of polarization of the incident light, for different angles of incidence as given by theory when the ordinary wave only traverses the crystal. But the readings of the polarizer circle give an experimental series of values of this same quantity and a comparison of these two series affords us a test of the theory. Since, in general, these two series of angles are not measured from the same zero point, there will, even if theory and experiment agree, be a constant difference between the two series depending on the difference of zeros. If the difference between corresponding values in the two series is not constant, but varies as the angle of incidence changes, we must infer that theory and experiment do not agree. We can test in the same manner the formula for the case in which the extraordinary wave only is propagated.

In practice, mainly in consequence of two difficulties, the experiments were conducted in a somewhat different manner. Unless certain rather elaborate conditions are fulfilled, the angle through which a NICOL'S prism is turned is not the angle through which the plane of polarization of the emergent light moves, and the difference between the two may, as I have shown (Phil. Mag., Nov., 1880, "On NICOL'S Prism"), be very considerable. Besides, when I began the experiments the only circle available was one graduated to 3′ of arc, which was not sufficiently accurate for my purpose. I allowed, therefore, plane polarized light to fall on my spar prism, and by turning it varied the angle of incidence until only one ray emerged. I observed the angle of incidence and the deviation, and from them calculated the azimuth of the plane of polarization of the incident light on the electro-magnetic theory. I then took a small cell with plane parallel glass sides and filled it with a weak solution of sugar; this I placed in the path of the incident light and thus produced a change in the position of its plane of polarization which brought the extraordinary ray into view again. This, by adjusting the angle of incidence on the spar prism, can again be made to vanish, and the angle of incidence and deviation being observed we can obtain a second value for the azimuth of the plane of polarization of the incident light. The angle between these two azimuths is the angle on the electro-magnetic theory through which the plane of polarization has been turned by the sugar cell. But this can be directly observed and the theory thus tested. Then I removed the sugar cell, altered somewhat the original plane of polarization, and again made the same observations, thus obtaining a series of values for the rotation produced by the cell corresponding to different angles of incidence on the spar prism.

The same observations were made using only the extraordinary ray. Thus the uncertainty arising from the want of adjustment and bad graduations of the polarizer circle was avoided.

The second difficulty was perhaps more serious. It was impossible to estimate with anything like sufficient accuracy the position of the spar prism for which the light of either ordinary or extraordinary ray was just quenched. To obviate this the apparatus was arranged as follows. The spar prism was mounted on the table of a spectrometer, kindly lent me by Professor STOKES, with a circle on silver and verniers reading to 10″. The instrument, and the method of adjusting the prism and focussing the telescope and collimator, have been described at length in my paper "On Plane Waves in a Biaxal Crystal" (Phil. Trans, 1879, p. 293). The sodium light was replaced by a strong source of white light, a powerful paraffin lamp, or the oxyhydrogen lime-light. A biquartz with the line of separation horizontal was placed between the polarizer and the collimator slit, and carefully adjusted by set screws, so that the light fell on it normally. Between the biquartz and the slit was placed a convex lens of about 20 centims. focal length, arranged so as to form an image of the biquartz on the slit of the collimator. The light from the slit fell on the spar prism, and two spectra, an ordinary and extraordinary, were formed and viewed by the telescope. Each of these spectra was divided horizontally into two parts, corresponding to the two parts of the biquartz, and when everything was adjusted the line of separation was seen clearly and distinctly.

Let us consider the ordinary image. Owing to the dispersion of the planes of polarization produced by the biquartz, the light of different colours in the incident wave is polarized in different planes, and a position can be found for the polarizer such that the plane of polarization of light of a certain colour on emerging from one-half of the biquartz is so related to the angle of incidence on the spar prism that the light of that colour is absent from the refracted beam. A dark band will be seen across one-half the spectrum in the place of this colour. Light of this colour emerging from the other half of the biquartz is in general not polarized in the same plane, and therefore in general, though a dark band will be formed in both halves of the spectrum, it will occupy different positions in the two. By turning the polarizer these bands appear to move in opposite directions across the field, and for one position of the polarizer the one can be brought vertically below the other. This position can be determined with great accuracy.

When this is the case the wave length of the light destroyed is clearly such that it has been rotated through 90° in opposite directions by the two halves of the biquartz. It is light then of a definite wave length, and we are thus able to place our prism with great accuracy in a position such that no light of one certain definite wave length is present in the ordinary wave. If, then, we are able to observe the angle of incidence and the deviation of the light of the same wave length in the extraordinary spectrum we shall have enough data to determine the azimuth of the plane of polarization of the incident light according to the electro-magnetic theory. It is easy enough to observe the angle of incidence. To find the deviation of the corresponding wave in the extraordinary spectrum, rotate the polarizer through about 90°; the dark bands will move out of the ordinary into the extraordinary spectrum, and the polarizer can be adjusted till they are brought to coincidence in it. When this is the case we know that it is light of the same wave length as before (viz.: that whose plane of polarization is turned through 90° by the biquartz), which is absent, and we have in the extraordinary spectrum a well-marked dark band, whose centre can easily be determined, and to which the needle point or cross wires of the observing telescope can be set with all the accuracy required. If we observe then the deviation of this dark band, we obtain the deviation in the extraordinary spectrum of the light which in the first part of our observation was wanting from the ordinary spectrum.

To escape the difficulty of having continually, when making observations for the determination of the position of the plane of polarization, to turn the polarizer through about 90° in order to get the deviation of the light in the extraordinary spectrum when the ordinary was quenched, or *vice versâ*, I divided the operation into two parts.

In the first I set the spar prism at a known angle of incidence and turned the polarizer until the dark bands coincided in the ordinary spectrum and then observed the deviation. I then turned the polarizer until the bands coincided in the extraordinary spectrum, and again observed the deviations; each of these observations was repeated five or six times and the mean taken. I then altered the angle of incidence by about 6° and made similar measurements.

In this manner I obtained a series of observations of angles of incidence ranging from 30° to 85°, with the corresponding deviations for both ordinary and extraordinary waves.

Let us call ϕ the angle of incidence, ϕ' that of refraction for the ordinary wave, ϕ'' for the extraordinary. From the observations the values of ϕ' and ϕ'' are easily determined by means of formulæ given by Professor STOKES (Brit. Ass. Report, 1862) and used by me in the paper already referred to.

If i be the angle of the prism ψ, ψ' the angles of emergence from and incidence on the second face, and D the deviation, we have

$$\psi = \mathrm{D} + i - \phi$$

$$\tan\frac{\phi'-\psi'}{2} = \tan\frac{i}{2}\tan\frac{\phi-\psi}{2}\cot\frac{\phi+\psi}{2}$$

$$\phi' + \psi' = i$$

and these equations give us ϕ'.

ϕ'' of course is found in the same manner.

I obtained thus a series of values of ϕ, ϕ', and ϕ''; now, of course, since ϕ' refers to the ordinary wave, if μ be the ordinary refractive index of the light used we should have $\frac{\sin\phi}{\sin\phi'} = \mu$, a constant, and this was found to be the case within small limits of error, showing that I had succeeded in quenching the same light throughout.

The value of μ was 1·662.

The values of $\phi'' - \phi'$ were also tabulated, and, of course, varied very slowly.

When this table had once been constructed, it was sufficient for the future to observe the angle of incidence; for knowing ϕ and μ, ϕ' is at once given by the formulæ $\sin\phi' = \sin\phi./\mu$ and ϕ'' by interpolation from the table. Thus the observations with the sugar-cell reduced to determining the angles of incidence at which the dark bands in the spectra coincided.

Each of these was determined five or six times and the mean taken.

We must now return to the theoretical considerations which enable us to express the azimuth of the plane of polarization of the incident light in terms of the angles of incidence and refraction and the position of the plane of polarization in the crystal.

Let us consider a plane-wave incident at an angle ϕ, let ϕ', ϕ'' be the angles of refraction for the two refracted waves respectively. The incident, reflected and refracted waves cut the plane face of incidence in the same line, let θ, $\theta_{,}$, θ' and θ'' be the angles between this line and the directions of the electric displacement in the incident reflected and refracted waves respectively.

Let a, $a_{,}$, a' and a'' be the amplitudes of the electric displacements in these directions.

Let q be the angle between the extraordinary wave normal and the corresponding ray.

Then (LORENTZ, 'Schlömilch,' xxii.; GLAZEBROOK, Proc. Camb. Phil. Soc., 1881) we have the equations

$$(a \cos \theta + a_{,} \cos \theta_{,}) \sin^2 \phi = a' \cos \theta' \sin^2 \phi' + a'' \cos \theta'' \sin^2 \phi'' \quad . \; . \; . \; . \; . \; . \quad (1)$$

$$(a \sin \theta + a_{,} \sin \theta_{,}) \sin \phi = a' \sin \theta' \sin \phi' + a'' \sin \theta'' \sin \phi'' . \quad . \; . \; . \; . \; . \; . \quad (2)$$

$$(a \cos \theta - a_{,} \cos \theta_{,}) \sin \phi \cos \phi = a' \cos \theta' \sin \phi' \cos \phi' + a'' \cos \theta'' \sin \phi'' \cos \phi'' . \quad (3)$$

$$\begin{aligned}(a \sin \theta - a_{,} \sin \theta_{,}) \sin^2 \phi \cos \phi = {} & a' \sin \theta' \sin^2 \phi' \cos \phi' \\ & + a''(\sin \theta'' \cos \phi'' + \tan q \sin \phi'') \sin^2 \phi'' \quad . \; . \; . \; . \quad (4)\end{aligned}$$

These equations express the conditions referred to previously and enable us to find $a_{,}$ a' a'' and $\theta_{,}$; θ', θ'' are known from the position with reference to the axis of the wave front in the crystal.

We can solve them in the general case, but for our present purpose it is sufficient to find the conditions that only one wave should be propagated in the crystal. Let us first take the ordinary wave; we may put $a''=0$ in the equations, and we get the condition

$$\tan \theta = \tan \theta' \cos (\phi - \phi') . \quad . \; . \; . \; . \; . \; . \; . \; . \; . \quad (5)$$

by eliminating $a_{,}$ a' and $\theta_{,}$.

This then is the condition which must hold between the position of the plane of polarization of the incident light and the angles of incidence and refraction in order that only the ordinary wave may traverse the crystal.

If we desire to have only the extraordinary wave, put $a'=0$ and we obtain

$$\tan \theta = \tan \theta'' \cos (\phi - \phi'') + \frac{\sin^2 \phi'' \tan q}{\cos \theta' \sin (\phi + \phi'')} . \quad . \; . \; . \; . \; . \quad (6)$$

In order to apply these formulæ we must find θ' θ'' and q in terms of the angles of incidence and refraction and constants.

Let the intersection of the incident, reflected and refracted waves with the face of incidence meet a sphere, centre O, in B (fig. 1).

Let the inward drawn normal to the face of incidence meet the same sphere in C, while the face itself cuts it in A B.

Let B I be the trace of the incident wave. B R of the refracted.

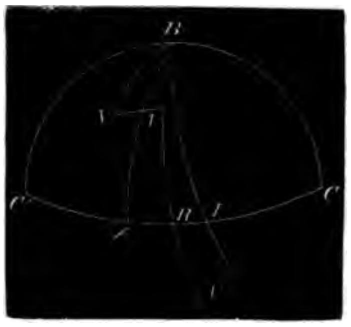
Let the optic axis cut the sphere in X. Join B X.

The prism used in the experiments was cut in such a way that X and R were on opposite sides of the arc B A, as in fig. 1.

Let B X $=\beta$ and let the angle A B X $=\lambda$. β and λ are known if the position of the face of incidence with reference to the optic axis is known.

Draw X V perpendicular to B R and take V V′ an arc of 90°. Then V V′ $=\frac{\pi}{2}$; also O V, O V′ are clearly the two possible directions of vibration in the wave front B R. O V is the direction of the extraordinary vibration O V′ of the ordinary.

Fig. 1.



Let us suppose that B R is the ordinary refracted wave corresponding to an incident wave B I. Then A B $V=\phi'$ and X B $V=\lambda+\phi'$; also B $V=\frac{\pi}{2}-\theta'$; and from the right angle and triangle X B V we have

$$\cos \text{X B V}=\tan \text{B V} \cot \text{B X}$$

$$\cot \theta'=\tan \beta \cos (\lambda+\phi') \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

But we have from (5)

$$\cot \theta=\cot \theta' \sec (\phi-\phi')$$

Therefore

$$\cot \theta=\tan \beta \cos (\lambda+\phi') \sec (\phi-\phi') \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (8)$$

If B R represent the extraordinary wave

$$\text{B V}=\theta'', \quad \text{A B V}=\phi'',$$

and we have

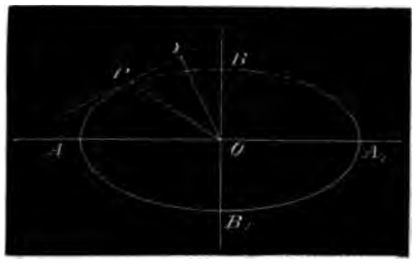
$$\tan \theta''=\tan \beta \cos (\lambda+\phi''). \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (9)$$

so that from (6) we obtain

$$\tan \theta=\tan \beta \cos (\lambda+\phi'') \cos (\phi-\phi'')+\sin^2 \phi'' \tan q / \cos \theta'' \sin (\phi+\phi'') \quad . \quad (10)$$

It remains now to find q, the angle between the extraordinary ray and the wave normal.

Fig. 2.



Let the figure (fig. 2) represent a section of the surface of wave slowness passing through the optic axis, O A, and the extraordinary wave normal, O P. Let O Y be perpendicular on the tangent plane at P, then O Y is, we know, the extraordinary ray. Let a and b be the principal refractive indices in direction O A and O B, and let $OP=r$, $OY=p$. Since OP and OY are respectively the directions of the wave normal and the ray, the angle $POY=q$, and $p=r\cos q$; let the angle $POA=\psi$.

Then

$$\frac{1}{p^2}=r^2\left\{\frac{\cos^2\psi}{a^4}+\frac{\sin^2\psi}{b^4}\right\}$$

Therefore

$$\sec^2 q=r^4\left\{\frac{\cos^2\psi}{a^4}+\frac{\sin^2\psi}{b^4}\right\}$$

Also

$$\frac{1}{r^2}=\frac{\cos^2\psi}{a^2}+\frac{\sin^2\psi}{b^2}$$

whence

$$\tan q=r^2\sin\psi\cos\psi\frac{a^2-b^2}{a^2b^2} \quad . \ . \ . \ . \ . \ . \ . \ . \ . \quad (11)$$

But since r is a radius vector of the surface of wave slowness, $r=\sin\phi/\sin\phi''$, and we have

$$\sin^2\phi''\tan q=\frac{a^2-b^2}{a^2b^2}\sin^2\phi\sin\psi\cos\psi$$

Again X V (fig. 1) $=90°-\psi$, and from the triangle X B V

$$\sin XV=\sin BX\sin XBV, \quad \text{or} \quad \cos\psi=\sin\beta\sin(\lambda+\phi'')$$

Also

$$\cos BX=\cos XV\cos BV, \quad \text{or} \quad \cos\beta=\sin\psi\cos\theta''$$

Thus

$$\sin^2\phi''\tan q=\frac{a^2-b^2}{2a^2b^2}\,\frac{\sin 2\beta\sin(\lambda+\phi'')\sin^2\phi}{\cos\theta''}$$

and equation (10) becomes

$$\tan\theta=\tan\beta\cos(\lambda+\phi'')\cos(\phi-\phi'')+\frac{a^2-b^2}{2a^2b^2}\,\frac{\sin 2\beta\sin(\lambda+\phi'')\sin^2\phi}{\sin(\phi+\phi'')\cos^2\theta''} \quad . \ . \quad (12)$$

but θ'' can be found in terms of ϕ, ϕ'' and constants, from the formula

$$\tan\theta''=\tan\beta\cos(\lambda+\phi''),$$

and θ is thus expressed in terms of ϕ, ϕ'' and constants.

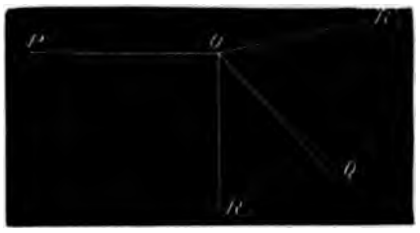
The value of a has been found already; it is the ordinary refractive index of the spar for the light used, and has been shown to be 1·662.

b is the extraordinary index, and may be taken with sufficient accuracy for the purpose from MASCART's or RUDBERG's determinations. Either of these give $b=1\cdot488$ as the value of the extraordinary index, corresponding to the value 1·662 for the ordinary.

In the spar prism used one face coincided very closely with a cleavage plane. The other face was inclined to this plane at an angle of 39° 17′ 20″, and the edge of the prism—that is, the intersection of the two faces—was nearly coincident with that of two cleavage planes.

Let O R_1, O R_2, O R_3 (fig. 3), be normal to the three cleavage planes; O P, O Q to the faces of the prism. The incident light fell on the face normal to O P. Then experiment proved that P, Q, R_3, R_1 were very closely indeed in the same zone; for the present we shall treat them as if they were accurately so, and this zone will therefore be the principal plane of the prism.

Fig. 3.



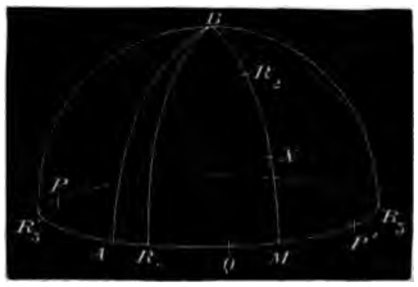
A series of observations on August 25, 1880, gave the values for the angles.

$$P\,O\,R_1 = 105°\ 54'$$
$$R_1\,O\,Q = 34°\ 48'$$
$$Q\,O\,R_3 = 40°\ \ 8'$$
$$P\,O\,R_3 = 180°\ 50'$$

Each observation was the mean of four or five closely concordant ones. The prism was reset and relevelled in November and another series of measurements taken, which agreed with the above to within 1′.

Let the optic axis and the edge of the prism meet the sphere in X and B respectively (fig. 4). Then B R_2 X is a great circle which bisects the arc R_1 R_3 in M say.

Fig. 4.



Let P O produced backwards meet the sphere in P′. Then

$$R_1R_2 = R_2R_3 = R_3R_1 = 74°\ 56'$$

and

$$XR_1=XR_3=XR_2$$

Also

$$\text{angle } R_1XM=60^\circ$$

$$R_1M=\tfrac{1}{2}R_1R_3=37^\circ\ 28'$$

whence

$$R_1X=44^\circ\ 37'$$

and

$$XM=26^\circ\ 15'\ 30'$$

Thus

$$BX=63^\circ\ 44'\ 30'' \quad . \ . \ . \ . \ . \ . \ . \ . \ . \ . \ . \ . \quad (13)$$

and BX is the angle denoted by β in the formulæ.

Again we have

$$R_3P'=0^\circ\ 50'$$

Therefore

$$MP'=36^\circ\ 38'$$

In the arc $P'R_1$ take $P'A=90^\circ$. Then B A is the trace of the face of incidence, and

$$\lambda=ABX=AM=90^\circ-MP'=53^\circ\ 22' \quad . \ . \ . \ . \ . \ . \ . \quad (14)$$

These values of β and λ were used in reducing the experiments.

The error produced by assuming P, R_1, R_3, and Q to be in the same zone will be discussed later.

The results of the experiments are contained in the following tables.

Table I. is the interpolation table used as described above to find ϕ'' from the value of ϕ, and gives the series of corresponding values of ϕ, ϕ' and $\phi''-\phi'$.

Table II. gives the values of ϕ, ϕ' and θ for the case in which the ordinary wave only traversed the crystal.

ϕ is directly observed, ϕ' and θ are found from the formula

$$\sin\phi'=\sin\phi/\mu$$

and

$$\cot\theta=\tan\beta\cos(\lambda+\phi')\sec(\phi-\phi')$$

where $\mu=1{\cdot}662$, $\beta=63^\circ\ 44'\ 30''$, $\lambda=53^\circ\ 22'$.

The values are arranged in pairs. In the first experiment recorded in each pair there was no sugar cell in the path of the light. In the second experiment the sugar cell was interposed. The fifth column gives the differences between the two values of θ thus found, and this, if the formula given by theory were correct, ought to be the rotation of the plane of polarization produced by the sugar cell.

Each recorded value of ϕ is the mean of five observations. At high angles of incidence the difference between two observations of the same value of ϕ was sometimes as great as 10′. The error produced in the value of θ by an error $\delta\phi$ in ϕ is, for these values of ϕ, less than $\delta\phi/2$, so that the extreme difference between the values of θ calculated from each of the five values of ϕ, of which the mean is given in the table, may be as great as 5′. At high angles of incidence the mean error in the value of θ, as given in the table, is considerably less than 2′.

At lower angles of incidence, 45° to 30°, the differences in the observed values of ϕ were much less, and rarely amounted to 5′, but then the error produced in θ by an error $\delta\phi$ in ϕ is not far short of $\delta\phi$, so that the probable accuracy of the values of θ given in the table remains much the same as before.

TABLE I.

ϕ.	ϕ'.	$\phi''-\phi'$.
$83^\circ\ 11'\ 0''$	$36^\circ\ 41'\ 10''$	$1^\circ\ 2'\ 40''$
78 10 30	36 4 50	1 0 50
68 11 30	33 57 40	56 20
65 34 35	33 13 10	54 40
56 49 0	30 14 10	50 20
52 28 40	28 30 10	48 0
48 32 30	26 48 10	46 10
38 17 50	21 53 40	41 10
31 5 15	18 5 50	37 20
30 36 0	17 50 10	37 0
24 42 0	14 33 40	33 0
21 3 10	12 29 0	30 0
17 45 0	10 34 10	26 50

TABLE II.

	ϕ.	ϕ'.	$\theta-90°$.	Rotation.
	° ′ ″	° ′ ″	° ′ ″	° ′ ″
1	80 47 0 70 59 0	36 26 10 34 40 10	0 33 20 4 55 30	4 22 10
2	78 14 0 69 29 0	36 5 20 34 18 0	1 29 20 5 54 10	4 24 50
3	73 43 0 66 6 20	35 16 40 33 22 30	3 30 10 7 47 50	4 17 40
4	70 57 40 63 57 50	34 39 50 32 43 0	4 56 30 9 11 50	4 15 20
5	66 6 20 59 51 50	33 22 30 31 21 30	7 47 50 11 58 10	4 10 20
6	64 12 30 58 9 30	32 48 10 30 44 20	9 2 0 13 11 50	4 9 50
7	59 45 0 54 8 30	31 19 0 29 11 10	12 3 20 16 9 30	4 6 10
8	57 54 20 52 24 30	30 38 40 28 28 30	13 22 50 17 28 10	4 5 20
9	52 16 40 47 1 40	28 25 10 26 7 10	17 34 10 21 35 50	4 1 40
10	46 50 40 41 30 50	26 2 10 23 30 10	21 44 10 25 50 0	4 5 50
11	41 41 30 36 4 10	23 35 30 20 45 0	25 41 40 29 54 20	4 12 40
12	36 5 40 30 18 50	20 45 40 17 40 50	29 53 20 34 0 10	4 6 50
13	30 36 0 24 25 0	17 50 10 14 24 10	33 48 10 37 54 20	4 6 10
14	24 42 0 17 45 0	14 33 50 10 34 10	37 43 20 41 54 30	4 11 10

The rotation produced by the cell was measured carefully by FIZEAU's method, and the mean of several observations in which the extreme difference was about 5′ gave the value 4° 6′ 15″.

Each of the numbers then in Table II., column 5, ought to be equal to 4° 6′ 15″, and this is evidently not the case. The differences begin by being too large, and allowing for the probable error of the experiment, they decrease fairly uniformly as the angle of incidence decreases until we reach an angle of incidence of about 52°.

As the angle of incidence is still further decreased there is a tendency to increase in the values of the rotation given by the table. The value 4° 12′ 40″, line 11, is pretty clearly too big, as also possibly is that in line 14. A very small displacement in the position of the sugar cell, so that the light traversed it somewhat obliquely, would give rise to an error of the kind here considered. The cell was usually adjusted by observing the beam of light reflected from its first face. This could be made with a little care to travel back through the biquartz, and in that case the light clearly fell normally on the cell.

The first twelve sets of observations recorded in Table II. were made on October 28, 1880, the last two a few days later.

Thus, unless there is some regular source of error in the experiments, we must conclude that the formula connecting the plane of polarization and the angle of incidence in the case in which only the ordinary ray traverses a crystal of Iceland spar, as given by the electro-magnetic theory, is only true approximately.

The method does not enable us to determine accurately by experiment the position of the plane of polarization of the incident light with reference to the face of the crystal on which it falls; we can however compare the rate of change of the position of this plane, as the angle of incidence varies, found from experiment with its value deduced from theory.

We arrive at the conclusion that this rate of change as given by theory is too rapid when the angles of incidence are large, that for angles of incidence of from 30° to 60° the theoretical and experimental rates agree, while for lower angles of incidence the theoretical rate is again possibly too great; the last inference however being a little uncertain.

Two other shorter sets of observations made about the same time confirm these statements exactly, and it does not seem worth while to print a table of the numbers actually arrived at.

TABLE III.

	φ.	φ″.	θ.	Rotation.
	° ′ ″	° ′ ″	° ′ ″	° ′ ″
1	82 55 0 69 21 30	37 42 10 35 13 10	0 48 30 4 12 40	3 24 10
2	77 24 20 67 0 0	36 58 10 34 33 40	1 37 0 5 18 0	3 41 0
3	73 4 40 64 57 30	36 16 0 33 56 20	2 35 10 6 21 40	3 46 30
4	69 3 30 61 37 30	35 8 20 32 50 30	4 21 0 8 17 10	3 56 10
5	66 42 0 59 57 0	34 28 10 32 15 20	5 27 10 9 19 30	3 52 20
6	64 46 50 58 17 0	33 52 50 31 38 20	6 27 40 10 26 0	3 58 20
7	61 29 40 55 37 0	32 47 50 30 35 50	8 22 0 12 19 0	3 57 0
8	55 43 0 50 27 30	30 38 10 28 25 50	12 14 50 16 14 0	3 59 10
9	50 18 0 45 15 0	28 21 50 26 2 30	16 21 0 20 27 20	4 6 20
10	45 17 0 40 17 0	26 3 20 23 35 50	20 26 0 24 34 40	4 8 40
11	40 9 20 35 4 30	23 32 0 20 53 10	24 41 0 28 51 20	4 10 20
12	35 7 30 29 50 30	20 54 0 18 1 20	28 48 50 32 58 30	4 9 40
13	29 40 0 24 4 0	17 58 10 14 44 50	33 3 20 37 12 30	4 9 10
14	24 15 0 17 45 0	14 51 10 11 1 0	37 4 50 41 23 0	4 18 10

To complete the investigation we should consider the effects of possible errors in the constants β and λ. This can be done more advantageously after we have tabulated the results of the experiments in which the extraordinary ray only was allowed to traverse the crystal.

Table III. contains these. θ as before is the azimuth of the plane of polarization of the incident light, which in this case is calculated from the formula (12)

$$\tan\theta = \tan\beta\cos(\lambda+\phi'')\cos(\phi-\phi'')$$
$$+\frac{a^2-b^2}{a^2b^2}\,\frac{\sin 2\beta\sin(\lambda+\phi'')\sin^2\phi}{\sin(\phi+\phi'')\cos^2\theta''}$$

where

$$\tan\theta'' = \tan\beta\cos(\lambda+\phi'')$$

$$a = 1{\cdot}662 \qquad b = 1{\cdot}488$$

$$\beta = 63^\circ\, 44'\, 30'' \quad \lambda = 53^\circ\, 22'$$

The observations recorded were made on November 16, 1880.

For the rotation of the sugar cell, measured by FIZEAU's method, the value $4^\circ\, 4'\, 20''$ was obtained as the mean of seven measures, the two extremes of which differed by $5'$.

If our theory then were correct, each of the differences given in Table III., column 5, should be $4^\circ\, 4'\, 20''$.

We see at once that this is very far from being the case. The values of the rotation commence by being much less than $4^\circ\, 4'\, 20''$ and, with the exception of Experiment 4, increase fairly regularly as the angle of incidence decreases.

As was the case with the ordinary ray, the rotations agree with experiment for an angle of incidence of about 50°; from that point onwards the theoretical rotation is too great.

Thus the theoretical rate of change in the position of the plane of polarization is too small for high angles of incidence, but increases as the angle of incidence decreases, and finally becomes too great.

Several other series of experiments lead to the same conclusions.

It remains then to discuss the effect of an error in the values of λ or β, and this is all the more necessary, for we know that the values taken are only closely approximate.

Let us take the ordinary wave first for which we have

$$\cot\theta = \tan\beta\cos(\lambda+\phi')\sec(\phi-\phi')$$

and consider the effect of decreasing β by a given amount.

The logarithm of $\cot\theta$ is thus decreased throughout by the same quantity.

Now the change produced in θ by a given change in $\log\cot\theta$ is greatest when θ is nearest to 45°, thus by any change in β the values of θ will be more altered in Experiment 14 than in Experiment 1.

But we have to consider the change in the difference between two consecutive values of θ.

When θ is near 90° $\log\cot\theta$ changes more rapidly for a given change in θ than when it is near 45°.

If then in Experiment 1 we subtract from each of the values of $\log\cot\theta$ a certain quantity, it will alter the value of θ in the second line by an amount considerably greater than the alteration it produces in the first line, and thus the difference between the two values of θ will be reduced.

If on the other hand we consider Experiment 14, each of the values of θ there will be more affected by this change than in Experiment 1, but the effect produced on the value of θ in the first line will be nearly the same as that produced on the value of θ in the second, and thus the difference between the two values will not be so much altered.

Thus a decrease in β will decrease everywhere the theoretical value of the rotations, but it will affect the high angles of incidence considerably more than the low. It will thus tend to bring the theory more into accordance with experiments.

Let us see how it will affect the extraordinary ray.

The term in the formula with $\frac{a^2-b^2}{2a^2b^2}$ for a factor is practically a small and slowly varying correction except for the very highest angles of incidence; let us consider it as constant with regard to β and see how a decrease in β affects the values of θ supposed to depend only on the term

$$\tan\beta\cos(\lambda+\phi'')\cos(\phi-\phi'')$$

Exactly the same reasoning applies to this as in the case of the extraordinary wave.

The values of θ in Experiment 14 will be most decreased, but the alteration in the value of θ in the second line of Experiment 1 will be much greater than the alteration of the value of θ in the first line, while the change in the values of θ in Experiment 14 will be much the same for the two. Thus, by decreasing β the differences will be decreased throughout, but the changes will be greatest when the angles of incidence are large.

Thus a decrease in β will increase the differences between the theory and experiment.

Hence considering the ordinary and extraordinary rays together, a change in β will not reconcile the facts observed.

Calculation shows that decreasing β by 30′ decreases the differences in Experiments 4, 11, and 14 by 6′, 4′, and 1′ 20″ respectively for the extraordinary ray, and the amounts are about the same for the ordinary.

We must now consider alterations in λ; putting $\tan\beta=K$ we have for the ordinary ray

$$\cot\theta=K\sec(\phi-\phi')\cos(\lambda+\phi')$$

therefore

$$\delta\theta=K\sin^2\theta\sin(\lambda+\phi')\sec(\phi-\phi')\delta\lambda.$$

Now when ϕ is large θ is nearly 90° and $\phi-\phi'$ is large.

Thus $\sin\theta\sin(\lambda+\phi')$ and $\sec(\phi-\phi')$ all increase with ϕ', and therefore $\delta\theta/\delta\lambda$ is greatest when ϕ is large.

Thus the alteration produced in θ is greater the greater the angle of incidence.

If then we decrease λ we shall reduce in every case the differences between the corresponding values of θ, but we shall reduce them most for high angles of incidence and thus tend to bring the theory more into accordance with experiment.

to coincide with the end of the needle point in the focus of the telescope. The telescope was then turned to view the images reflected from the faces of the prism, and the levelling screws of the prism adjusted until the same coincidence was established.

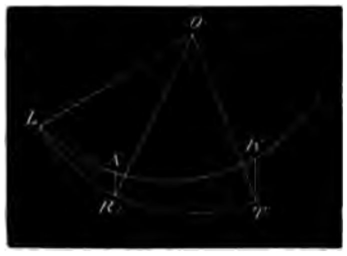
Now observation showed that when P and Q were level in this manner R_1 and R_3 were both a very little too low.

The vertical angular distance between the end of the needle and the mark across the slit, seen by reflection from R_1 or R_3, could be estimated by setting the needle on the slit, reading the vernier of the telescope and then moving the telescope with the needle until the horizontal distance between it and the slit appeared about equal to the vertical distance between the end of the needle and the mark.

This horizontal distance is found at once by again reading the vernier of the telescope.

I found as the mean of several closely concordant observations that when the angle of incidence of the light on the face R_1 was 20° 58′ the image formed by reflexion was 10′ too low, and the same exactly was the case with the image formed by reflexion at the same angle from R_3.

Fig. 5.



Let L O (fig. 5) be the direction of the incident light from the mark on the slit, O T the reflected ray, O R_1 the normal to R_1.

L R_1 T is a great circle, let L N K be the principal plane of the prism, T K and R_1 N being perpendicular to L N K.

$$LR_1=\phi=R_1T$$

Let the angle $TLK=\gamma$, $R_1N=x$.

The experiment has shown that $TK=10'$.

From the triangles T L K, R_1 L N

$$\sin x=\sin\gamma\sin\phi$$
$$\sin 10'=\sin\gamma\sin 2\phi$$

and

$$\phi=20^\circ\ 58'$$

whence

$$x=6'$$

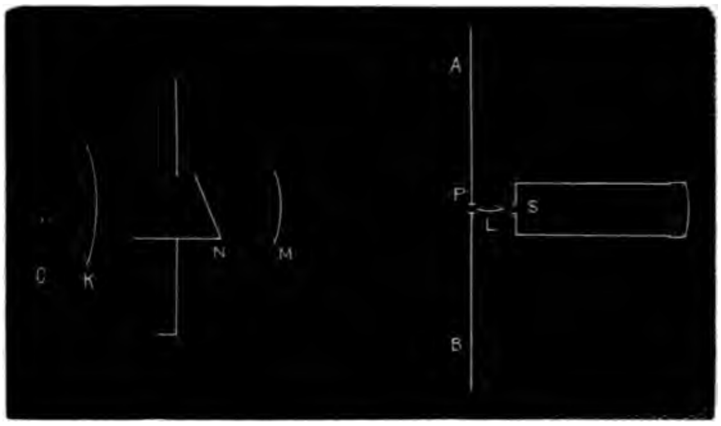
Thus R_1 is 6′ below the great circle L K, and so also is R_3.

set screws all the adjustments requisite for accurate measurement of the position of the plane of polarization of the light emerging from the Nicol can be accomplished.

Before asking permission, therefore, to lay the preceding results before the Royal Society I waited until these circles were ready, and have repeated the experiments described above, using one of them instead of the sugar-cell to produce the rotation in the plane of polarization of the light.

The only additional adjustment necessary was to make sure that the axis of rotation of the polarizing Nicol was parallel to the incident light. The Nicol was placed at about a metre behind the slit of the collimator and adjusted by looking along the axis of the collimator until the centre of the face of the Nicol appeared to be in the axis. In the figure (7), which gives the apparatus in plan, S is the slit of the collimator, and N the Nicol. Between the slit and the Nicol was placed a screen, A B, with another slit, P, adjusting it, so that P lay also in the axis of the collimator, and between P and S a lamp, L, so as to illuminate both slits. The lamp was so adjusted that the light from it fell along the axis of the collimator in one direction, while in the other after passing through the slit P, it fell on the face, N, of the Nicol and was there reflected. A convex lens was placed between P and N in such a position that the rays reflected from the face of N, formed an image of the slit on the screen A B. On rotating the Nicol round its axis this image moved over the screen, and if the axis of rotation is parallel to the axis of the collimator, as it should be, the centre of the image will describe a circle round P as centre. A horizontal scale was affixed to the screen along A B and the position of the Nicol adjusted until the image of the slit, as the Nicol was turned, cut this scale in points equidistant from P on either side of it.

Fig. 7.



This made it certain that the axis of the Nicol was in the same vertical plane as that of the collimator. But the axis of the collimator had been levelled with a spirit level, and by altering one of the feet on which the circle carrying the Nicol rested its axis was rendered horizontal. Thus the axis of the Nicol was rendered parallel to that of the collimator.

Behind the NICOL was placed a large lens, K, of somewhat short focal length, to act as a condenser, and at its focus the source of light, C, used in the experiment.

The lens M, the screen and lamp L, were of course removed, and when the narrow parallel beam of light from the lamp, C, after passing through the NICOL fell on the slit of the collimator, I knew that it traversed the NICOL in a direction parallel to the axis of rotation.

By means of a long handle and a HOOK's joint the tangent screw of the circle of the polarizer could be turned by the observer when his eye was at the telescope.

The experiments were conducted in a somewhat different order to that adopted in the first set. The analyzing prism was placed so that the light fell on it at a known angle of incidence and the NICOL turned by hand until the two black bands appeared fairly close together in the ordinary spectrum. The polarizing circle was then clamped and turned by means of the handle until the bands were brought exactly into coincidence.

The reading N_0 of the NICOL circle was taken and also the deviation D_0 of the dark band in the ordinary spectrum. The NICOL circle was then turned, keeping the angle of incidence on the analyzing prism the same, until the dark bands were seen in the extraordinary spectrum and coincidence was established as before; the reading N_E of the NICOL circle was taken, and also the deviation D_E of the band in the extraordinary spectrum. The analyzing prism was then moved so as to alter the angle of incidence, and another set of observations taken, only in the reverse order. A series of values of the angles of incidence and the quantities N_0, N_E, D_0, D_E were thus observed. Each single observation was repeated five times and the mean taken and used in the calculations.

For a certain known angle of incidence, D_0 is the deviation of the light that is quenched in the ordinary spectrum. It is also however the deviation in the ordinary spectrum of the light that is missing from the extraordinary, when the coincidence between the black bands is established there.

D_0 is therefore the angle of deviation we must use in calculating the position of the plane of polarization of the incident light, when the ordinary ray only is transmitted. The experimental value of the angle determining this position, measured from some unknown zero, is N_E, for this gives us the position of the NICOL when the extraordinary wave is quenched.

Of course the theoretical values of the positions of the plane of polarization might have been determined as before from the values of the angle of incidence and ordinary refractive index, by the use of the interpolation Table I.

I wished, however, to make this series of observations entirely independent of the first, and so recalculated the interpolation table.

Since the position of the zero of the polarizer circle with reference to the analyzing prism is unknown, an unknown constant will, as before, come into our tables.

Let us determine it so that the theoretical and experimental values of the position of the plane of polarization in the first experiment on the ordinary wave agree.

Table IV. gives the values of ϕ, D_0+i, N_E, ϕ', θ_0 and the differences between θ_0 and N_E.

The last two columns give respectively the differences between the consecutive values of θ_0 and consecutive values of N_E.

The differences of θ_0 correspond to the rotation in the last column of Table II., and the differences of N_E to the constant experimental value of that rotation, viz., 4° 6′ 20″. The last two columns, therefore, enable us to compare these experiments with those recorded in Table II.

D_0+i is given, i being the angle of the prism because it occurred in this form in the formulæ, and is just as easily found at once as D_0.

Table V. gives the similar values for the case in which the extraordinary wave only is transmitted.

The zero from which N_0 is measured is of course that from which N_E has been measured.

As has already been said, each of the numbers in the columns $D+i$ and N is the mean of five observations.

The observations of the deviation rarely differed by as much as 20″, so that the mean is probably accurate to 5″. At the lower angles of incidence that is in the neighbourhood of the position of minimum deviation this would produce an error of about 1′ in the value of θ. When the angle of incidence is about 45° the error introduced into the value of θ by an error of 5″ in D is practically inappreciable.

Thus the theoretical values of θ are probably very accurate, the possible error being greatest when the angle of incidence is small, and then probably it is considerably less than 2′. The differences in the observed values of N were greater; except in the case of the first two sets recorded, when owing to the high angle of incidence very little light got through the prism and the field was very dark—the greatest difference between two observed experimental values of the same quantity was 5′. The mean error of the observations is rather under 1′ 30″, so that the recorded values of N_0 and N_E are probably accurate within this limit.

For the first two values of N_0 the extreme differences among the observations were as much as 10′, but a greater number of observations was taken. The mean error for them was 3′. In the case of the first two observations of N_E the difficulty was not so great; the extreme differences were 4′, and the mean error about 1′ 30″.

We proceed now to discuss the results of the table.

If the agreement between theory and experiment were complete, the differences recorded in the third column from the end in each table would be zero throughout.

In each case as there recorded, they increase for a time as the angle of incidence decreases and then decrease again. We of course must remember that our numbers do not give absolutely the difference between theory and experiment; there is a unknown constant to be considered which we have arbitrarily determined so as to make the difference in the first line of Table IV. zero. It may quite well be that we are

TABLE IV.—Ordinary wave only transmitted.

	ϕ.	D_0+i.	ϕ'.	θ_0-90.	$N_E-90°$.	θ_0-N_E.	Rotation theory.	Rotation experiment.
	° ′ ″	° ′ ″	° ′ ″	° ′ ″	° ′ ″	′ ″	° ′ ″	° ′ ″
1	82 57 40	87 19 22	36 40 0	− 0 5 50	− 0 5 50	0 0	..	..
2	75 14 40	81 24 40	35 35 0	2 45 40	2 31 15	15 25	2 51 30	2 37 5
3	67 12 55	76 32 50	33 41 30	7 6 40	6 43 50	22 50	4 21 0	4 12 35
4	59 59 55	73 11 14	31 24 0	12 0 10	11 27 50	32 20	4 53 30	4 44 0
5	54 43 10	71 17 26	29 24 40	15 44 30	15 13 0	31 30	3 44 20	3 45 10
6	49 5 40	69 43 36	27 3 0	20 0 10	19 30 35	29 35	4 15 40	4 17 35
7	43 46 55	68 43 10	24 35 50	24 6 30	23 35 50	30 40	4 6 20	4 5 15
8	38 41 40	68 9 0	22 5 20	27 58 20	27 32 50	25 30	3 51 50	3 57 0
9	32 51 0	67 58 42	19 2 30	32 14 30	31 54 30	20 0	4 16 10	4 21 40
10	27 38 10	68 19 54	16 12 0	35 49 30	35 31 20	18 10	3 35 0	3 36 50
11	21 17 30	68 34 28	12 36 50	39 50 20	39 36 30	13 50	4 0 50	4 5 10

TABLE V.—Extraordinary wave only transmitted.

	ϕ.	D_E+i.	ϕ''.	θ_E.	N_0.	$N_0-\theta_E$.	Rotation theory.	Rotation experiment.
	° ′ ″	° ′ ″	° ′ ″	° ′ ″	° ′ ″	′ ″	° ′ ″	° ′ ″
1	82 57 40	85 31 20	37 42 20	0 48 30	0 37 0	−11 30	..	..
2	75 14 40	79 38 20	36 33 20	2 10 15	2 8 50	− 1 25	1 21 45	1 31 50
3	67 12 55	74 47 28	34 37 40	5 11 15	5 24 50	13 35	3 1 0	3 16 0
4	59 59 55	71 25 8	32 16 30	9 15 15	9 34 50	19 35	4 4 0	4 10 0
5	54 43 10	69 28 57	30 14 40	12 58 0	13 17 45	19 45	3 42 45	3 42 55
6	49 5 40	67 51 28	27 50 50	17 17 0	17 38 6	21 6	4 19 0	4 20 21
7	43 46 55	66 43 56	25 21 10	21 38 30	31 58 20	19 30	4 21 30	4 20 14
8	38 41 40	65 59 43	22 46 0	25 55 30	26 12 10	16 40	4 17 0	4 13 50
9	32 51 0	65 33 18	19 41 0	30 39 0	30 49 50	10 50	4 43 30	4 37 40
10	27 38 10	65 33 18	16 47 20	34 38 15	34 45 30	7 15	3 59 15	3 55 40
11	21 17 30	66 9 22	13 7 20	39 6 30	39 12 0	5 30	4 28 15	4 26 30

wrong in so doing. The table, however, shows us conclusively that we cannot so determine this constant as to bring theory and experiment into exact agreement.

The last two columns in each table enable us to compare these results with those of our former experiments.

The one gives us the rotation of the plane of polarization when only one wave traversed the crystal according to the electro-magnetic theory, the other the measured value of the same rotation.

Consider Table IV. first. At high angles of incidence the theoretical value of rotation is greater than the experimental. As the angle of incidence decreases the two become more nearly equal and agree within the limits of the error of experiment between angles of incidence ranging from 55° to 45°. As the angle of incidence still further decreases the theoretical value of the rotation becomes decidedly less than the experimental.

In the main this agrees with Table II. At high angles of incidence the rotation given by theory is greatly in excess of that given by experiment, the two agree between 55° and 40°, but for the lower angles of incidence Table II. shows a slight increase in the theoretical value as compared with the experimental.

Turning now to the last two columns in Table V., we see that the theoretical rotation is at high angles of incidence less than the experimental, that as the angle of incidence decreases the two tend to become equal and agree very closely for angles of incidence between 55° and 45°, while from that point onwards the theoretical rotation is bigger than the experimental. The change in the relative values of the two is very regular, while the actual change of sign in their difference occurs between the values $49° \, 5' \, 40''$ and $43° \, 46' \, 45''$ of the angle of incidence.

Referring to Table III. we see that this is exactly the state of affairs there indicated. The theoretical rotation is at first less than the experimental and increases gradually as the angle of incidence decreases, becoming the greater between the values $50° \, 27' \, 30''$ and $45° \, 15'$, almost the same limits as above, of the angle of incidence, and continuing so throughout the rest of the arc examined.

With the exception then of one or two observations recorded at the end of Table II., the results of the two series of experiments, the one made during the autumn of 1880, the other during the summer of 1881, agree and lead us to the conclusion that the laws arrived at by the electro-magnetic theory connecting the planes of polarization of the incident and refracted rays in the case of refraction at the surface of a uniaxal crystal are not exact but are probably close approximations to the truth.

These same laws have been arrived at by NEUMANN,* MACCULLAGH,† KIRCHKOFF‡ and others on various assumptions as to the nature of the ether; and in the case of an isotropic refracting media agree with FRESNEL'S formulæ for the relative positions of the planes of polarization of the incident and reflected rays, though not for the refracted.

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* Abhand. der Akad. der Berlin, 1835.

† Irish Trans., 1837.

‡ Abhand. der Akad. der Berlin, 1876.

The experiments having all been conducted with the same piece of spar do not afford data for deciding whether the differences observed are functions only of the angle of incidence, or whether, as is more likely, they depend partly on it and partly on the position of the face of incidence with reference to the axis of the crystal. Though in no case large, they are certainly considerably greater than the possible errors of experiment.

The experiments have been conducted by Lord RAYLEIGH'S kind permission in one of the rooms of the Cavendish Laboratory, Cambridge, and the apparatus used is chiefly the property of the Laboratory.

In conclusion, I would refer to a paper on the same subject by F. E. NEUMANN ("Beobactungen über den Einfluss der Krystaleflächen auf das reflectirite Licht." POGG. Ann., xlii.).

He polarized light by passing it through a tourmaline, and then allowed it to fall on a prism of Iceland spar at a known angle of incidence, observing the position of the plane of polarization when only one ray passed through the crystal. The light of a lamp was used and the crystal prism achromatised approximately for the ordinary ray. The prism was capable of rotation about a normal to the face on which the light fell so as to alter relatively to the axis of the crystal the position of the plane of incidence. The theoretical values of the position of the plane of polarization were calculated from formulæ given by NEUMANN (Abhandlungen der Akad. der Berlin, 1835), and MACCULLAGH (Irish Transactions, 1837), which are identical with those deduced from the electro-magnetic theory. In the eight observations recorded, the differences between the calculated and observed values of the azimuth of the plane of polarization range from $-8'$ to $+4'$.

Each observed value is the mean of 40 observations, but it is not stated how far the observations differ among themselves; observations were only made for a small number of values of ϕ, the angle of incidence, two at most for each position of the plane of incidence relatively to the axis of the crystal, and the values of ϕ chosen were 40°, 45°, 50° and 55°.

Now, it will be seen on referring to my tables, that for these values of ϕ the differences there recorded are small. Nothing is said in NEUMANN'S paper as to the method adopted to determine the values of ϕ' and ϕ'', the angles of refraction of the ordinary and extraordinary rays. In fact, they are both indeterminate, seeing that the experiment was made with white light. These are the only other experiments made to test the theory so far as it concerns the refraction of light.

The same paper of NEUMANN'S contains a series of experiments on the position of the plane of polarization of light reflected from the surface of the crystal under various circumstances, the results of which are compared with theory.

The comparison is also extended to some earlier experiments of SEEBECK's (POGG. Ann. xxi., xxii., and xxxviii.). As far as one can judge from the details given, the differences are hardly greater than the possible errors of the experiment, for in all cases lamp light was used and the position of the plane of polarization determined by the quenching of one of the rays in a double image prism, and both these circumstances preclude great accuracy. On the other hand, the number of individual observations was very large, so that the mean deserves considerable weight.

XII. *On the Results of Recent Explorations of Erect Trees containing Animal Remains in the Coal-formation of Nova Scotia.*

By J. W. DAWSON, *LL.D., F.R.S., C.M.G.*

Received October 11, 1881,—Read January 12, 1882.

[PLATES 39–47.]

THE occurrence of remains of land animals in the interior of erect fossil trees is, so far as yet known, confined to certain horizons in the coal-field of the South Joggins in Nova Scotia. These remains were first discovered by Sir CHARLES LYELL and the writer in the summer of 1851. They were found in fragments of the sandstone filling an erect *Sigillaria* which had fallen from the cliff near Coal Mine Point. As other erect trees occurred in the beds from which this was supposed to have fallen, search was made by the writer in subsequent visits for additional trees; but up to 1876 only three of those which became accessible by the wasting of the beds were found to yield animal fossils. These, however, afforded many additional specimens, and several new species of Batrachians and Millipedes. The results of these explorations were published at various times in the Journal of the Geological Society of London,* in a work entitled 'Air-breathers of the Coal Period,' and in 'Acadian Geology;' and Dr. SCUDDER described the new species of Millipedes in the Memoirs of the Boston Society of Natural History.

The beds containing the productive trees being thus well known, and being exposed in a cliff and in a reef extending into the sea, it seemed probable that many others might be obtained by quarrying operations of no great difficulty. In 1878 the subject was brought under the notice of the Council of the Royal Society, and a grant of £50 was made from the Government Fund to aid in the extraction of these trees and the collection of their contents. With the aid of this grant, a thorough survey and examination has been made of the cliff and reef by Mr. ALBERT T. HILL, C.E., by Mr. W. B. DAWSON, C.E., and by myself, with the kind aid of B. B. BARNHILL, Esq., Superintendent of the Joggins Coal Mines. By these means, along with the removal of fallen débris and sand from the outcrop of the beds, twenty additional trees were discovered and were extracted by cutting and blasting; affording many additional specimens and much information respecting the conditions of accumulation of the beds and the manner of entombment of the animal remains.

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* Vols. x., xvi., xviii., xix.

Three annual reports have been made to the Society, detailing the plan and progress of the work. I may here merely state that, after preliminary clearing and exposure of the accessible trees, more especially in the reef extending from the shore and uncovered at low tide, the precise position of each was marked on the plan and section. The trees were then carefully taken out and their contents were examined. The portions containing animal remains were preserved in as large pieces as possible, and were boxed on the spot, the material of each tree being kept by itself. On being taken to Montreal, the whole of the material was cleaned and examined and carefully split up, each surface being scrutinised with the lens under a strong light. The fossils found were marked, keeping together the bones belonging to each skeleton, and were exposed as far as possible with fine chisels and needle points. As the work proceeded, drawings and photographs of the more important bones were made, more especially in the case of those which ran any risk of being damaged in the development of neighbouring or underlying fragments. The pieces belonging to each animal were then attached to cards or placed in separate drawers for study. This preliminary labour necessarily required much time, and though the accessible trees were exhausted in 1879, the final revision of the specimens and the microscopic examination of the bones and teeth have been completed only in the present year.

I may add that, unless additional specimens are exposed by falls of the cliff, further material of this kind can be obtained only by mining in the 6-inch coal supporting the trees, and its roof.

In the following pages I propose to notice as shortly as is consistent with clearness, the new facts obtained from the study of these interesting fossils, under the following heads:—

1. Geological relations of the beds containing the fossiliferous trees.
2. Character and contents of the trees.
3. Description of the included animal remains.

1. GEOLOGICAL RELATIONS OF THE BEDS.

The beds in question belong to Group XV. of Division 4 of the section of the South Joggins, tabulated by Sir W. E. LOGAN and the writer.* The detailed sequence of the beds more immediately connected with the fossiliferous trees, as noted in the course of the recent explorations, is as follows, in descending order; the dip of the beds being S. 30° W., at an angle of 32°.

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* 'Acadian Geology,' pp. 156 to 192.

to the encroachments of the sea. East of this the coast recedes, still presenting a high cliff in which the beds crop out one by one to the surface. In front of the point the great sandstone, No. 1, and the minor bed, No. 8, extend in reefs seaward, and owing to the great rise and fall of the tide in Cumberland Bay a length of about 330 yards of these reefs is exposed at low tide (see map, section, and view of the coast, Plates 46 and 47).

From information obtained by HENRY POOLE, Esq., it would appear that the cliff recedes at the rate of about 25 feet in 40 years, so that trees are rarely exposed in the bank, and those existing on the reef, and which are the most accessible, represent the effects of denudation extending over at least five centuries.

The relative positions of all the trees observed are seen in the plan and section, from which it would appear that they occur in groups. There is reason to believe, however, that these erect trees are only survivors of a much more dense forest, of which the weaker and more perishable trees had been overthrown. The positions of the trees extracted before 1878 could only be indicated approximately.

The manner of accumulation of the beds enclosing the erect trees is rendered evident by their character and contents, and has been noticed in my 'Acadian Geology,' p. 190, *et seq.* The details ascertained by the recent excavations may be stated as follows :—

The underclay, No. 13, represents a loamy soil on which *Sigillariæ* must have flourished for a long time, filling it with their roots and rootlets. The result was the accumulation of the coal, No. 12, which is filled with flattened and carbonised bark of these trees, as is the black shale constituting its roof. Bark of *Lepidofloyos*[*] is however largely associated with that of *Sigillaria* in these beds. Leaves of *Cordaites* also occur in this coal, and its mineral charcoal shows under the microscope bast fibres of the inner bark of *Sigillaria*, with scalariform, uniporous, and reticulated tissues, probably belonging to the wood of *Sigillaria, Lepidofloyos, Calamites,* and *Cordaites.*[†] Leaves of the latter genus, as I have shown (*op. cit.*), constitute a large part of some thin coals at the Joggins, and some portion of the ligneous matter, which in former papers I have referred to other genera, may, since the discoveries of GRAND-'EURY, possibly be referred to *Cordaites.* The more compact portion of the coal when sliced shows shreds of epidermal tissue with a few rounded bodies, probably spores of ferns or lycopods.[‡]

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[*] *Ulodendron* of some English palæobotanists.

[†] 'Acadian Geology,' p. 168; and paper on "Coal Accumulation," Jour. Geo. Soc.

[‡] I cannot admit that the large trunks of silicified and calcified wood of the genus *Dadoxylon* (*Araucarioxylon*), so abundant in the coal-formation of Nova Scotia, belonged to *Cordaites.* Their foliage is more probably represented by the leafy twigs of *Walchia* or *Araucarites* found with them. It is also true that some ribbed trees with the markings of *Sigillaria* have wood of the structure attributed by GRAND-'EURY and RENAULT to *Cordaites.* Where the dividing lines between *Sigillaria, Cordaites,* and *Dadoxylon* will finally be fixed remains somewhat uncertain. As I have elsewhere argued, however, it is evident that under the names *Sigillaria* and *Cordaites* are included cryptogamous and gymnospermous trees of very different grades.

After a quiet accumulation of vegetable matter, sufficient to give six inches of pure coal, the area must have subsided or been overflowed with water, probably brackish, in which *Naiadites* and Entomostracans established themselves, but in which for a time very little sediment was deposited, the dead plants remaining on the surface of the submerged swamp, and possibly others drifted to the locality, forming, with a little fine argillaceous matter, the material of carbonaceous shales. In the meantime the stronger and larger trunks of the *Sigillaria* forest remained erect, and around their bases there gradually accumulated layers of mud, constituting the shale, No. 10, including drifted plants, while it is not unlikely that the abundant remains of *Sphenophyllum* and *Pinnulariæ* represent aquatic vegetation growing on the surfaces of the accumulating mud. At this time the greater number of the trees had either not yet become hollow or were too tall to receive any sediment. A few, to be noticed in the sequel, were however either wholly or in part filled with clay.

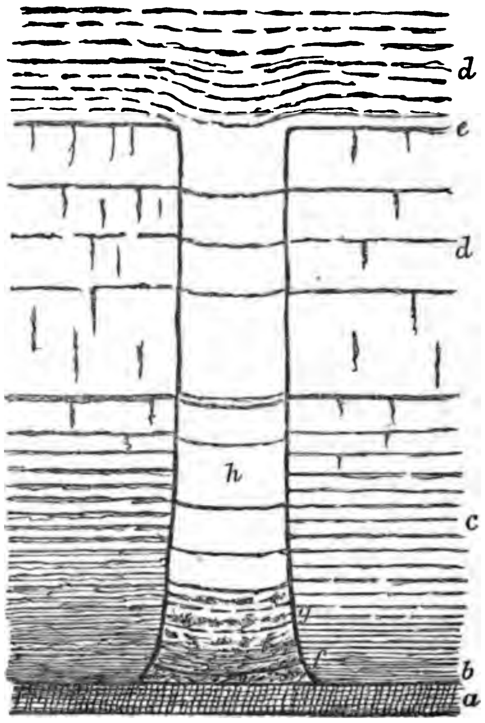
By gradual silting up, possibly aided by a slight elevation, the area again became capable of supporting land plants, as evidenced by the erect *Calamites* and *Calamodendra* which rise from the surface of the shale, and by the *Stigmariæ* in the succeeding sandy deposit. The surface however now became subject to periodical or occasional inundations bearing sand, at first fine and argillaceous, but afterwards coarser. In the intervals of these inundations the thickets of *Calamites* and ferns which occupied the ground were tenanted by Batrachians, Millipedes, and Land Snails.

In the meantime the larger and stronger *Sigillariæ*, which had remained erect, while sand was accumulating around their trunks, became hollow through decay, the strong outer rind alone remaining, while the inner bark and woody axis fell to pieces and dropped into the bases of the hollow cylinders. In this condition these hollow trees would constitute deep wells and pits in the soil, their openings more or less masked with herbage or with shreds of bark projecting above the surface. They no doubt served as places of retreat to Millipedes and Land-Snails; but to the small reptiles heedlessly passing over the surface they were pitfalls into which they fell, and being unable to escape, perished.

In connexion with this, it should be observed that the conditions of the case excluded all animals unable to creep or walk on land. Hence the assemblage of species in these trunks is of a special character, and includes none of those more aquatic forms of Batrachians which have been discovered in the aqueous deposits of the period. Further, only the smaller animals of the locality would be entrapped, larger species being little likely to fall into openings so narrow.

How long any of the hollow trees remained open it is impossible to say; but there is, as might be expected, evidence of successive stages and different modes in their filling. Some short bases of trees, resting on the coal and not extending into the shale, are mere disks of mineral charcoal, and may represent trees which had gone to decay even before the first submergence of the coal. Others seem to have been broken off before the close of the deposition of the shale, and are filled with that material. Others remained open till the deposition of the overlying sandstone, and must have been still about 9 feet high, when finally filled. These are the most productive of animal remains. The normal character of the deposits in such taller trees is represented in fig. 1. At the base is a layer of mineral charcoal representing the débris of the trunk itself, and sometimes containing a sandstone cast of the pith. Above this is a variable quantity of more or less indurated sandy matter, in layers, which, owing to pressure, are often concave above. This is blackened with organic matter, and contains fragments of bark and comminuted plants, with occasional leaves of *Cordaites* or fragments of stems of *Calamites*. It contains the greater part of the animal remains, though a few of these occur in the sandstone above. This deposit, which is the productive layer of the several trees, evidently consists of matter which fell into the hollow trunks or was washed in by rains, while the trees were still open at the surface of the soil. Above this productive layer are layers or sometimes a continuous deposit of sand, by which the trees were finally filled, and which contains no organic remains except fragments of drifted plants. Thus the productive portion of each tree is found near its base. After the final filling and the deposition of additional sediment, the greater compressibility of the matter contained in the trees

Fig. 1.



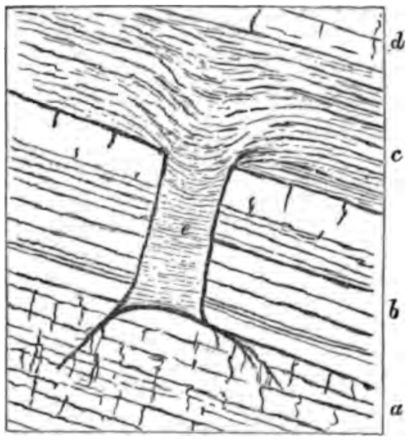
Section of erect tree.

(*a*) Coal. (*b*) Coaly shale. (*c*) Gray shale. (*d*) Sandstone. (*e*) Land surface.
(*d*) Argillaceous sandstone. (*f*) Mineral charcoal. (*g*) Carbonaceous deposit, with animal remains.
(*h*) Sandstone, filling interior of trunk.

caused this to settle somewhat, so that a funnel-shaped depression occurs over some of the taller trunks (figs. 1 and 2).

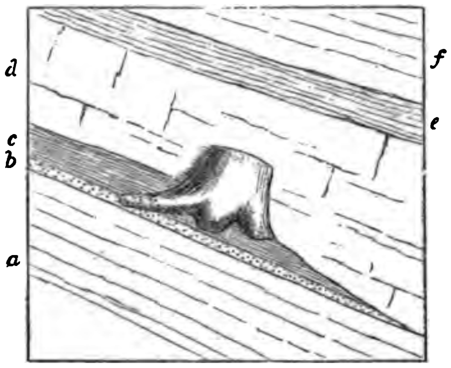
As opportunity occurred in the progress of the work, erect trees were extracted from other beds in the section, in the hope of discovering another fossiliferous forest; but with little positive result. In a trunk, about 15 inches in diameter, and standing in Section XXVI. of Division 4, Mr. A. T. HILL had found in 1876 several shells of *Pupa vetusta.* This tree, which was uniformly filled with compact argillo-arenaceous matter, was taken out, and a few additional *Pupæ* obtained, but no other remains; and no other trees could be seen in the bed (fig. 2). Another tree standing in Section XXIV., and 2 feet in diameter, was interesting in consequence of its well-marked Stigmarian roots, and of its standing on the sloping edge of a partially denuded shale (fig. 3). It contained no fossils.

Fig. 2.



Erect tree in Section XXVI., containing shells of *Pupæ* and showing funnel-shaped depression.
(*a*) Underclay with roots and rootlets. (*b*) Sandstone and shale. (*c*) Shale. (*d*) Sandstone.
(*e*) Shale filling tree and containing shells of *Pupa* in lower part.

Fig. 3.



Erect tree in Section XXIV.
(*a*) Sandstone and shale. (*b*) Sandstone. (*c*) Shale. (*d*) Sandstone. (*e*) Shale. (*f*) Sandstone.
This tree had probably originally extended to the shale (*e*), or nearly so.

2. Characters and Contents of the Erect Trees.

It may be well in the first instance to present in a summary form the characteristics and contents of each tree extracted, taking them in order, from the shore cliff to the end of the reef, 1,063 feet in all, and including those extracted prior to 1878 with the others. The positions of the several trees may be seen on the plan and section.

No. 1. Lyell's tree of 1851, which comes first both in order of time and position. Diameter, about 17 inches; height, unknown; distance from crop of coal, No. 12, in bank, about 99 feet. Contained remains of *Dendrerpeton Acadianum*, *Hylonomus Wymani* or *Lyelli*, and *Pupa vetusta*.

No. 2. Extracted in 1860. Diameter, 1 foot 8 inches; height, 9 feet; distance from No. 1 about 73 feet; surface obscurely ribbed. Contained *Dendrerpeton Acadianum* and *Pupa vetusta*.

No. 3. Extracted in 1879. Diameter, 1 foot 8 inches; height, 2 feet 6 inches; distance about 20 feet from No. 2. Filled with argillaceous matter, and a little mineral charcoal at base. No animal remains observed.

No. 4. Extracted in 1879. A crushed coaly bark covering a mound of mineral charcoal. No animal remains observed. Distant about 20 feet from No. 3.

No. 5. Extracted in 1877. Diameter, 2 feet 6 inches; height remaining, about 4 feet. Surface with flat ribs. Distance from No. 4, 93 feet, and from the face of the cliff in 1879, 58 feet. This tree was extracted under unfavourable circumstances, owing to the interference of débris from the foot of the cliff, and high tides. It afforded portions of a skeleton of *Dendrerpeton Acadianum*, bones of *Hylonomus Lyelli*, *Pupæ*, and fragments of Millipedes. On one surface was a trail running round the circumference, indicating the efforts of an imprisoned Batrachian to escape. A few much decayed bones on this surface appeared to belong to *Dendrerpeton*.

No. 6. Extracted in 1879. Diameter, 2 feet; height remaining, less than a foot. This was probably the base of a tree, of which the upper part had been removed by the sea, leaving only the lowest layer. Afforded some much decayed bones, apparently of *Hylerpeton*. This tree was distant only a few feet from No. 5.

No. 7. Extracted in 1879. Distant from No. 5, 66 feet. Merely the base remaining. Consists of mineral charcoal without fossils.

No. 8. Extracted in 1879. Diameter, 1 foot 6 inches; remaining height, 1 foot. Consisted entirely of black carbonaceous matter with bones. It afforded several skeletons of *Hylonomus* and *Dendrerpeton*, the first observed remains of *Hylerpeton*, and many *Pupæ* and Millipedes. The skeletons in this tree were less disturbed than in any other I have examined; but owing to the long exposure of the base of the tree, after removal of the upper part, the material was very soft and much mud-stained, and the bones in a crumbling condition. This tree and Nos. 7, 9, 10 and 11 constitute a group near to each other, and of which owing to recession of the cliff only the bases remained.

No. 9. Extracted in 1878. Short coaly base of a tree with a few bones of *Hylerpeton*, much decayed.

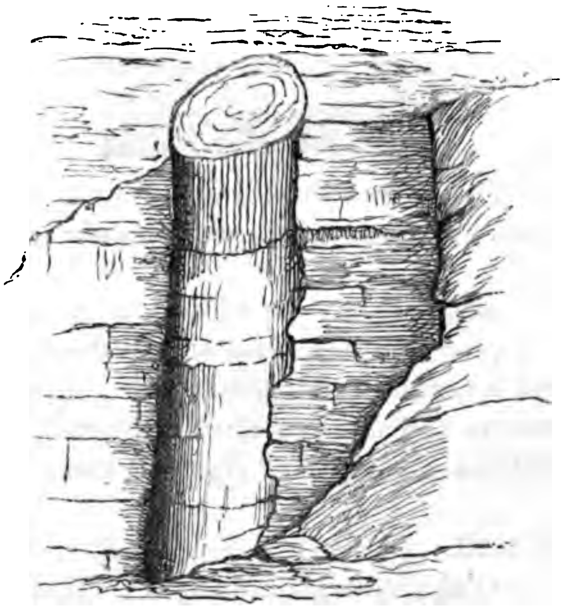
No. 10. Extracted in 1879. Diameter, 1 foot 6 inches. A short coaly base with much mineral charcoal. Detached bones of *Hylerpeton;* teeth and scales of *Sparodus;* remains of larvæ of insects (?).

No. 11. Extracted in 1879. Diameter, 15 inches, much crushed; height, about 2 feet; filled with argillaceous matter; unproductive. Distant 8 feet from No. 7, and exactly opposite No. 10.

No. 12. Extracted in 1878. Diameter, 2 feet 4 inches; height remaining, 3 feet 6 inches; distant 20 feet from No. 11. Surface marked with broad ribs and rows of leaf-scars. In upper part gray sandstone, in bottom 4 inches of dark-coloured material which afforded bones of *Dendrerpeton* and *Hylonomus,* also Millipedes and *Pupæ.* Some of the cuticle of *Dendrerpeton* is preserved. This tree had been inclined to the south-west, and was consequently somewhat flattened, so that its diameter in one direction was double that in the other, and the lower side had been crushed or bent inwards. In the base there were two inches of mineral charcoal, and over this about 2 inches of hard and dark-coloured laminated matter, which afforded many *Trigonocarpa* and fragments of plants. It also contained the whole of the animal remains above referred to.

No. 13. Extracted in 1878 (fig. 4). Diameter, 1 foot 6 inches; height, 9 feet;

Fig. 4.



Tree No. 13, as it appeared when partially exposed in the reef.

distant 100 feet from No. 12; surface roughly ribbed. From the position and great height of this tree, we found it most convenient to undermine it and to extract its basal part, leaving the remainder intact in the reef (Plate 45, fig. 141). The base contained about a foot of dark-coloured matter, with remains of *Hylerpeton, Hylonomus, Smilerpeton, Pupæ,* and Millipedes. This tree must have been a pit at least 8 feet in depth when the reptiles fell into it.

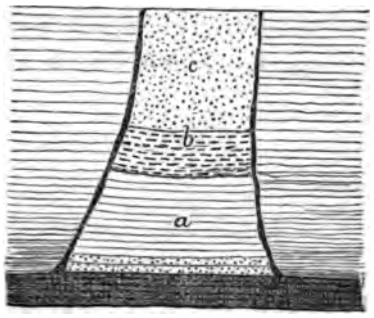
No. 14. Extracted in 1879. Diameter, 2 feet; height, 3 feet 6 inches; distant 13 feet from No. 13. Though there were in the base of this tree 3 inches of carbonaceous matter and mineral charcoal, no animal remains were found in it.

No. 15. Extracted in 1878. Diameter, 1 foot 6 inches; height remaining, 3 feet; distant 111 feet from No. 14. Consisted throughout of uniform gray sandstone without animal remains.

No. 16. Extracted in 1878. Diameter, 15 inches; height remaining, 4 feet 6 inches; distant 94 feet from No. 15. At the base had 6 inches of productive matter containing remains of seven individuals, of genera, *Smilerpeton, Dendrerpeton, Hylerpeton, Hylonomus,* and many *Pupæ* and Millipedes.

No. 17. Extracted in 1878. Diameter, 2 feet; height remaining, 2 feet. Close to No. 16. This tree had been filled in a somewhat peculiar manner (fig. 5). In the bottom were 2 feet of gray shale without animal remains, as if at the time when shale was in process of deposition around the trunk some fissure or opening had admitted a quantity of this material. Subsequently, the usual thin deposit of productive carbonaceous matter occurred, and then the sandstone filling. This tree contained bones of *Hylerpeton* and *Hylonomus,* with remains of Millipedes.

Fig. 5.



Tree No. 17, section showing material filling the interior.

(*a*) Gray shale filling lower part. (*b*) Carbonaceous matter containing animal remains. (*c*) Sandstone.

No. 18. Extracted in 1878. Diameter, 2 feet 5 inches; height, 7 feet; distant 44 feet from No. 17. Obscure ribs and leaf-scars. Filled with argillaceous matter, and destitute of animal remains. In the base was a layer of mineral charcoal, holding a sandstone cast of the medullary cylinder, 2 inches in diameter. The trunk spreads at base to 3 feet, and extends its *Stigmaria* roots on the roof of the coal. A prostrate and flattened trunk of *Lepidofloyos* was found immediately below it.

No. 19. Extracted in 1876. Diameter, 1 foot 6 inches; basal part, about a foot thick and partially compressed, alone seen; the upper part having been removed by the sea. The productive matter in its base afforded 13 skeletons of *Dendrerpeton, Hylonomus, Hylerpeton,* and *Fritschia,* besides Millipedes and *Pupæ.*

No. 20. Extracted in 1878. Diameter, 1 foot; distance from No. 18, 126 feet; Filled with shaly matter not productive.

No. 21. Extracted in 1878. Diameter, 1 foot 8 inches; height remaining, 4 feet; distant from No. 20 26 feet. About 5 inches of carbonaceous matter in the base, containing remains of *Hylonomus, Hylerpeton* and *Smilerpeton.* This tree is much crushed by lateral pressure at the base, and the bones obtained from it were in a fragmentary condition.

No. 22. Extracted in 1878. Diameter, 2 feet 3 inches; height remaining, 3 feet. Filled with sandstone to base, without fossils.

No. 23. Extracted in 1878. Diameter, 1 foot 6 inches; base only preserved. Close to No. 22. Some carbonaceous matter in the base believed to be productive, but accidentally mixed with the contents of No. 16.

No. 24. Extracted in 1878. Diameter, 15 inches; height remaining, 3 feet; distant 8 feet from No. 23. A little carbonaceous matter in hard laminæ of sandstone in base. Contained a few scattered scales and bones of *Dendrerpeton,* and some jointed objects supposed to be insect larvæ.

No. 25. Not extracted. About 18 inches in diameter, and several feet in length. Distant 136 feet from No. 24. This trunk is prostrate and partially flattened. It is visible only at very low tides, and I had no opportunity to extract it. Being prostrate, and so far distant from No. 24, it may possibly mark the limit of the erect trees in this direction.

It will be observed that of the erect trees catalogued above, ten were altogether unproductive. Of these, some had evidently been broken down and filled while the area was still submerged. Others, on the contrary, had remained inaccessible to animals till suddenly filled by the final irruption of sand. The intermediate conditions were those favourable to the entombment of land animals. Fifteen trees were more or less productive: a remarkable proportion when the combination of circumstances necessary to this result is considered. The greater part of the remains have however been obtained from nine or ten of the trees catalogued; but some of the others were only bases of trunks from which more productive portions may have been removed by the sea. The more productive trees are intermixed with the others, and there seems no local restriction in this matter, unless the scarcity of the erect trees toward the end of the reef may indicate a margin of the forest in that direction.

The trees range in diameter from a foot to nearly 3 feet, the ordinary diameter being about 18 inches. The measurements of diameter were made at a short distance above the base, where the trunks became approximately cylindrical. The extreme diameter of the largest tree at the base was 3 feet. The measurements of height refer to the actual condition of the trees. The maximum height of trees which had lost nothing by denudation is 9 feet; but several of those now of less height may, previously to the modern denudation of the beds, have been as tall as any of the others.

So far as can be ascertained, all the trees affording amphibian remains as well as the others standing erect with them, belong to the genus *Sigillaria* in its wider sense, or to the family *Sigillariæ*. Those retaining markings have the characters of the broad-ribbed species of *Sigillaria* proper, like *S. reniformis* and *S. Brownii*, which seem to have been among the largest trees of the coal-period, and those best fitted, by the density and indestructibility of their outer bark, for retaining an erect position. Trunks and branches referrible to the subdivisions *Rhytidolepis* and *Favularia*,* and also trunks of *Lepidofloyos*, occur in a flattened condition in the black shale; and these as well as *Cordaites* must have constituted portions of the same forest, though none of them endured long enough to become repositories of amphibian remains. All the erect trees, so far as observed, had roots of the Stigmaria type.

It is deserving of remark in this connexion, that the circumstances of the growth and entombment of this forest entirely contradict those theories as to *Sigillaria* and *Stigmaria* which suppose that these plants grew in water or on submerged areas. In the present instance the surface on which the trees grew, now represented by the 6-inch coal, must have been underlain by several feet of peaty matter with prostrate trunks of trees, and no remains of aquatic animals. The forest contained not only *Sigillariæ* of different species but *Lepidofloyos, Cordaites,* and *Ferns.* When the area was submerged this vegetation was killed, while plants like *Sphenophyllum*, which could grow in water, apparently took its place, and were in turn replaced by *Sigillariæ* and *Calamites* when the area again became land. Further, when the reptiles fell into the erect trees the surface on which they walked, though subject to inundations, must ordinarily have been several feet above the drainage level, otherwise the hollow trees would have remained full of water.

No remains of any aquatic animals were found in the trees holding amphibian remains, unless certain minute annulated bodies, doubtfully compared by Dr. SCUDDER to leeches, can be regarded as of this character. The vegetable matters present consisted, first of the decayed material of the interior of the trunk itself, and, secondly, of fragments which had fallen into or been drifted into the trees after they became hollow. Most of these were pieces of bark and wood, but leaves of *Cordaites* were not uncommon, and there were numerous specimens of the little nutlets which I have elsewhere named *Trigonocarpum sigillariæ*,* and which must be fruits either of *Sigillaria* or of *Cordaites*. Pinnules of *Alethopteris lonchitica* were also occasionally met with, and fragments of *Calamites*. In one tree there occurred so large a mass of aerial roots of the type of *Psaronius*, as to render it probable that these roots may have grown in the interior of the decaying tree.

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* See 'Acadian Geology,' p. 432–3.

With reference to the mode of occurrence of the animal remains, it is to be observed that from the manner of their entombment all the bones of each specimen must be present in the matrix. On the other hand, having dropped asunder on decay of the soft parts, and having been liable to disturbance by water dripping into the trunks, they are often much displaced. In some instances, also, they have evidently fallen into the crevices of loose vegetable matter, afterwards consolidated. Besides this, the original inequality of the surfaces of deposit has been increased by the effect of pressure in depressing the centre, so as to give a basin-form to the layers and to produce a certain amount of displacement at the sides. The effect of this is increased by the circumstance that some of the amphibia seem to have crept close to the sides of the cavity in which they were imprisoned, and to have died in that position, so that their bones lie close to the bark, and in the portion of the deposit most bent by pressure. It is thus difficult to collect the whole of the bones belonging to a skeleton, and impossible to expose some of them without destroying others. I have been obliged, in consequence, to endeavour to secure and expose the more important portions of the skeleton in as perfect condition as possible, and in many instances it has proved possible to recover only a portion of the bones. The state of preservation of the remains is very different in different layers. In some cases considerable portions of the cuticle remain in a carbonised state ; this has occurred when carcases have been quickly covered with moist débris, or permanently water-soaked, so that the soft parts could become carbonised instead of decaying away. These portions of cuticle are unfortunately not laid out flat, so as to show the form of the animal, but much folded and crumpled ; and it would seem that where the cuticle has been thus preserved the bones have been specially liable to decay. The bones are often much softer than the matrix and of a brownish colour, but in some instances they are quite white and in excellent preservation. On a few of the surfaces they have been partially removed by the percolation of water. Their cavities are usually occupied with calcite, but sometimes with pyrite. When sliced they generally show their microscopic structure in a good state of preservation, though the presence of hard grains of pyrite in the softer bone often makes it difficult to prepare satisfactory slices. In a few cases bones of small amphibia and shells of *Pupa vetusta* have been found included in the mineral charcoal which forms the lowest layers in the trees, as if these animals had crept into the interstices of the fragments of decaying wood and bark.

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* 'Acadian Geology,' p. 477.

3. DESCRIPTION OF THE ANIMAL REMAINS.

Batrachians.

As already stated, the circumstances attending the entombment of vertebrate animals in erect trees were of such a nature as to exclude the more aquatic forms. The species found in these repositories, therefore, constitute a special and peculiar assemblage, representing the more terrestrial types of the batrachian or amphibian life of the period.

The whole of the amphibian animals found in the twenty-four trees extracted may be included in twelve species, of which two are of doubtful character, owing to the imperfection of their remains. Of the remaining ten, eight belong to the family which I have separated* under the name *Microsauria,* and two are referable to the *Labyrinthodontia,* though perhaps to a special subdivision of that group. The whole may be included in the order *Stegocephala* of COPE.

The *Microsauria* are characterised by somewhat narrow crania, smooth cranial bones, simple or non-plicated teeth, well-developed limb bones and ribs, elongated biconcave vertebræ, bony plates and scales on the thorax and abdomen, and horny scales on the back and sides.

Though probably to be included in the group *Stegocephala,* as defined, and predominantly Batrachian in their affinities, they presented in form, clothing, and probably in habits of life, a close approximation to the lacertians. Their predominating terrestrial habits are evidenced by the circumstance that in nearly all the species the length of the femur coincides very nearly with that of the mandible, while their lacertian form is indicated by the fact that in death their crania and skeletons lie on one side and not on the back or front.

The following genera of this group are represented in the erect trees of the South Joggins:—

1. *Hylonomus.*—Form lizard-like, with the posterior limbs somewhat large in proportion to the anterior. Size, small. Mandibular and maxillary teeth numerous, small, conical, pointed. Palatal teeth minute. Abdominal scales oval. Four species, namely, *H. Lyelli, H. Wymani, H. multidens, H. latidens.*

2. *Smilerpeton.*—Form somewhat elongated and limbs short. Mandibular and maxillary teeth wedge-shaped, with cutting edges. Palatal teeth numerous, some of them large. Abdominal scales oval. One species, *S. aciedentatum.*

3. *Hylerpeton.*—Body stout with strong limbs. Mandibular and maxillary teeth strong, not numerous, grooved at apex. Palatal teeth numerous and some of them large. Thoracic plate broad. Abdominal scales pointed or oat-shaped, Two species, *H. Dawsoni* and *H. longidentatus.*

4. *Fritschia.*—Body lizard-like. Limbs large and well ossified. Mandibular

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* 'Air-breathers of the Coal Period.'

and maxillary teeth conical, grooved at apex. Abdominal scales slender and rod-like. One species, *Fritschia curtidentata.*

5. *Amblyodon.*—A genus characterised by stout cylindrical teeth, blunt at the apices; but otherwise imperfectly known.

The *Labyrinthodontia* are represented in these trees by the genus *Dendrerpeton* alone.

Dendrerpeton may be characterised as having a lizard-like form of body, with the anterior and posterior extremities nearly equal; the skull somewhat elongate with small orbits, and the nostrils placed at the front. The cranial bones sculptured. The teeth plicated at the base, more especially on their inner sides. A series of large teeth on the palate. The body was covered above with imbricated horny scales and had lappets or pendants at the sides. The abdomen was protected by thin bony scales semi-elliptical or oat-shaped in form, and arranged in a chevron pattern. There was probably also a thoracic plate. Two species, *D. Acadianum* and *D. Oweni.*

Of the above species six were more or less perfectly known previous to the recent explorations, but additional material has been obtained illustrating some of their characters. In the following notes these new facts will be stated, with more full descriptions of the new species.

1. *Hylonomus Lyelli*, Dawson (Plate 39, figs. 1 to 14, and fig. 27; also Plate 45, fig. 140).

[Journal of Geological Society of London, vol. xvi., 1859, p. 268. 'Air-breathers of the Coal Period,' 1868, p. 40. 'Acadian Geology,' 3rd edition, 1880, p. 370.]

This species is by much the most abundant in the erect trees examined. It is the type of the genus *Hylonomus* and of the family *Microsauria.* Its characters may now be given somewhat completely, as follows:—

General form lizard-like, with the hind limbs rather larger than the fore-limbs. Length when mature, 5 to 6 inches.

Head somewhat elongate; bones of skull smooth or with microscopic striæ, perfectly united, except at parietal foramen. Occipital condyle double, and apparently bony. Teeth simple, conical, numerous, about forty in each mandible, and nearly equal, except that a few of the anterior ones are rather larger than the others. The teeth are anchylosed to the jaw in a furrow protected by an external bony plate.

Vertebræ with cylindrical bodies, slightly concave at the ends. When partly exfoliated they appear hourglass-shaped, in consequence of the internal cartilage having the form of two cones attached by their apices. Zygapophyses conspicuous above; neural arches united to the bodies of the vertebræ, and with broad neural spines. Dorsal vertebræ with strong lateral processes. Caudal vertebræ apparently simple and cylindrical. Number of vertebræ in neck and trunk about thirty.

Ribs long and curved, with capitulum and tubercle, cartilaginous within.

Anterior limb slender, humerus with distinct keel; radius and ulna separate; toes, four or five.

Posterior limb with well-developed femur, with a trochanter and bifid articulatory surface at distal end; tibia and fibula shorter, separate; toes five, somewhat long and slender.

Pelvis large, principally composed of two large broad triangular ilio-ischiac bones.

Tail probably shorter than the body; thirteen vertebræ seen in one specimen.

Lower surface protected with a thoracic plate and numerous oval bony scales, with delicate lines of growth externally, and concave internally with thickened edges. Upper surface protected with imbricated horny scales. In front two rows of horny tubercles and plates, with epaulettes composed of bristle-like fibres projecting from the skin (Plate 39, fig. 27; Plate 45, fig. 140).

The dimensions of the largest individual found are as follows:—

Length of	head	about	2	centimetres.
„	neck	„	1·3	„
„	trunk	„	7	„
„	posterior limb to heel		3	„
„	mandible		1·8	„
„	rib		1·3	„
„	humerus		1·4	„
„	femur		1·8	„
„	tibia		1·2	„
„	body of vertebra		2·5	millimetres.

Teeth, five in 1 millimetre.

A smaller individual gave the following measurements:—

Length of	mandible	7	millimetres.
„	rib	6	„
„	humerus	5	„
„	femur	7	„

Under the microscope the bone of *Hylonomus Lyelli* shows well-marked cells and canaliculi, the cells being oval in the cranial bones and more elongated in the limb bones. The dentine of the teeth presents very fine tubes radiating from the pulp cavity, and covered with a thin plate of structureless enamel at the outer surface.

In such specimens of this animal as are preserved in sufficient perfection to show its attitude, the head and trunk lie on one side, showing that the body was compressed laterally; a form fitting it to pass among dense vegetation. Coprolitic matter associated with its remains contains fragments of insects.

2. *Hylonomus Wymani*, DAWSON (Plate 39, figs. 15 to 17).

[Journal of Geological Society, *l.c.* 'Air-breathers of the Coal Period,' p. 52.
'Acadian Geology,' p. 378.]

As compared with the last species this was smaller in size, more elongated in form, had the teeth less numerous (about twenty-two in the mandible), and shorter and more obtuse in form. There are six to seven in 1 millimetre.

Length of skull	8	millimetres.
„ mandible	5	„
„ rib	5·5	„
„ femur	6	„
„ humerus	5	„

This species is much more rare than the former; but quantities of minute bones, probably belonging to it, occur in the coprolitic matter.

Other characters of this species will be found in the memoir and work above cited.

3. *Hylonomus multidens*, s.n. (Plate 39, figs. 23 to 26).

This animal is known only by portions of bones of the head and a few other fragments. The scattered bones of the extremities are inseparable from those of *H. Lyelli* occurring with it. As compared with that species, the bones of this are smoother and more delicate. The teeth are more numerous and slender. The crushed distal end of a femur or humerus found near the skull indicates that the limbs were well developed.

Length of mandible		11	millimetres.
„ skull	about	15	„
„ femur	„	9	„

Teeth, five to six in 1 millimetre.

4. *Hylonomus latidens*, s.n. (Plate 39, figs. 18 to 22).

Of this species fragments of three specimens were found in three distinct trees. It seems to have been somewhat stouter built than *H. Lyelli*, but with the limbs shorter in proportion. Its generic affinities are perhaps somewhat doubtful, as it presents in some respects characters intermediate between *Hylonomus* and *Hylerpeton*.

Mandibular and maxillary teeth broadly conical, about twenty in each mandible—three in 1 millimetre; anterior mandibular teeth somewhat larger than the others, and bent or hooked. Vomer or palate with minute teeth. Thoracic plate large. Scales of abdomen oval, but somewhat narrow, and tending to be oat-shaped,

Length of mandible (imperfect)	9	millimetres.
,, humerus	7	,,
,, vertebra	2	,,
,, tibia (?)	5	,,
,, thoracic plate	1·5	centimetre.
,, six caudal vertebræ	8	millimetres,

5. *Smilerpeton aciedentatum*, DAWSON (Plate 40, figs. 28 to 45).

[*Hylonomus aciedentatus*, DN., Journal of Geological Society, *l.c.* 'Air-breathers of the Coal Period,' p. 49. 'Acadian Geology,' p. 376.]

This animal was originally referred by me to the genus *Hylonomus;* but additional specimens and more careful comparison oblige me to place it in a new genus.

Its most remarkable distinction is the form of the mandibular and maxillary teeth, which are of a peculiar wedge-shape, being broad and oval at base and narrowed to a longitudinal edge at top. Thus, when viewed from the side they appear narrow and blunt, but when the jaw is broken across, and they are viewed from the rear or front, they appear broad and sharp-edged. The effect of this arrangement is that the jaw is armed with a closely-placed series of chisels or wedges, giving an almost continuous edge. At the end of the mandible some of the teeth are longer and more conical.

Another important character is that the palatal and vomerine bones seem to have bristled with teeth, mostly of very small size; but there are also some larger palatal teeth, of which some are sharply pointed and others blunt with furrowed points.

The vertebræ are of the same type with those of *Hylonomus;* but some which appear to be caudal have a pointed spine above, indicating perhaps a flattened tail. The ribs are short and stout.

Only a few fragments of the limbs have been found; they would seem to have been less developed than in *Hylonomus,* and perhaps this animal was less gifted with powers of walking on land than any of the other *Microsauria.* This would accord with the fact that its bones are much scattered, and occur chiefly in sandy layers, as if introduced in times of rain or of partial inundation.

Some fragments indicate thoracic plates, and there are numerous abdominal scales of oval form. Above, the body appears to have been clothed with small tubercles and horny scales, and to have had cuticular pendants like those of *Dendrerpeton.*

In a large specimen the mandible measures 2·2 centimetres, the femur 2 centimetres, and a rib the same. The following are measurements from a smaller specimen:—

Length of	mandible	1·5 centimetre.
„	femur	1·5 „
„	humerus (?).	1·3 „
„	vertebra	3·5 millimetres.
„	rib	1 centimetre.

There are five teeth in 2 millimetres.

Under the microscope the bone shows elongated cells and a somewhat fibrous structure, and there is much cartilage associated with it. The teeth are composed of radiating tubes of ivory of remarkable coarseness, and with distinct lateral canaliculi; but the coarse tubes do not reach the surface to which only their terminal canaliculi extend, and there appears to be a thin superficial layer of dense and brilliant enamel having externally a microscopic vertical striation.

The above characters are taken from two specimens approaching each other in dimensions. A third specimen was of much smaller size, and had longer and more slender palatal teeth. It was originally set apart as a second species, under the name *S. acutidentatum;* but as the bones secured are few, and it may possibly be a young individual of the present species, I have thought it best to leave it undescribed in the meantime.

6. *Hylerpeton Dawsoni*, OWEN (Plate 41, figs. 62 to 85).

[OWEN, Journal of Geological Society, vol. xviii., p. 241. DAWSON, 'Air-breathers of the Coal Period,' p. 55. 'Acadian Geology,' p. 380.]

The specimen on which OWEN founded this genus and species was obtained by me in 1861. Unfortunately it consisted of only a few fragments of bone, the principal of which was a mandible, with some of the teeth remaining, and a fragment of a maxilla. The trees subsequently examined have afforded bones belonging to four additional individuals, and enable the following description to be given.

Bones of skull slightly striated, but not sculptured as in *Dendrerpeton.* Lower jaw with distinct ascending ramus or coronoid process, a feature not known in any other of these amphibia, but observed by COPE in his genus *Brachydectes,* which may be allied to *Hylerpeton,* but is known only by the jaws and teeth. Teeth, twelve in each ramus of the mandible, bluntly conical, slightly striated at the apex. Pulp-cavities large and longitudinally striated at the sides, though the teeth are not folded. Maxilla furnished with similar teeth, one of which near the front is larger than the others. Palatal teeth numerous, small and conical, with a few large teeth at the sides.

Vertebræ short, cylindrical, well ossified, with well developed zygapophyses and neural spines; ribs strong and much curved, with well developed division of the proximal ends; pelvis imperfect, but apparently large, with broad ilium.

Humerus half the length of the mandible; radius half as long as humerus; femur very large and stout, nearly as long as the mandible; leg bones and phalanges correspondingly stout.

The thoracic plate is indicated only by some fragments. The abdominal scales are narrow and pointed (oat-shaped), smooth externally and with a ridge at one side within.

The following are dimensions of the largest specimen :—

Length of	mandible	4·4 centimetres.
„	largest tooth	5 millimetres.
„	femur	3·5 centimetres.
„	tibia	2 „
„	humerus	2 „
„	radius	1·5 „
„	vertebra	6 millimetres.
„	rib	3 centimetres or more.
„	scales	5 to 7 millimetres.

Under the microscope the bone presents a coarser structure than that of *Hylonomus*, the bone-cells being large and of elongated form. The dentine of the teeth has coarse tubes with canaliculi, the appearance being very similar to that in *Smilerpeton*, though the tubes are scarcely so large.

It seems evident that *Hylerpeton Dawsoni* was an animal somewhat stout and broad in form, with a large and pointed head and massive limbs. It seems to have fed in part at least on Millipedes, as remains of these are found in coprolite associated with its bones. For some reason also, the specimens of this species seem to have been among the earliest introduced into the erect trees. Perhaps they sought their myriapodous food near these hollow trunks, or were in the habit of breaking up decayed wood in search of Myriapods.

7. *Hylerpeton longidentatum*, s.n. (Plate 42, figs. 86 to 109).

[Preliminary Notice, American Journal of Science, December, 1876.]

Two specimens of the species occur in the collections, neither of them perfect. Their characters warrant us in placing the animal in the genus *Hylerpeton*, but it is very distinct from the previous species, more especially in the length and slenderness of its teeth, and in the breadth of its thoracic plate.

Head much elongated, with the bones minutely pitted, and with delicate microscopic striæ, but not sculptured. Mandibular and maxillary teeth long and acute, pointing backwards, with the apex of their inner sides finely striated; twenty or more in each ramus of the lower jaw; palatal bones with several long slender teeth and many minute teeth. The mandibles found are not complete, but there are indications that there was an ascending process as in *H. Dawsoni*, but less developed. The narrowness of the dentary bone in Plate 42, fig. 86, is caused in part by the lower posterior edge being bent inward at (*a*), and by the posterior end being broken off above.

Vertebræ short and stout, and apparently well ossified. Ribs long, with double head and much curved.

Humerus longer than the femur, which is short and stout, if the bone taken for it is rightly determined.

Abdominal scales narrow, oat-shaped; thoracic plate large, broadly oval.

Length of mandible	4	centimetres.
„ vertebra	5	millimetres.
„ rib	2·5	centimetres.
„ humerus	1·5	„
„ femur (?)	1·2	„
„ tibia	8	millimetres.
„ mandibular teeth	3	„

Six to seven teeth in 1 centimetre.

Under the microscope, the bone shows somewhat elongated cells similar to those of *H. Dawsoni*, but smaller. The teeth in cross-section near the base, present a somewhat complex structure, though the dentine is not folded. Next the pulp cavity there are straight radiating cells or tubes, coarse and with lateral canaliculi. This structure, which resembles that of *H. Dawsoni*, extends about half way to the surface, when the large tubes cease and the canaliculi form a dense network. On the outside of this is a thick layer of enamel with fine straight canaliculi. A section nearer the apex of the tooth would probably show a less thickness of the second or intermediate layer.

8. *Fritschia curtidentata*, s.n. (Plate 43, figs. 110 to 128).

[*Hylerpeton curtidentatum*, Preliminary Notice, American Journal of Science, *l.c.*]

This species is represented in the collections by two specimens—one well preserved and probably adult, the other smaller and less perfect. In 1876, when I possessed but one specimen, I referred it to *Hylerpeton;* but it differs from that genus in the absence of palatal teeth and in the abdominal armour, which consists of long slender rods instead of scales. This kind of protective structure, as is well known, occurs in HUXLEY'S genus *Ophiderpeton*—a creature otherwise very different—and in COPE'S *Sauropleura*, which in the parts known has much resemblance to the present genus.*

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* Certain species of *Urocordylus* described by HUXLEY and FRITSCH would seem to present forms of scales intermediate between the oat-shaped and rod-like types.

I have dedicated the genus to Professor ANTON FRITSCH, who has so ably illustrated the carboniferous and permian Batrachians of Bohemia.

Bones of the head very smooth, having only a few microscopic punctures. Teeth conical, somewhat obtuse, striated at the inner side of the apices; there are about thirty in each ramus of the mandible, and about twenty-seven in the maxillary bone. As in the other *Microsauria,* they are implanted in a furrow.

Vertebræ short and well ossified, three in a centimetre. Ribs strong, curved, about 1 centimetre in length.

Limbs robust, the bones better ossified than in any of the other species of *Microsauria.* Humerus round, and with a distinct keel on the shaft. Femur with well-formed articulating surfaces. Toes of hind foot probably five, central ones long and slender.

Thoracic plate of moderate size and somewhat rounded. Abdomen protected by needle-like rods, which are very numerous, and were probably arranged *en chevron.*

Length of mandible (not quite perfect) . .	2·1	centimetres.
,, maxilla	2	,,
,, rib.	1	,,
,, humerus	2	,,
,, femur.	2·4	,,
,, radius and tibia . . . . about	1	,,
,, toe of hind foot	7	millimetres.

Eight teeth in 5 millimetres.

Under the microscope the bone shows small rounded cells with numerous canaliculi. The cells in the limb bones are a little longer than the others. The teeth are simple, and consist of dentine traversed by moderately fine and somewhat tortuous tubes; these are much coarser than in *Hylonomus,* but finer than in *Hylerpeton.*

9. *Dendrerpeton Acadianum,* OWEN (Plate 40, figs. 46 to 51; and Plate 44, figs. 129 to 137, except fig. 131).

[OWEN, Journal of Geological Society, vol. ix. DAWSON, 'Air breathers of the Coal Period,' p. 17; 'Acadian Geology,' p. 362.]

Portions of several specimens of this species are in the new material, but do not add much to the knowledge of its characters, which have been fully given by OWEN from the somewhat complete specimens obtained by me in 1859.

Some interesting fragments of cuticle were found in association with one of the skeletons, which show very well the scaly covering of the back and the lappets and edge-scales of the sides. These last probably formed a border or margin to the bony scales covering the abdomen (Plate 40, figs. 46 to 51).

A specimen of the skull was also obtained showing its internal surface, and exhibiting clearly the sutures of the several bones. This I have submitted to Professor Cope, and give a diagrammatic representation of it (Plate 44, fig. 132), showing his interpretation of the several bones, which apparently accord in number and arrangement with those of *Labyrinthodontia*, but differ from those of the *Microsauria*, in so far as the crushed condition of these latter enable an opinion to be formed.

Under the microscope, the bone of *Dendrerpeton* presents large and somewhat rounded or oval cells with numerous canaliculi filling the intermediate spaces. The teeth have the dentine folded at the base, but sometimes only on the inner side. The tubes of the dentine are simple and fine, in this resembling those of *Baphetes* and other Labyrinthodonts.

The abdomen of *Dendrerpeton* was protected by numerous broadly sub-oval scales, arranged *en chevron*, but I have not been able to detect with certainty any thoracic plate, though I have one specimen which I think may be a part of such a plate.

Other characters of this species will be found in the memoir and works above cited.

10. *Dendrerpeton Oweni*, Dawson (Plate 44, figs. 131, 138, 139).

[Journal of Geological Society, vol. xviii., p. 469. 'Air-breathers of Coal Period,' p. 32. 'Acadian Geology,' p. 368.]

This species is represented by four specimens in the new material, and these serve to establish its distinctness from the last mentioned, in its smaller size, its more delicate cranial sculpture, and its longer and more curved teeth. Its abdominal scales are also narrower and more pointed, approaching in this to the oat-like form of those of *Hylerpeton*. So far as known, the scaly covering of the back of this species was similar to that of *D. Acadianum*.

Other characters of the species will be found in the memoir and works above cited.

11. *Sparodus*, sp. (?) (Plate 40, figs. 52 to 56).

In the coaly matter or mineral charcoal at the base of tree No. 10, appeared a few fragments of an animal which may possibly belong to the above-named genus of Fritsch, though I am by no means certain of this identification or of the real nature of the animal.

The skull is represented by a fragment of a maxillary or intermaxillary bone, with blunt conical teeth. It is smooth or marked merely with microscopic dots. There is also a fragment which may be a palatal bone studded with minute teeth.

A few vertebræ associated with the above bones are long and narrow, with large zygapophyses and long neural spines. Length of body about 3 millimetres.

With these remains are a few bony scales different from those of any other species found in these trees, and more resembling scales of Ganoid Fishes. They are somewhat rectangular in form, enamelled on the surface and beautifully sculptured with waving lines.

In the same trunk were found some teeth and bones referable to *Hylerpeton Dawsoni,* and it is not impossible that the remains above referred to may have belonged to some creature devoured by that animal, and which would not otherwise have obtained admission to the interior of an erect tree. The tree itself had been removed by the sea, all but a little of the base, and this was in a very unsatisfactory state, so that doubt might even exist as to the limit between the deposit in the interior of the tree and that under its base.

12. *Amblyodon,* gen. nov. (Plate 40, figs. 57 to 61).

In tree No. 16 were found a few teeth and bones which do not seem referable to any of the genera above named, though pretty certainly belonging to a member of the group of *Microsauria.*

A fragment of a jaw 1 centimetre in length has ten cylindrical teeth, simple and smooth, with large pulp cavities and rounded regularly at the apices. With these are four vertebræ of the usual type, measuring together 1 centimetre. Fragments of cranial bones also occur and are obscurely pitted. There is also what seems to be the shaft of a limb bone and a few oval scales. A flat, somewhat rhombic bone with a style at one side may possibly be a thoracic plate or possibly a parasphenoid.

The material is too scanty for any satisfactory description of this animal, but I have named it provisionally *Amblyodon problematicum.*

13, *Coprolitic Matter.*

This occurs in several of the trees, not in masses of regular form but in indefinite patches. It is of a gray or buff colour, and usually highly calcareous. It is often filled with comminuted bones not determinable; but evidently of small Batrachians and probably of *Hylonomus.* Fragments of chitinous matter also abound in some of the coprolitic masses. In most cases they seem to belong to Millipedes, but in a few examples insect remains occur. They are not determinable; but in one specimen was a well-preserved fragment of a head apparently of a small neuropterous insect showing one of the compound eyes. Fragments of shells of *Pupa* are found in and near some of the coprolitic masses, and I think it probable that these pulmonates formed a part of the food of some of the Batrachian species.

Some doubt must of course exist as to whether the substances contained in the coprolite represent the ordinary food of the amphibia or only that to which they had access while imprisoned in the erect trees. The facts so far as they go would indicate that the larger amphibian species preyed upon the smaller, and the latter on Insects, Millipedes and *Pupæ.*

14. *Land Snails.*

The additional facts obtained in relation to these I have detailed in a paper communicated to the American Journal of Science.* *Pupa vetusta* proves to be by far the most abundant species. A very few crushed specimens of *Zonites priscus* were detected. The only new species found is a minute *Pupa* of different type from *P. vetusta,* and which I have named *P. Bigsbii.* These three species, with two described by BRADLEY from the coal formation of Illinois (*Pupa Vermilionensis* and *Dawsonella Meeki*) and a third from Ohio, recently described by Professor WHITFIELD (*Anthracopupa Ohioensis*) are so far the only known carboniferous pulmonates. The shell which I have named *Strophites grandaœa,*† from the Devonian of New Brunswick, is probably a still older representative of this group. Details with respect to them will be found in the paper in the American Journal of Science above referred to, and in that of Professor WHITFIELD in the same Journal for February, 1881.

15. *Millipedes, &c.*

Much additional material of this nature was obtained, but mostly in a very fragmental condition. On careful examination, I was able to refer nearly all the specimens to the species of *Xylobius* and *Archiulus,* already distinguished by Dr. SCUDDER in the material from the trees previously excavated. A number of other fragments, which seemed to indicate additional forms and species of arachnids, have been placed in Dr. SCUDDER'S hands, and also some jointed objects which may be remains of larvæ of insects.

CONCLUSION.

It may be interesting to note the whole number of individual animals represented by the amphibian remains found, and the trees in which they were contained, as an indication of their distribution and relative abundance.

Microsauria.

1. *Hylonomus Lyelli*	12 specimens.	Trees No.	5, 8, 12, 13, 16, 19.
2. *H. Wymani*	7 ,,	,,	1, 8, 17, 19, 21.
3. *H. multidens*	1 ,,	,,	17.
4. *H. latidens*	3 ,,	,,	12, 13, 21.
5. *Smilerpeton aciedentatum* .	3 ,,	,,	8, 13, 21.
6. *Hylerpeton Dawsoni* . . .	6 ,,	,,	6, 8, 9, 10, 16, 19, 21.
7. *H. longidentatum*	3 ,,	,,	16, 17, 19.
8. *Fritschia curtidentata* . .	3 ,,	,,	19, 21.
9. *Amblyodon problematicum* .	1 ,,	,,	16.

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* November, 1880.

† Ibid.

Labyrinthodontia.

10. *Dendrerpeton Acadianum* .	9 specimens.	Trees No. 1, 2, 5, 8, 12, 13, 19, 24.
11. *D. Oweni*	4 ,,	,, 8, 16, 19.

Incertæ Sedis.

12. *Sparodus*, sp. (?)	1 specimen.	Tree No. 10.

Total individuals, 53.

The negative result that, under the exceptionally favourable conditions presented by these erect trees, no remains of any animals of higher rank than the *Microsauria* and *Labyrinthodontia* have been found deserves notice here. It seems to show that no small quadrupeds of higher grade inhabited the forests of Nova Scotia at the period in question. This is perhaps confirmed by the remarkably Lacertilian characters assumed by the *Microsauria* of the period, which seem to have occupied the place now taken by the smaller true reptiles. That there were larger Labyrinthodonts than those found in the erect trees we know from the *Baphetes planiceps* of the Pictou coal,* and from the remarkable footprints of *Sauropus Sydnensis*† and *Sauropus unguifer*.‡ It is to be observed also that as some of the amphibian animals found in the erect trees are represented only by single specimens, there may have been still rarer species, which may be discovered should other trees be exposed. Nor must we forget that the fauna of those swamps and low-lying plains of the carboniferous period, to which our knowledge is at present limited, may not fully represent that of the uplands of the period.

With reference to the probability of the discovery of additional remains in the beds to which this paper relates, I may state that new trees will no doubt be exposed from time to time by the gradual wasting of the cliff. Otherwise additional specimens can be procured only by regular mining operations carried on in the 6-inch coal and its roof. These would of course be costly, and the small amount of coal afforded by the 6-inch seam would contribute very little towards defraying the expense.

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* Journal of Geological Society, vols. x. and xi.; 'Air-breathers of the Coal Period;' 'Acad. Geology,' p. 359.

† 'Acadian Geology,' p. 358.

‡ 'Geological Magazine,' vol. ix.

(Received March 8, 1882.)

Note I.—On Horny Scales and other Appendages of Carboniferous Amphibians.

I have discussed these at some length in my 'Air-breathers of the Coal Period,' and as few new facts occurred in the more recent explorations, I have merely adverted to them in the text. It has however been suggested to me that some more detailed reference to them would be desirable.

I have referred to this kind of cuticular covering as being found in connexion with the bones of *Hylonomus Lyelli* and *Dendrerpeton Acadianum* and *D. Oweni.* It may have been present in other species, but of course was likely to be preserved only in rare instances. The examples figured in Plates 40 and 45 may serve to give an idea of the perfect manner in which it has sometimes retained its characters, though unfortunately, from the uneven and irregular surface of the deposits in the interior of erect trees, it is always folded and crushed, so that it does not retain its original form.

It is to be observed that in the species referred to, the thorax and abdomen were protected with bony plates and scales. The horny scales and plates seem to have been confined to the upper parts.

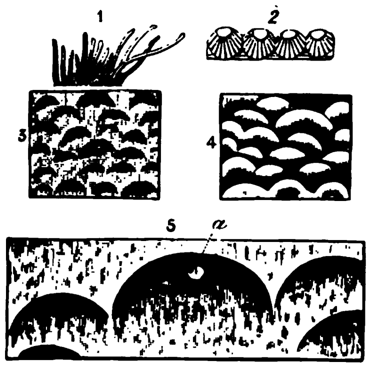
The horny scales and appendages are entirely different in appearance from the bony plates and scales. The latter are usually white or gray in colour, and present under the microscope true bony structures. The former are black, shining, and coaly in appearance, and are inseparable from the cuticle along with which they are preserved. The ordinary horny scales are semicircular, imbricated, or scattered sparsely over the surface of the skin. They are of different sizes, and the larger often show minute round pores, probably mucous or perspiratory pores. These scales are most conspicuous toward the upper and anterior parts of the body. Elsewhere they often degenerate into microscopic tubercles implanted in the skin. Along the sides, and perhaps near the margin of the upper scaly portion of the skin, there are in some specimens larger angular scales, apparently free at the margins, and forming a sort of Vandyke edging. In front the skin projects into long pendant lappets, terminated by similar angular points, and covered with oval scales, not imbricated, and each having a pore in its centre. These appear to have been present both in *Dendrerpeton* and *Hylonomus.* In the latter, as stated in the text, there were on the back and shoulders thick ridge-like and conical tubercles, having their surfaces sculptured with furrows, and *H. Lyelli* has two rows of flat horny bristle-like processes forming frills or epaulettes.

Portions of the scaly cuticle when carefully separated from the stone and mounted in balsam, are sometimes sufficiently thin to be studied as transparent objects. Viewed in this way, under a moderate power, the skin appears of a rich brown colour, and presents an areolar or cellular aspect, the scales appear dark brown, becoming black at their distal edges, and their pores appear as round transparent spots; viewed as an opaque object, the scaly skin appears black and shining, and the edges of the scales seem to be very thin and to bend upward as if free from the skin. The bristle-like appendages of *Hylonomus* are also translucent, when mounted in balsam; but even under a high power show only a faint indication of longitudinal fibrous structure (see woodcut, figs. 1 to 5). The thicker plates, when sliced, show, near the base, a few curved canals, probably vascular, but the upper part appears quite compact, and under a high power merely shows faint indications of tortuous fibres or tubes. Even the thickest show no bony structures whatever, and have throughout a carbonaceous or bituminous appearance.

When burned, the cuticle and horny scales give a strong flame, and emit a bituminous and ammoniacal odour, their chemical characters being those of highly bituminous coal or jet.

These portions of cuticle and horny scales are of rare occurrence, and appear to have owed their preservation to being embedded in wet fragments of bark and other vegetable matter, perhaps possessing a tanning quality; small loose bodies of similar character have been found which may have been horny plates detached from the skin by decay, but it is impossible to say to what they belonged. One of these is a semi-circular plate, about half an inch in diameter, and studded with conical tubercles. It may have been the armature of the snout of one of the *Microsauria*.

For additional descriptions and illustrations, see 'Air-breathers of the Coal Period,' p. 34, and Plates 1, 4 and 5; and 'Acadian Geology,' pp. 369–373.



SCALES AND APPENDAGES OF *Hylonomus* AND *Dendrerpeton*.

Fig. 1. Ornamental horny processes of *Hylonomus Lyelli*, × 10.
,, 2. Horny tubercles of the same, × 10.
,, 3. Cuticle and scales of *Dendrerpeton* (transparent), × 10.
,, 4. The same (opaque), × 10.
,, 5. Portion of the same, × 50, showing cutaneous pore at (*a*).

(Received April 10, 1882.)

I have much pleasure in appending the following note, which I consider a most important addition to my paper, showing that two species of Scorpions have been entombed with the other tenants of the erect trees.

Note II.—On Additional Remains of Articulates obtained by Dr. DAWSON *from Sigillarian Stumps in the Coal-field of Nova Scotia.** *By* Dr. SAMUEL H. SCUDDER.

The fragments sent to me for study, like those formerly received, consist in great part of myriapodal remains, often of single segments, and generally in a more or less crushed and flattened condition. In this respect they are not so well preserved as some of those previously studied, and obtained from erect trees in the same locality. Although all the species formerly separated occur in this collection, very little can be added to the statements then made. Two specimens occur of *Xylobius sigillariæ*, five of *X. similis;* three are somewhat doubtfully referred to *X. fractus*, eight to *X. Dawsoni*, and ten to *Archiulus xylobioides*. A single specimen of *X. Dawsoni*, showing four or five continuous segments, seems to prove that the elevated transverse ridge on each segment in this species was crowned by a single series of minute warts or raised points, not very closely set. A few specimens of different species exhibit the marks which were formerly interpreted as *foramina repugnatoria*, but are now presumed to be the casts of bases of spines,† thus bringing these species into more definite and probable relations to the carboniferous myriapods of Mazon Creek, though they plainly belong to a distinct group. Whatever spines they had must have been very small, slight, and wholly insignificant in comparison with those of the bristling *Archipolypoda* of the Morris beds. Careful search has been made for any other of those special features which distinguish the *Archipolypoda* from recent *Diplopoda*, but in vain, beyond the single but not unimportant point that the ventral plates, in *Archiulus* at least, are very broad and probably almost equally extensive in lateral expansion with the dorsal plates, a feature found nowhere in modern *Diplopoda*.

This is, perhaps, most clearly shown in two new species of *Archiulus*, discovered among these remains, and to which are referred a dozen or more specimens. One of these species is of about the same size with *A. xylobioides*, but has perfectly flat segments showing only a very slight and narrow transverse ridge at the anterior margin, occupying not more than one-fourth of the segment. The other is a smaller species, and has shorter and more simple segments, made slightly concave by the gentle elevation of both front and hind margins, but with no anterior ridge. In neither of these species could any trace of spines be found.

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* For descriptions of the remains previously discovered, see Mem. Bost. Soc. Nat. Hist., vol. ii., pp. 231-239, 561-562 (1873, 1876).

† Mem. Bost. Soc. Nat. Hist., vol. iii., pp. 148, 149 (1882).

Besides the myriopodal remains, there are a dozen fragments that must probably be referred to Scorpions. Of some of them there can be no doubt. The remainder are mere bits of integument showing the surface sculpture, but often with no natural borders whatever. In the character of the surface there is such difference as to indicate more than a single species. For though we should certainly expect to find considerable differences between the various parts of one and the same individual, the diversity here is too great, both in amount and nature, to render it at all probable that the difference may fairly be explained in such a manner. The better fragments exhibit a considerable portion of the stouter part of the body, enough to show its general form at least, and these point also to the probable existence of two species, of nearly the same size, but differing in form and sculpture; the more fusiform-shaped species having a less roughened surface than is found in the more parallel-sided form. The latter agrees tolerably with the carboniferous genus found near Mazon, Ill., called *Mazonia* by MEEK and WORTHEN, and certainly belongs to the same group of Scorpions; but in view of the remarkable addition to our knowledge of the carboniferous Scorpions in the promised publication of the researches of Mr. PEACH of the geological survey of Scotland, further study of these remains will best be postponed. In the meantime, they add another form of strictly land life to those already found in these remarkable repositories of fossils, and perhaps illustrate the utility of the bony and horny armour of the smaller Batrachians of the period, which may have had to contend with these active and venomous Arachnidans.

The occurrence of seven species of Millipedes in a few decayed trees in one locality, in connexion with similar discoveries in other parts of the world, tends to strengthen the probability of the suggestion, already made by Dr. DAWSON, that the animals of this type may have culminated in the Palæozoic period.

(Received September 4, 1882.)

Note III.—On the Footprints of Batrachians observed in the Carboniferous Rocks of Nova Scotia.

Though it is impossible to identify with certainty the footprints of particular species of the Batrachians of the coal-formation, and though most of the animals which have left impressions of this kind are much larger than those found in the erect trees, yet to give completeness to this memoir, it may be well to notice the indications of this kind which have been observed; more especially as they have not previously been brought together into one view.

For convenience, I shall refer the larger footprints, probably those of Labyrinthodonts of considerable dimensions, to the genus *Sauropus,* already established by LEA, and the smaller impressions, due perhaps to smaller Labyrinthodonts or to Microsaurians, and usually showing a longer stride and more slender toes, to the genus *Hylopus.* Besides these, there are the curious trails named *Diplichnites,*[*] and which may have been made by large unknown serpentiform batrachians.

Impressions of both types first appear in the lowest carboniferous or HORTON series, corresponding to the "Tweedian series" of TATE, the "calciferous sandstones" of MCLAREN, and the carboniferous slate and Coomhala grit of JUKES. No bones of Batrachians have as yet been found in these beds, but the footprints indicate the presence at the beginning of the Carboniferous Period, and before the deposition of the lower carboniferous limestones, of both large and small species similar to those of the coal-formation. At the other extremity of the carboniferous system, footprints have been found on the sandstones of the upper coal-formation or permo-carboniferous series.

1. *Sauropus unguifer,* DAWSON.

[Geological Magazine, vol. ix. 'Acadian Geology,' 3rd edition, supplement, p. 62.]

This is the largest footprint yet found in the coal-formation of Nova Scotia. The length of the hind-foot, on one of the slabs having the largest and most distinct impressions,[†] is six inches, and its greatest breadth about five inches; the stride is from 12 to 13 inches, the distance of the two rows of tracks being about seven inches, and the hind-foot covering the impression of the fore-foot, which from other slabs would appear to have been smaller and shorter. The impressions show four ordinary toes, and a fifth outer toe armed with a long claw or spur, which sometimes trailed on the ground, and, when the foot was planted, was plunged into the mud. It was no doubt an aid to the animal in ascending inclined surfaces of mud.

[*] Am. Journal of Science, 1873.

[†] Collected by SANDFORD FLEMING, Esq., C.E., and now in the Museum of the Geological Survey.

Very fine series of footprints of several individuals of different sizes, and showing different gaits, and in one instance the act of wallowing in soft mud, have been found in sandstone at Fillmore's Quarry, River Philip, Nova Scotia, and are now in the Museum of the Geological Survey. In some of the series of tracks there is no mark of the belly or tail. In others the belly has left impressions marked with longitudinal grooves, as if there were abdominal furrows or rows of scales. Descriptions of the principal specimens will be found in the publications above referred to,

2. *Sauropus Sydnensis*, DAWSON.

['Acadian Geology,' 3rd edition, p. 358, and woodcut.]

This animal had shorter and broader feet, with five toes and no indication of the peculiar claw of the previous species. The breadth of the foot was about three inches, the width of the body about six inches, and the stride about eight inches. The original specimen was obtained by R. BROWN, Esq., F.G.S., in the coal formations at Sydney, Cape Breton, and is now in the Museum of the McGILL University,

Sauropus antiquior, DAWSON.

This species is based upon a series of footprints found by Mr. F. M. JONES, of Halifax, at Parrsboro, and now I believe in the Provincial Museum at Halifax, where I have seen the specimen. The horizon is probably that of the HORTON series. The footprint is about three and a-half inches wide, and scarcely half as much in apparent length. It shows four subequal toes, and an outer toe diverging from the others, and showing indications of a short claw. The shortness of the impressions in this species and in *S. Sydnensis* gives them a digitigrade aspect, while those of *S. unguifer* have a plantigrade appearance, varying however in different impressions,

Hylopus Logani, DAWSON.

['Air-breathers of the Coal Period,' p. 5, fig. 1. 'Acadian Geology, 3rd edition, p. 353.]

The original specimen of this footprint was found by Sir W. E. LOGAN at Horton Bluff in 1841, and was the first evidence of the existence of Batrachians in the Carboniferous Period. The specimen obtained by LOGAN is fully described in the works above cited. The impression has been made on a firm surface, and shows merely the marks of four claws or narrow toes. Each impression is about one inch in length; the distance between the right and left footmarks is about three inches, aud the stride about four inches. There is no mark of the belly or tail. In 1881 a somewhat larger series of impressions, which should, however, probably be referred to the same species, was found in the same beds by Mr. PINEO, of Hantsport, and is now in the Museum of the McGill University. It shows indications of a fifth toe; and while the length of the foot is less than two inches, the stride is about eight inches, or more than four times the length of the foot.

5. *Hylopus Hardingi*, Dawson.

['Air-breathers of the Coal Period,' fig. 2, 'Acadian Geology,' 3rd edition, p. 356, fig. 139.]

This specimen, discovered by the late Dr. Harding, of Windsor, in the lower carboniferous shales of Parrsboro, indicates an animal of about the same size with *H. Logani*, and possibly nearly allied to it, but with five distinct and subequal toes which are long and slender. The footprints are about an inch in length, and those of the fore and hind feet are separate and of about equal size and similar form. The most remarkable feature of this series is the great length of the stride, which is nearly five times the length of the foot, and twice as much as the distance between the rows of tracks, apparently indicating that the animal stood as high on its legs as an ordinary Mammal.

6. *Hylopus Caudifer*, Dawson.

['Air-breathers of the Coal Period,' fig. 3.]

This is a slab with a series of footprints less than an inch in length and five-toed. The rows are distant from each other three inches and a-quarter, and the stride is three inches. There are at intervals marks of a tail trailed behind. This impression is in my own collection, and is from the middle coal-formation of the South Joggins, on gray ripple-marked sandstone.

In addition to the above, many obscure impressions have been found, which no doubt indicate several additional species. It is observable that in all the members of the carboniferous series, these footprints have been found most plentifully in the vicinity of those old ridges of land based on the older formations, which as I have shown in my 'Acadian Geology,' traversed the areas of deposition in Nova Scotia in the Carboniferous Period. On these isolated patches of land the Batrachians may have continued to exist throughout the period, undisturbed by the oscillations of elevation and depression which affected the lower levels.

It is evident that the smaller footprints to which I have referred under the generic name *Hylopus*, may have been produced by animals akin to those whose remains are found in the erect trees, though of somewhat larger dimensions; and it is instructive to observe that at the beginning of the Carboniferous Period there must have existed animals of this kind comparable in development of limb with the most highly endowed in this way of the *Microsauria*, and of greater bulk than those whose bones are found in the erect trees.

If we inquire as to the footprints of larger size included in the genus *Sauropus,* we know by the evidence of osseous remains that some large Batrachians inhabited the areas of coal deposition in Nova Scotia. More especially *Baphetes planiceps* (OWEN) must have been an animal of sufficient dimensions to have produced footprints like those of *Sauropus unguifer.* A second species of *Baphetes* is probably indicated by a jaw-bone found at the South Joggins, and which I have named provisionally *B. minor.* The *Eosaurus Acadianus* of MARSH must have been a large animal; but we do not yet know if it was capable of walking on the land.

On the whole, the evidence of footprints serves to indicate that both *Labyrinthodontia* and *Microsauria* existed in Nova Scotia throughout the Carboniferous Period, and that very many of the larger and important species still remain to be discovered. It seems remarkable that, while remains of Fishes are so abundant in the carboniferous shales of Nova Scotia and New Brunswick, nothing is as yet known as to the more aquatic types of Batrachians found so abundantly in the carboniferous strata elsewhere. This may either indicate a local paucity of these creatures, or may be a mere accident of preservation or discovery.

EXPLANATION OF THE PLATES.

(Figures not otherwise designated are of the natural size.)

PLATE 39.

Hylonomus Lyelli (figs. 1 to 14).

Fig. 1*. Skull and portion of skeleton.

Fig. 2*. Skull of the same. × 2.

Fig. 3*. Skeleton. (*a*) Fore limb. (*b*) Vertebræ and ribs, the former with bony coating stripped off and showing hourglass-shaped casts of interior. (*c*) Femur. (*d*) Pelvis. (*e*) Remains of femur and tibia.

Fig. 4. Section of tooth. × 500.

Fig. 5. Cells of bone. × 500.

Fig. 6. Portion of maxilla with teeth.

Fig. 7. Teeth. × 25.

Fig. 8. Mandible and teeth.

Fig. 9. Maxilla. × 5.

Fig. 10. Bones of foot.

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* Figures marked with an asterisk are from drawings by J. H. EMERTON. The remainder are from photographs by INGLIS and camera drawings by the author.

Fig. 11. Rib.

Fig. 12. Skull, crushed laterally. (*o*) Orbit. (*m*) Mandible. (*n*) Nasal bones.

Fig. 13. Skeleton, showing portions of limbs, vertebræ and ribs.

Fig. 14. Femur and thoracic plate.

Hylonomus Wymani (figs. 15 to 17).

Fig. 15. Mandible. × 8.

Fig. 16. Teeth. × 25.

Fig. 17. Vertebra. × 5.

Hylonomus latidens (figs. 18 to 22).

Fig. 18. Teeth. × 25.

Fig. 19. Section of jaw and teeth. × 5.

Fig. 20. Vertebra. × 2.

Fig. 21. Fragment of humerus. × 2.

Fig. 22. Mandible with teeth. × 5.

Hylonomus multidens (figs. 23 to 26).

Fig. 23. Mandible, maxilla, and cranial bones. × 3.

Fig. 24. Outline of another skull. × 2.

Fig. 25. Teeth. × 25.

Fig. 26. End of femur. × 2.

Hylonomus Lyelli (fig. 27).

Fig. 27. Diagram showing probable arrangement of ornamental horny scales (enlarged). (*a*) Horny scales enlarged. (*b*) Bony scales, natural size and magnified.

PLATE 40.

Smilerpeton aciedentatum (figs. 28 to 45).

Fig. 28. Section of tooth. × 500.

Fig. 29. Cells of bone. × 500.

Fig. 30. Teeth, lateral aspect. × 25.

Fig. 31. Teeth, longitudinal aspect. × 25.

Fig. 32. Section of tooth. × 50.

Fig. 33. Shaft of femur? × 2.

Fig. 34. Intermaxillary and teeth. × 25.

Fig. 35. Sections of teeth. × 25.

Figs. 36, 37. Palatal teeth. × 25.

Fig. 38. Mandible. × 2.

Fig. 39. Large palatal teeth. × 25.

Fig. 40. Fragment of femur. × 2.

Fig. 41. Rib. × 2.

Fig. 42. Palate. × 2.
Fig. 43. Caudal vertebra.
Fig. 44. Long palatal tooth. × 25.
Fig. 45. Bony scale. Natural size and magnified.

Dendrerpeton Acadianum (figs. 46 to 51).

Fig. 46*. Portion of cuticle, wrinkled and scaly. Fig. 46A. Lateral points. Fig. 47B. Lappets and pendants.
Fig. 47*. Ends of smaller pendants. Magnified.
Fig. 48*. Pendant. × 2.
Fig. 49*. Two points and scaly surface. × 4.
Fig. 50. Scaly surface. × 5.
Fig. 51. Pendant. × 2.

Sparodus, sp. (figs. 52 to 56).

Fig. 52. Tooth. × 25.
Fig. 53. Four of the smaller teeth. × 25.
Fig. 54. Three bony scales. × 5.
Fig. 55. Fragment of limb bone. × 2.
Fig. 56. Vertebra. × 2.

Amblyodon, sp. (figs. 57 to 61).

Fig. 57. Tooth. × 25.
Fig. 58. Section of tooth. × 5. Fig. 58A. Same. × 25.
Fig. 59. Fragment of thoracic plate.
Fig. 60. Shaft of limb bone.
Fig. 61. Rib.

PLATE 41.

Hylerpeton Dawsoni (figs. 62 to 85).

Fig. 62*. Section of palate with teeth. Fig. 62A. Same. × 2.
Fig. 63*. Section of large tusk. Fig. 63A. Same. × 2.
Fig. 64*. Mandible and teeth. Fig. 64A. Same. × 2.
Fig. 65*. Vertebra. End view.
Fig. 66*. Fragment of rib?
Fig. 67*. Femur.
Fig. 68*. Humerus and vertebra.
Fig. 69*. Head of humerus.
Fig. 70*. Rib.
Fig. 71. Scales.
Fig. 72. Large scale.
Fig. 73. Bones of foot.
Fig. 74. Tooth. × 25.

Fig. 75. Dentine of tooth. × 500.
Fig. 76. Cell of bone. × 500.
Fig. 77. Scales. × 2.
Fig. 78. Head of rib.
Fig. 79. Rib.
Fig. 80. Tooth, showing ribbed interior and smooth surface.
Fig. 81. Fragment of vertebra. × 2.
Fig. 82. Section of body of vertebra. × 2.
Fig. 83. Section of tooth. × 50.
Fig. 84. Fragment of humerus—ribs and scales.
Fig. 85. Mandible of a small specimen. × 5.

PLATE 42.

Hylerpeton longidentatum (figs. 86 to 109).
Fig. 86*, 87*. Mandible and teeth. 86A. Same. × 2.
Fig. 88*. Fragment of palate with teeth. 88A. Same. × 2.
Fig. 89*. Thoracic armour.
Fig. 90*. Fragments of limb bones (these possibly belong to next species).
Fig. 91*. Limb bone and rib.
Figs. 92*, 93*, 94*. Vertebræ.
Fig. 95*. Fragment of scale.
Fig. 96. Section of tooth. × 500.
Fig. 97. Tooth. × 25.
Figs. 98, 99. Teeth showing grooved points. × 25.
Fig. 100. Cross section of tooth. × 25.
Fig. 101. Scale. × 2.
Fig. 102. Head of rib. × 2.
Figs. 103, 104. Palatal teeth. × 25.
Fig. 105. Limb bones. × 2.
Fig. 106. Cells of bone. × 500.
Fig. 107. Rib.
Fig. 108. Teeth and mandible. × 8.
Fig. 109. Mandible and thoracic plate. × 2.

PLATE 43.

Fritschia curtidentata (figs. 110 to 128).
Fig. 110*. Portion of mandible. Fig. 110A. Same. × 2.
Fig. 111*. Maxilla and teeth. Fig. 111A. Same. × 2.

Fig. 112*. Humerus, ulna, and tarsal bone.
Fig. 113*. Femur, distal end.
Fig. 114*. Humerus?
Fig. 115*. Limb bones.
Fig. 116*. Rib and limb bones.
Fig. 117*. Vertebræ.
Fig. 118*. Portion of palate.
Fig. 119. Tooth, showing grooved point. × 25.
Fig. 120. Tooth. × 25.
Fig. 121. Section of tooth. × 500.
Fig. 122. Cells of bone. × 500.
Fig. 123. Bones of anterior limb.
Fig. 124. Ribs, vertebræ, and bony rods.
Fig. 125. Various bones. (*a*) Maxilla. (*b*) Humerus. (*c*) Bony rods. (*d*) Femur. (*e*) Rib.
Fig. 126. Vertebræ.
Fig. 127. Mandible, curved by pressure.
Fig. 128. Various bones. (*a*) Maxillary. (*b*) Humerus and limb bones.

PLATE 44.

Dendrerpeton Acadianum (figs. 129 to 130 and 132 to 137).

Fig. 129. Interior of skull and mandible.
Fig. 130. Maxilla. × 2.
Fig. 132. Plan of constituent bones of skull represented in fig. 129. (*N*) Nasals. (*F*) Frontals. (*SOr*) Super-orbitals. (*P*) Parietals. (*PP*) Post-parietals. (*Pt*) Pterotics. (*SOc*) Super-occipitals. (*E*) Epiotic.
Fig. 133. Section of tooth. × 250.
Fig. 134. Cells of bone. × 500.
Fig. 135. Tooth. × 25. Showing plaited surface.
Fig. 136. Fragments of super-orbital and parietal, showing sculpture. × 5.
Fig. 137. Five teeth. × 5.

Dendrerpeton Oweni (figs. 131, 138, 139).

Fig. 131. Maxillary, showing sculpture and teeth. × 2.
Fig. 138. Teeth. × 25.
Fig. 139. Foot. × 2.

PLATE 45.

Hylonomus Lyelli.

Fig. 140. Skin and horny plates, tubercles, and scales of this species; from a micro-photograph. × 5. The specimen represents a fragment from the shoulder.

Erect Trees.

Fig 141. Tree No. 13, as standing in the reef; from a photograph by T. C. WESTON, Esq., of the Geological Survey of Canada.

PLATE 46.

View of Coal-mine Point, South Joggins, Nova Scotia; from a photograph by J. C. WESTON, Esq.

PLATE 47.

Plan of Low-tide Beach off Coal-mine Point, and section to illustrate the position of the trees containing remains of land animals, by W. B. DAWSON, M.A., Assoc. Mem. Inst. C.E.

XIII. *Experiments to determine the Value of the British Association Unit of Resistance in Absolute Measure.*

By Lord RAYLEIGH, *F.R.S., Professor of Experimental Physics in the University of Cambridge.*

Received February 15,—Read March 9, 1882.

[PLATE 48.]

THE present paper relates to the same subject as that entitled "On the Determination of the Ohm in Absolute Measure," communicated to the Society by Dr. SCHUSTER and myself, and published in the Proceedings for April 12, 1881—referred to in the sequel as the former paper. The title has been altered to bring it into agreement with the resolutions of the Paris Electrical Congress, who decided that the ohm was to mean in future the absolute unit (10^9 C.G.S.), and not, as has usually been the intention, the unit issued by the Committee of the British Association, called for brevity the B.A. unit. Much that was said in the former paper applies equally to the present experiments, and will not in general be repeated, except for correction or additional emphasis.

The new apparatus (Plate 48) was constructed by Messrs. ELLIOTT on the same general plan as that employed by the original Committee, the principal difference being an enlargement of the linear dimensions in the ratio of about 3 : 2. The frame by which the revolving parts are supported is provided with insulating pieces to prevent the formation of induced electric currents, and more space is allowed than before between the frame and those parts of the ring which most nearly approach it during the revolution. It is supported on three levelling screws, and is clamped by bolts and nuts to the stone table upon which it stands. The ring is firmly fastened by nuts to two gun-metal pieces which penetrate it at the ends of the vertical diameter, and which form the shaft on which it rotates. The lower end of the bottom piece is rounded, and bears upon a plate of agate, on which the weight of the revolving parts is taken. A little above this comes the driving pulley (9 inches in diameter), and above this again the screw and nut by which the divided card is held. The top piece is hollow, forming a tube with an aperture of $1\frac{1}{4}$ inches, and is held by a well-fitting brass collar attached to the upper part of the frame. On this bearing the force is very small, so that the considerable relative velocity of the sliding surfaces has no ill effect. Notwithstanding its great weight, the ring ran very lightly, and the principal resistance to be overcome was that due to setting air in motion.

In the original apparatus the ring is very light, in fact scarcely strong enough to stand the forces to which it is subjected in winding on the wire. In order to avoid this defect, and also on account of its larger size, the new ring was made very massive. Cast solid, with lugs at the ends of what was to be in use the horizontal diameter, it was cut into two equal parts along a horizontal plane. The two parts were then insulated from one another by a layer of ebonite, and firmly joined together again at the lugs by bolts and nuts, after which the grooves, &c., were carefully turned. As it was intended to use two coils of wire in perpendicular planes, two rings were prepared. The smaller ring fitted into the larger, the end pieces passing through holes along the vertical diameters of both. But for a reason that will presently be given, only the larger ring was used in the present experiments.

In the spring of 1881 the larger ring was wound in Messrs. ELLIOTTS' shop under the superintendence of Dr. SCHUSTER and myself, and the necessary measurements were taken. On mounting the apparatus a few days later in the magnetical room of the Cavendish Laboratory, and making preliminary trials, we were annoyed by finding a very perceptible effect upon the suspended magnet even when the wire circuit was open. The currents thus indicated might have been due to a short circuit in the wire, or more probably (considering that the wire was triple covered, and that the winding had been carefully done) had their seat in the ring itself. Experiment showed that the insulation between the two parts of the ring, as well as between the wire and each part, was very good, so that no currents could travel round the entire circumference; but on consideration it appeared not unlikely that currents of sufficient intensity might be generated in those parts of the ring which lie nearest to the ebonite layer. The width of the ring (in the direction of its axis) was 4 inches, and the least thickness—that at the bottom of the grooves—about $\frac{3}{8}$ inch, so that the operative parts may be compared to four vertical plates $\frac{3}{8}$ inch thick, 4 inches broad, and (say) 6 inches high. In these plates currents will be developed during the rotation, whose plane is perpendicular to that of the currents in the wire.

The unwished for currents could doubtless have been much diminished by saw cuts in a vertical plane extending a few inches upwards and downwards from the insulating layer, but it appeared scarcely safe to assume that the ring would retain its shape under such treatment. It would have been wiser to have tried the effect of spinning the ring alone before winding on the wire, but we were off our guard from the fact that the old ring gave no perceptible disturbance.

Theory having shown that these currents, if really formed in the manner supposed, could be satisfactorily allowed for, we decided to proceed with the experiment. At the worst, the differential effect between wire circuit closed and wire circuit open could only be in error by a quantity depending upon the square of the speed, and therefore capable of elimination upon the evidence of the spinnings themselves; while if the view were correct that the disturbing currents were principally in a plane perpendicular to that of the wire, even the correction for induction would not be much affected. A special experiment, in which the ring (with wire circuit open) was oscillated backwards and forwards through a small angle in time with the natural vibrations of the magnet, allowed us to verify the plane of the currents. A marked effect was produced when the plane of the ring was east and west, but nothing could be detected with certainty when it was north and south—the opposite of what would happen with the wire circuit closed. After this, no doubt could remain but that most of the disturbance was due to currents in the ring, and subsequent spinnings after the removal of the wire have proved that no sensible part of it was caused by leakage through the silk insulation. The existence of this disturbance, however, so far modified our original plan as to induce us to omit the second ring as giving rise to too great a complication.

The suspended magnet was made of four pieces of steel attached to the edges of a cube of pith and of such length (about ½ inch) as to be equivalent in their action to an infinitely small magnet at the centre of the cube. Before the pieces were put together the approximate equality of their magnetic moments was ascertained. The resultant moment was between six and seven times as great as that used in our former experiments. In virtue of the greater radius of the coil, this important advantage was obtained without undue increase of the correction for magnetic moment, which amounted to about ·004, only twice as great as before. The effect of mechanical disturbances, such as air currents, was still further reduced by diminishing the size of the mirror, particularly in its horizontal dimension. On both accounts the influence of air currents was probably lessened about 15 times, and, in fact, no marked disturbance was now caused by the proximity of a lamp to the magnet box.* In consequence of these changes, however, it was found necessary to introduce an inertia ring in order to bring the time of vibration up to the amount (about 5½ seconds from rest to rest) necessary for convenient observation. The diameter of the ring was about ¾ inch, and the whole weight of the suspended parts was not too great to be borne easily by a single fibre of silk. A brass wire passing between the spokes of the ring prevented the needle from making a complete revolution.

The enlarged scale of the apparatus allowed us to introduce a great improvement into the arrangement of the case necessary for screening the suspended parts from the mechanical disturbance of the air caused by the revolution of the coil. A brass tube of an inch in diameter was not too large to pass freely through the hollow axis. At its lower extremity (fig. 1) it was provided with an outside screw, to which the magnet box was attached air-tight. By unscrewing the box, whose aperture was large enough to allow the inertia ring to pass, the suspended parts could be exposed to view, and by drawing up the brass tube they could be removed altogether, so as to allow the coil to be dismounted, without breaking the fibre. The upper end of the fibre was attached to a brass rod sliding in a socket at the upper end of the tube, by which the height of the magnet could readily be adjusted. The whole was supported on three screws

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* See pp. 115–132 of the former paper.

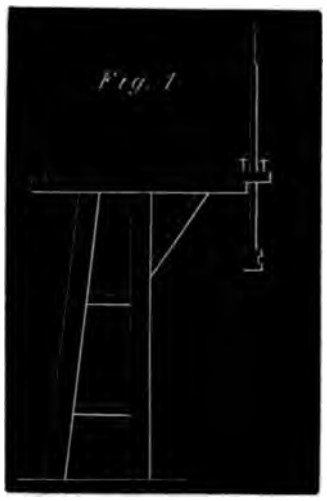
Fig. 1

passing through the corners of a brass triangle attached to the tube not far above the place where it emerged from the hollow axis. . The points of the screws rested upon the same overhanging stand as in the former experiments (p. 113).*

The larger diameter of the tube made the system so rigid that no mechanical disturbance of the kind formerly met with was to be detected at the highest speed to

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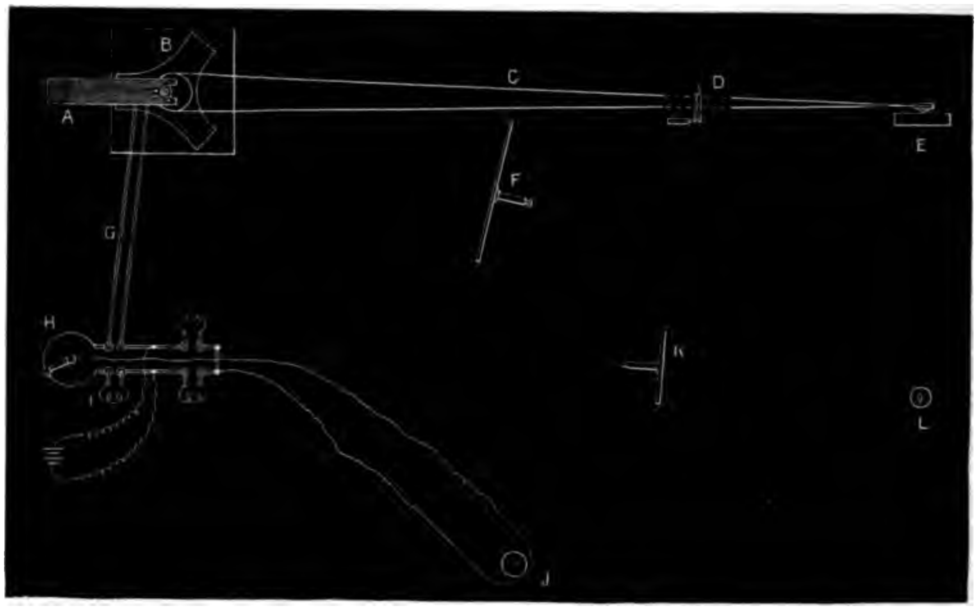
* June, 1882. The general disposition of the apparatus is shown in fig. 2.



A. Stand for suspended parts.
B. Frame of revolving coil.
C. Driving cord.
D. Electro-magnetic fork and telescope.
E. Water engine.
F. Principal telescope and scale.

G. Copper connecting bars.
H. FLEMING's bridge.
I. Platinum-silver standard.
J. Bridge galvanometer.
K. Telescope and scale of auxiliary magnetometer.
L. Auxiliary magnetometer needle and mirror.

which we could drive the coil. Even a tap with the finger-nail upon the magnet-box produced but a small disturbance.

No change was required in the arrangements for regulating and determining the speed of the coil, which worked, if possible, more perfectly than before, in consequence of the greater inertia of the revolving parts. The divided card was, however, on an enlarged scale, and the numbers of the teeth in the various circles were so arranged that each circle was available for a distinct pair of speeds according as it was observed through the slits in the plates carried by the electric fork or over the top of the upper plate. The speeds actually used corresponded to 80, 60, 45, 35, and 30 teeth, seen through the slits, *i.e.*, about 127 times per second.

The greater resistance of the copper coil (23 instead of 4·6) rendered necessary a modification in the method of making the comparisons with the standard. The whole value of the divided platinum-iridium wire on FLEMING's bridge being only $\frac{1}{20}$ ohm, a change of temperature in the copper of not much more than a degree would exhaust the range of the instrument. To meet this difficulty it was only necessary to add resistances to the copper circuit so as to compensate approximately the temperature variations, for it is evident that it can make no difference whether the change of resistance of the entire revolving circuit is due to a rise of temperature, or to the insertion of an additional piece. The platinum-silver standard was therefore prepared so as to have a resistance (about 24 ohms) greater than any which we were likely to meet with in the copper, and the additional pieces were relied upon to bring the total within distance. As at first arranged, the additional resistance was inserted at the mercury cups, instead of a contact piece of no appreciable resistance. During the comparison with the standard it was transferred to another part of the circuit.

In the course of May, 1881, a complete series of spinnings were taken, the arrangements and adjustments being (except as above-mentioned) in all respects the same as with the old apparatus. Five different speeds were used, and each of them on three different evenings. The work of observing was also distributed as before, Dr. SCHUSTER taking the readings of the principal magnetometer, and Mrs. SIDGWICK the simultaneous readings of the auxiliary magnetometer, while I observed the divided card and regulated the speed. At each speed on each evening four readings were taken with wire circuit closed, two with positive and two with negative rotation, and in like manner four readings were taken with the wire circuit open. Observations on the zero with the coil at rest were for the most part dispensed with, as it was thought that the time could be better employed otherwise ; in fact, the mean of the two not very different positions of equilibrium obtained with positive and negative rotation when the wire circuit was open, gives all that is wanted in this respect. In the actual reductions we only require the *difference* of readings with positive and negative rotations.

It was hoped that these observations would have been sufficient, but on the introduction by Dr. SCHUSTER of the various corrections for temperature, for the beats between the two forks, and for the outstanding bridge-wire divisions, the necessity for which disguises the significance of the numbers first obtained, it was found that the agreement of the results corresponding to a given speed was by no means so good as we had expected in view of the precautions taken and the accuracy of the readings. What was worse, there was evidence of a decided progression, as if the absolute resistance of the standard had gradually diminished during the time occupied by the spinnings.

It is not impossible that there really was some change in the standard which had then been newly prepared; but the discrepancies were not, as according to this view they ought to have been, proportional to the speeds of rotation. I am inclined rather to attribute them to shiftings of the paper scales. The principal magnetometer scale was composed of three lengths of 50 centims. each, cemented with indiarubber to a strip of deal. The compound scale thus formed was examined by Dr. SCHUSTER in March, 1881. Between the graduations of the first and of the middle piece there was a gap of about $\frac{1}{4}$ millim., and another of nearly the same magnitude between the middle and the third piece. When I re-examined the scale in July, the gap at 500 divisions had increased to $\frac{9}{10}$ millim., and that at 1000 to $\frac{1}{2}$ millim. Curiously enough, there were no observable errors in the equality of the divisions of the three parts taken separately; but the changes above-mentioned are sufficient to throw considerable doubts upon the value of the first series of spinnings. They have, however, been reduced by Dr. SCHUSTER, and the result is given below for the sake of comparison.

To be free for the future from uncertainties of this kind, I replaced the paper scale by a long glass thermometer tube by CASELLA, graduated into millimetres. The divisions were fine and accurately placed, but the imperfect straightness of the tube has rendered necessary certain small corrections in the final results. Probably a straight strip of flat opal would have been an improvement.

The second series of spinnings was made in August, 1881, and this, it was fondly hoped, would be final. To guard against possible change in the platinum-silver coil a careful comparison with the standard units was previously instituted by Mrs. SIDGWICK, of which the details are given later. As we had unfortunately lost the advantage of Dr. SCHUSTER's assistance, the observations at the principal magnetometer devolved upon Mrs. SIDGWICK. The much easier post at the auxiliary magnetometer was usually occupied by Lady RAYLEIGH; occasional assistance has been rendered by Mr. A. MALLOCK and by Mr. J. J. THOMSON.

In the conduct of the second series one or two minor changes were introduced. In order to know the temperature of the standard tuning-fork more accurately, a thermometer was placed between its prongs and read at the same time as the number of beats was taken. The insertion of the small resistances necessary to bring the copper coil within range of the standard was also arranged in a different manner. Some trouble had been experienced in getting a sufficiently good fit between the contact pieces used in the first series and the mercury cups. It is necessary that the stout copper terminals should press down closely upon the bottoms of the cups, and also that the mercury should not be liable to escape at high speeds from the effect of centrifugal force. Bits of indiarubber tubing were placed round the copper legs, by which a fair fit with the sides of the cups was effected; but I thought that it would be an improvement to revert to a single contact piece for the mercury cups of no sensible resistance, whose fit could be carefully adjusted, and to insert the extra resistances at the connexion of the other (outer) ends of the component coils. For this purpose binding screws were employed, pressing firmly together the flat copper terminals of the copper wire and of the German-silver resistance pieces. It is almost unnecessary to say that these short lengths of German-silver wire were doubled upon themselves before being coiled, and that the pieces were not touched between a spinning and the associated resistance comparisons. Used in this way the screwed up contacts seemed unobjectionable, even though the surfaces were not amalgamated.

On each night and for each speed a set of twelve spinnings was made, six with wire circuit open, and six with wire circuit closed. It was usual to take, first, two of the former (one with positive and one with negative rotation); secondly, to compare the resistances of the revolving circuit and the standard; thirdly, after inserting the contact piece and adjusting the indiarubber strap by which it was held down, to make the six closed contact spinnings; fourthly, to compare the resistances again; and lastly, to complete the open contact readings. Each spinning, it will be understood, involved the reading of several elongations (about six for the open contact and ten for the closed), from which the position of equilibrium was deduced.

Table II. (p. 691) gives all the results of the second series, except one for 35 teeth on August 27th, which was rejected on the ground that it exhibited such large *internal* discrepancies, as to force us to the conclusion that the contact piece had been inserted improperly. It will be seen that the agreement is good except on August 29th, in which case the deflections are as much as four or five tenths of a millimetre too small. These discrepancies, though not very important in themselves, gave me a good deal of anxiety, as they were much too large to be attributed to mere errors of reading, and seemed to indicate a source of disturbance against which we were not on our guard.

The least unlikely explanations seemed to be (1) a change in the distance of the mirror from the scale, which unfortunately had not been remeasured at the close of the spinnings, though this would require to reach 3 millims.; (2) imperfect action of the contact piece from displacement of mercury or otherwise; (3) a change of level in the axis of rotation. The anomalous result of August 27th seemed to favour (2), while on behalf of (1) it must be said that the stand of the telescope and scale as well as the support for the suspended parts of the principal magnetometer were of wood. It was just conceivable that under the influence of heat or moisture some bending might have occurred.

On my return to Cambridge in October we proceeded to investigate these questions with the closest attention. As repeated direct measurements of the distance of the mirror and scale were inconvenient, measuring rods (like beam compasses) were provided to check the relative positions of the telescope stand and of the upper end of the suspending fibre with regard to fixed points on the walls of the room. But no changes comparable with 3 millims. were detected, even under much greater provocation than could have existed during the August spinnings. The next step was to examine the action of the contact piece. For this purpose the coil was balanced against the standard as usual, except that the contact piece was inserted and connexion with the bridge made at the other ends of the double coil. It was presently found that the resistance *did* depend upon the manner in which the contact piece was pressed, and that to an extent sufficient to account for the August discrepancies. Eventually it was discovered that one of the legs of the contact piece, which by a mistake had been merely rivetted and not soldered in, was shaky.

After this there could be no reasonable doubt that the faulty contact piece was the cause of our troubles. In all probability the leg became loose on the 27th, in which case the earlier results would be correct. Moreover, the final means are not very different, whether the spinnings of August 29th are retained or not. This being the case, we might perhaps have been content to let the matter rest here; but in view of the importance of the determination, and the desirability as far as possible of convincing others as well as ourselves, we thought that it would be more satisfactory to make a third and completely independent series of spinnings.

In this series the faulty composite contact piece was replaced by a horse shoe of continuous copper, and a check was instituted upon the distance between mirror and scale. The opportunity was also taken to make a minor improvement in connexion with the auxiliary magnetometer. The somewhat unsteady table on which the telescope and scale had stood was replaced by one of stone, and the arrangements for illumination were improved by throwing an image of a gas flame on the part of the scale under observation. The same number of readings were made as in the second series, but we found it more expeditious to take the six open contact spinnings together. At the beginning of the evening it was desirable to commence with these open contact spinnings in order to give more time for the coil to acquire the temperature of the room, which always rose somewhat, although the lamps and gas were lit a couple of hours beforehand. Later in the evening we sometimes took the closed contact readings for two speeds consecutively, in order that the intermediate resistance comparison might serve for both. In other respects the arrangements were unaltered.

Full details of the observations and reductions are given below. It will be sufficient here to mention that the maximum discrepancy between any two deflections at the same speed amounts only to $\frac{7}{100}$ of a millimetre, so that the agreement on different nights is more perfect than could have reasonably been expected. At the lowest speed the above-mentioned discrepancy is less than one part in 3000, and at the highest speed less than one part in 6000. No spinnings in the third series were rejected, except on one or two occasions when it appeared *at the time of observation*, from the behaviour of the auxiliary magnetometer, that there was too much earth disturbance. The spinnings were then suspended, and the observations already obtained were not reduced.

At the close of the spinnings, Mrs. SIDGWICK made a further comparison of our platinum-silver coil with the standard units.

The value arrived at for the B.A. unit (·9865 ohm) differs nearly three parts in a thousand from that which we obtained with the original apparatus. This difference is not very great, and may possibly be accounted for by errors in the measurement of the coil (see p. 114 of former paper). If a coil be imperfectly wound, the mean radius, as determined by a tape, is liable to be too great. At any rate, this discrepancy sinks into insignificance in comparison with that which exists between either of these determinations and that of Professor KOHLRAUSCH,* according to whom the B.A. unit would be as much as 1·0196 ohms. With respect to the method employed by KOHLRAUSCH, I agree with ROWLAND† in thinking it difficult, and unlikely to give the highest accuracy; but how in the hands of a skilful experimenter it could lead to a result 3 per cent. in error, is difficult to understand. The only suggestion I have to make is that possibly sufficient care was not taken in levelling the earth-inductor. Although estimates are given of the probable errors due to uncertainties in the various data, nothing is said upon this subject. In consequence, however, of the occurrence of the horizontal intensity as a square in the final formula, in conjunction with the largeness of the angle of dip, the method is especially sensitive to a maladjustment of this kind. I calculate that a deviation of the axis of rotation from the vertical through 21′ in the plane of the meridian, would alter the final result by 3 per cent.‡

According to ROWLAND'S determination, the value of the B.A. unit is ·9912 ohm. The method consists essentially in comparing the integral current in a secondary circuit, due to the reversal of the battery in a primary circuit, with the magnitude of the primary current itself. The determination of the secondary current involves the use of a ballistic galvanometer, whose damping is small, and whose time of vibration can be ascertained with full accuracy; and it is here, I think, that the weakest point in the method is to be found. The logarithmic decrement is obtained by observation of a long series of vibrations, and it is assumed that the value so arrived at is applicable to the correction of the observed throw. I am not aware whether the origin of damping in galvanometers has ever been fully investigated, but the effect is usually supposed to be represented by a term in the differential equation of motion proportional to the momentary velocity. This mode of representation is no doubt applicable to that part of the damping which depends upon the induction of currents in the galvanometer coil, under the influence of the swinging magnet. If this were all, a correction for damping would be accurately effected on the basis of a determination of the logarithmic decrement, made with the galvanometer circuit closed in the same manner as when the throw is taken. In all galvanometers, however, a very sensible damping remains in operation even when the circuit is open, of which the greatest part is doubtless due to aerial viscosity; and it is certain that the retarding force arising from viscosity is not simply proportional to the velocity at the moment, without regard to the state of things immediately preceding.

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* POGG. Ann., Ergänzungband VI. Phil. Mag., April, 1874.

† American Journal, April, 1878.

‡ See p. 684.

In particular, the force acting upon the suspended parts as they start suddenly from rest in the observation of the throw, must be immensely greater than in subsequent passages through the position of equilibrium, when the vibrations have assumed their ultimate character. I calculate that in the first quarter vibration (*i.e.*, from the position of equilibrium to the first elongation) of a disc vibrating in its own plane and started impulsively from rest, the loss of energy from aerial viscosity would be 1·373 times that undergone in subsequent motion between the same phases. From this it might at first appear that in this ideal case the logarithmic decrement observed in the usual manner would need to be increased by more than a third part in order to make it applicable to the correction of a throw from rest; but in order to carry out this view consistently we should have to employ in the formula the time in which the needle would vibrate if the aerial forces were non-existent instead of the actually observed time of vibration. Now since the action of viscosity is to increase the time of vibration, the second effect is antagonistic to the first, so that probably the error arising from the complete neglect of these considerations is very small.

There is another point in which it appears to me that the theory of the ballistic galvanometer is incomplete. It is assumed that the magnetism of the needle in the direction of its axis is the same at the moment of the impulse as during regular vibrations. Can we be sure of this? The impulse is due to a momentary but very intense magnetic force in the perpendicular direction, and it seems not impossible that there may be in consequence a temporary loss of magnetism along the axis. If this were so, the actual impulse and subsequent elongation would be less than is supposed in the calculation, and too high a value would be obtained of the resistance of the secondary circuit in absolute measure. In making these remarks I desire merely to elicit discussion, and not to imply that ROWLAND's value is certainly four parts in a thousand too high.

Determinations of the absolute unit have been made also by H. WEBER,* whose results indicate that the B.A. unit is substantially correct. In the absence of sufficient detail it is difficult to compare this determination with others, so as to assign their relative weights.

The value of the B.A. unit in absolute measure is involved in the two series of experiments executed by JOULE on the mechanical equivalent of heat.† The result from the agitation of water is 24868, while that derived from the passage of a known absolute current through a resistance compared with the B.A. unit was 25187. The latter result is on the supposition that the B.A. unit is really 10^9 C.G.S. If we inquire what value of the B.A. unit will reconcile the two results, we find—

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* Phil. Mag., Jan., Feb., March, 1878.

† Phil. Trans., Part II., 1878. Brit. Ass. Rep., 1867; Reprint, p. 175.

$$1 \text{ B.A. unit} = \cdot 9873 \text{ ohm},$$

in very close agreement with the measurement described in the present paper. It should be remarked that in the comparison of the two thermal results some of the principal causes of error are eliminated; and it is not improbable that an experiment in which heat should be simultaneously developed in one calorimeter by friction, and in a second similar calorimeter by electric currents, would lead to a very accurate determination of resistance, more especially if care were taken so to adjust matters that the rise of temperature in the two vessels was nearly the same, and a watch were kept upon the resistance of the wire while the development of heat was in progress.

[*June*, 1882.—Since this paper was sent to the Society, Mr. GLAZEBROOK has worked out the results of a determination of the B.A. unit in absolute measure by a method not essentially different from that adopted by ROWLAND. The final number is practically identical with that of the present paper; and the agreement tends to show that the difference between ourselves and ROWLAND is not to be attributed to the use of a ballistic galvanometer.

Reference should have been made to the results of LORENTZ.* He finds as the value of the mercury unit *defined* by SIEMENS

$$1 \text{ mercury unit} = \cdot 9337 \frac{\text{earth quadrant}}{\text{second}}$$

The corresponding number calculated from the results of the present paper with use of the value of the specific resistance of mercury lately found (Proc. Roy. Soc., May 4, 1882) is ·9413. If we invert the calculation, we find that according to LORENTZ the value of the B.A. unit would be ·9786 absolute measure. The method of LORENTZ is ingenious, and apparently capable with good apparatus of giving a result to much within 1 per cent. Mrs. SIDGWICK and myself are at present making a trial of it.]

It will be desirable here to consider briefly some of the criticisms of KOHLRAUSCH and ROWLAND upon the method of the original British Association Committee, which has been adopted in the present investigation without fundamental alteration. The difficulty, remarked upon by KOHLRAUSCH, of obtaining a rapid and uniform rotation, has not been found serious, and I believe that no appreciable error can be due either to irregularity of rotation or to faulty determination of its rapidity. It has also been brought as an objection to the method that a correction is necessary on account of the magnetic influence of the suspended magnet upon the revolving circuit. The theory of this action is, however, perfectly simple, and the application of the correction requires only a knowledge of the ratio of the magnetic moment to the earth's horizontal force. If the magnetic moment is very small, the correction is unimportant; if larger, it can on that very account be determined with the greater ease and accuracy. It is probable that in the original experiments too feeble a magnetic moment was used, and that in consequence the suspended parts were too easily disturbed by non-magnetic causes; but this might have been remedied without increasing objectionably the correction in question. At any rate the larger coil of the new apparatus allows the use of any reasonable magnetic moment.

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* POGG. Ann., 1873.

Perhaps the least advantageous feature in the method is the necessity for creating a violent aerial disturbance in the immediate neighbourhood of a delicately suspended magnet and mirror. If, however, any deflection occurs in this way, very little error can remain when the open contact effect is subtracted from the closed contact effect. The difficulty of avoiding a sensible deflection, due to currents in the ring, when the wire circuit is open, is connected with a special advantage—*i.e.*, the possibility of assuring ourselves that there is no leakage from turn to turn of the coil. In the method followed by ROWLAND, for instance, such a leakage would lead to error, and could not be submitted to any direct test.

The correction for self-induction cannot be made very small without a disadvantageous reduction of the whole angular deflection; but so far as the wire is concerned it can be calculated *à priori*, or determined by independent experiment, with the necessary accuracy. There is reason, however, to think that the best method of treatment is to determine this correction from the spinnings themselves, combining the results of widely different speeds so as to obtain what would have been observed at a small speed. At small speeds it is certain that all effects of self-induction and of mutual induction between the wire circuit and other circuits in the ring will disappear.

Measurements of coil.

The mean radius of the coil, being the fundamental linear measurement of the investigation, must be found with full accuracy. There has been some difference of opinion as to the best method of effecting this. The greatest accuracy is probably attained by the use of the cathetometer. The measurement of the circumference of every layer by a steel tape has the advantage that the subject of measurement is three times as large, and is much less troublesome. The disadvantage is that if a layer be not quite even, there is danger of measuring the maximum rather than the mean outside circumference. In the present investigation the coil was so large that the tape could be employed without fear.*

Each of the component coils marked A and B had $18 \times 16 = 288$ windings, but in consequence of variations in the thickness of the triple silk covering, there was a difficulty in getting exactly 18 turns into each layer. In the eleventh layer of A it was necessary to be content with 17 turns, and to place an extra turn on the outside, so as to form the commencement of a seventeenth layer—a circumstance which of course was taken into account in calculating the mean. The number thus arrived at, after correction for the thickness of tape, is the mean *outside* circumference. What we require is the mean circumference of the axis of the wire; it may be derived from the first by subtraction of half the difference between the tape readings for the first layer, and for the bottom of the gun-metal groove.

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* The original Committee also employed the tape method. Their measurement of the length of the wire when unwound was not in order to find the mean radius, as SIEMENS and KOHLRAUSCH suppose, but to verify the number of turns.

The results obtained by Dr. SCHUSTER and myself when the coils were wound are:

	Coil A.	Coil B.
Mean of readings in millims.	1489·3	1487·5
Correction for tape	·6	·6
Mean outside circumference	1488·7	1486·9
Correction for thickness of wire . .	3·4	3·4
Mean circumference	1485·3	1483·5
Mean radius	236·39	236·11
Mean circumference of A and B . . .	1484·4	
Mean radius of A and B (a)	236·25	
Axial dimension of section in millims. .	19·9	19·9
Radial	15·9	15·4
Distance of mean planes ($2b'$)	65·95	

Two or three readings were taken of the circumference of every layer, and to prevent mistakes in the number of turns, the plan described by MAXWELL,* of simultaneously winding string on wooden rods, was followed. Without some such device, there is great risk of confusion.

In estimating the degree of accuracy obtainable, we must remember that the circumference of each layer is measured before the outer layers are wound on; any change produced by the pressure of these outer layers is a source of error. We had already observed a tendency in the measurements to be less during the unwinding of a coil than during the winding, and we fully intended to remeasure the coil after the spinnings were completed. This was done on December 6, 1881, by MRS. SIDGWICK and myself. As we expected, somewhat smaller readings (by about $\frac{3}{4}$ millim.) were obtained for the circumference of the middle layers. The results were:

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* 'Electricity and Magnetism,' II., § 708.

	Coil A.	Coil B.
Mean radius	236·31	236·02
Mean of both	236·16	

or nearly one part in 2000 less than before. Of the two values, it would appear that the latter is more likely to represent the actual condition of the coil during the spinnings, and is therefore entitled to greater weight. If we give weights in the proportion of two to one, we get

Mean radius = 23·619 centims.*

Calculation of GK.

We have

$$\mathrm{GK}=2\pi^2n^2a\sin^3\alpha\left\{1+\tfrac{1}{6}\frac{c^2}{a^2}+\tfrac{5}{8}\frac{b^2-c^2}{a^2}\sin^2\alpha\cos^2\alpha-\tfrac{1}{8}\frac{b^2}{a^2}\sin^2\alpha\right\}$$

in which

a = mean radius = 23·625 (1st measurement)

b = axial dimension of section = 1·990

c = radial dimension of section = 1·565

n = total number of turns = 576

$2b'$ = distance of mean planes = 6·595

$$\sin\alpha=a\div\sqrt{(a^2+b'^2)}$$

From these data we find

$$\begin{aligned}\log 2\pi^2n^2 &= 6{\cdot}81617\\ \log a &= 1{\cdot}37337\\ \log\sin^3\alpha &= \bar{1}{\cdot}98744\\ \log\{\ldots\} &= \bar{1}{\cdot}99995\\ \hline \log \mathrm{GK} &= 8{\cdot}17693\end{aligned}$$

But if we substitute the adopted value of a, *i.e.*, 23·619 centims., we have by subtraction of ·00011

$$\log \mathrm{GK} = 8{\cdot}17682$$

Calculation of L.

We may write

$$\mathrm{L}=16^2\times18^2\,(\mathrm{L}_1+\mathrm{L}_2+2\mathrm{M}),$$

where L_1, L_2 are the coefficients of self-induction of the two parts, and M the coefficient of mutual induction without regard to the number of turns. L_1 and L_2 may be calculated from the formula

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* [August, 1882. At the time of use the tape was compared with a measuring rod, which again has been compared with a standard metre verified by the Standards Department of the Board of Trade. For the purposes of this investigation the differences observed are altogether negligible. I may add that the clock with which the standard tuning-fork was compared (see p. 137 of former paper) was rated from astronomical observations.]

$$L=4\pi a\left[\log_e\frac{8a}{r}+\tfrac{1}{12}-\tfrac{4}{3}(\theta-\tfrac{1}{4}\pi)\cot 2\theta-\tfrac{1}{3}\pi\operatorname{cosec}2\theta-\tfrac{1}{6}\cot^2\theta\log_e\cos\theta-\tfrac{1}{6}\tan^2\theta\log_e\sin\theta\right]$$

in which r is the diagonal of the section, and θ the angle between it and the plane of the coil. With this formula and with the dimensions as measured when the coil was wound, we get

$$L_1 \text{ (for A) } = 1029{\cdot}3 \text{ centims.}$$
$$L_2 \text{ (for B) } = 1031{\cdot}9 \text{ centims.}$$

It would not be difficult to calculate an approximate correction for the curvature of the coil, but this is scarcely necessary. (See p. 119 of former paper.) Adding the above, we have

$$L_1+L_2=2061{\cdot}2 \text{ centims.}$$

The value of M was found from the tables given as Appendix I. to § 706 of the new edition of MAXWELL'S 'Electricity.' If we suppose each coil condensed into the centre of its section, we find $M=4\pi\times 33{\cdot}061$. A more exact calculation by the formula of interpolation explained in Appendix II. gives $M=4\pi\times 33{\cdot}140$, so that

$$2M=832{\cdot}88 \text{ centims.}$$

The final result is accordingly

$$L=16^2\times 18^2\times 2894{\cdot}1=2{\cdot}4004\times 10^8 \text{ centims.}$$

These calculations of the coefficients of induction have been made independently by Mr. NIVEN and myself, and are so far reliable; but we must not forget that the accuracy of the result depends upon the accuracy of the data, and that in the present case the diagonal of the section (r) on which the most important part of L depends is an element subject to considerable relative uncertainty. It is probable that the effective axial dimensions of the section is somewhat less than the width of the groove, and therefore that the real value of L may be a little greater than would appear from the preceding calculation.

Theory of the ring currents.

If the circuits are conjugate, the currents in the wire and in the ring are formed in complete independence of one another, a circumstance which simplifies the theory very materially. In the same notation as was used in the former paper (p. 105), and with dashed letters for the ring circuit, we have as the equation determining the angle of deflection (ϕ) when the wire circuit is closed,

$$\tan\phi+\tau\frac{\phi}{\cos\phi}=\frac{\frac{1}{2}GK\omega}{R^2+L^2\omega^2}\{R+L\omega\tan\phi+R\tan\mu\sec\phi\}$$
$$+\frac{\frac{1}{2}G'K'\omega}{R'^2+L'^2\omega^2}\{R'+L'\omega\tan\phi+R'\tan\mu\sec\phi\}$$

When the wire circuit is open the equation determining the angle of deflection (ϕ_0) is

$$\tan\phi_0+\tau\frac{\phi_0}{\cos\phi_0}=\frac{\frac{1}{2}G'K'\omega}{R'^2+L'^2\omega^2}\{R'+L'\omega\tan\phi_0+R'\tan\mu\sec\phi_0\}$$

Since τ is an extremely small quantity it is unnecessary to keep up the distinction between $\tau\phi/\cos\phi$ and $\tau\tan\phi$. By subtraction

$$(1+\tau)(\tan\phi-\tan\phi_0)$$
$$=\frac{\frac{1}{2}GK\omega}{R^2+L^2\omega^2}\{R+L\omega\tan\phi+R\tan\mu\sec\phi\}$$
$$+\frac{\frac{1}{2}G'K'\omega}{R'^2+L'^2\omega^2}\{L'\omega(\tan\phi-\tan\phi_0)+R'\tan\mu(\sec\phi-\sec\phi_0)\}$$

The last term is small, and we may neglect ($\sec\phi-\sec\phi_0$) in combination with $R'\tan\mu$.

Moreover

$$\frac{\frac{1}{2}G'K'\omega}{R'^2+L'^2\omega^2}=\frac{(1+\tau)\tan\phi_0}{R'+L'\omega\tan\phi_0}$$

so that

$$(1+\tau)(\tan\phi-\tan\phi_0)=\frac{\frac{1}{2}GK\omega}{R^2+L^2\omega^2}\{R+L\omega\tan\phi+R\tan\mu\sec\phi\}$$
$$+(1+\tau)(\tan\phi-\tan\phi_0)\frac{L'\omega\tan\phi_0}{R'+L'\omega\tan\phi_0}$$

If now we write (GK) for $GK/(1+\tau)$, we get

$$\tan\phi-\tan\phi_0=\frac{\frac{1}{2}(GK)\omega}{R^2+L^2\omega^2}\{R+L\omega\tan\phi+R\tan\mu\sec\phi\}\{1+\frac{L'\omega}{R'}\tan\phi_0\}$$

The effect of L′ would therefore be to *increase* disproportionately the deflections at high speeds, *i.e.*, contrary to the effect of L. It appears, however, that in these experiments it could not have been sensible. At the highest speed $\tan\phi_0$ was about $\frac{1}{670}$, and ω about 26 per second, so that $\omega\tan\phi_0$ would be about $\frac{1}{26}$. The value of L′/R′ is difficult to estimate with any accuracy. But the value of L/R for the wire circuit is about ·01 second, and that for the ring circuit must be much less, so that the terms involving L′ may safely be omitted.

The quadratic in R then becomes

$$R^2-R\frac{\frac{1}{2}(GK)\omega(1+\tan\mu\sec\phi)}{\tan\phi-\tan\phi_0}+L^2\omega^2-\tfrac{1}{2}(GK)L\omega^2\frac{\tan\phi}{\tan\phi-\tan\phi_0}=0$$

whence

$$R=\frac{\frac{1}{2}(GK)\omega}{\tan\phi-\tan\phi_0}\left[\tfrac{1}{2}(1+\tan\mu\sec\phi)+\sqrt{\left\{\tfrac{1}{4}(1+\tan\mu\sec\phi)^2-U(\tan\phi-\tan\phi_0)^2\right\}}\right]\cdot$$

where

$$U=\frac{2L}{(GK)}\left\{\frac{2L}{(GK)}-\frac{\tan\phi}{\tan\phi-\tan\phi_0}\right\}$$

L *by direct experiment.**

Although the calculated value of L was the result of two independent computations, I considered that it would be satisfactory still further to verify it by an experiment with WHEATSTONE'S balance. The statement of this method and the final formula, as given on p. 116 of the former paper, being approximate only, it will be convenient here to repeat them with the necessary corrections.

The four resistances in the balance are two equal resistances (10 units each), that of the copper coil P, and a fourth resistance Q (nearly equal to P) taken from resistance boxes, of which P is the only one associated with sensible self-induction. When P and Q are equal, there is no permanent current through the galvanometer; but if the galvanometer circuit be first closed and then the battery current be made, broken, or reversed, the needle receives an impulse, whose magnitude depends upon L.

If x denote the change of current in the branch P, the action of self-induction is the same as that of an electromotive impulse in that branch of magnitude Lx, and the effect upon the galvanometer is that due to this electromotive impulse acting independently of the electromotive force in the battery branch.

In order now to get a second quantity with which to compare the induction throw, the resistance balance is upset in a known manner. If while Q remains unaltered, P be increased to $P+\delta P$, there is a steady current through the galvanometer, which we may regard as due to an electromotive force $\delta P.x'$ in the branch $P+\delta P$, x' being the current through the branch. If θ be the deflection of the needle under the action of the steady current, α the angular throw, and T the time of swing from rest to rest, we have by the theory of the ballistic galvanometer as the ratio of the instantaneous to the steady electromotive force

$$\frac{T}{\pi}\,\frac{2\sin\frac{1}{2}\alpha}{\tan\theta},$$

subject to a correction for damping; so that this expression represents the ratio of $Lx:\delta P.x'$. If the induction throw be due to the make or break of the battery circuit, x represents simply the current in the branch P. In the case where the battery current is reversed, we may write $2Lx$ for Lx, understanding by x the same as before. As this method was the one actually adopted, we will write the result in the appropriate form

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* In consequence of the necessity which ultimately appeared of introducing an arbitrary correction proportional to the square of the speed of rotation, the result of the present section does not influence the final number expressing the B.A. unit in absolute measure. The method, however, is of some interest, and (it is believed) has not been carried out before with the precautions necessary to secure a satisfactory result.

$$\frac{Lx}{\delta P.x'}=\frac{T}{\pi}\frac{\sin\frac{1}{2}\alpha}{\tan\theta}.$$

In the formula as originally given by MAXWELL, and as stated in the former paper, the distinction between x and x' (the currents before and after the resistance balance is upset) was neglected. This step is legitimate if δP be taken small enough, to which course however there are experimental objections. In order that $\tan\theta$ might be of suitable magnitude, it was found necessary to make the ratio of $\delta P : P$ equal to about $\frac{1}{800}$, a fraction too large to be neglected.

In carrying out the experiment it was found more convenient to insert the additional resistance in the branch Q, leaving P unaltered. By the symmetry of the arrangement it is evident that this alteration is immaterial, and that we may take the formula in the form

$$\frac{L}{Q}=\frac{L}{P}=\frac{\delta Q.x'}{Q.x}\frac{T}{\pi}\frac{\sin\frac{1}{2}\alpha}{\tan\theta},$$

x being the current in the branch Q when the resistance balance is perfect, x' the diminished current when the additional resistance δQ is inserted.

The principal difficulties in carrying out the experiment arose from variation in the battery and in the resistance balance. From these causes the results of two days' experiments were rejected, as unlikely to repay the trouble of reduction. On the last day (December 3, 1881) the first difficulty was overcome by using three large DANIELL cells (charged with zinc sulphate) in multiple arc. As precautions against rapid change of temperature the copper coils were wrapped thickly round with strips of blanket and deposited in a closed box. The delicacy of our arrangements was such that about $\frac{1}{1000}$ of a degree centigrade would manifest itself, so that it was hopeless to try to maintain the resistance balance absolutely undisturbed. The mode of applying a suitable correction will presently be explained. On December 3, partly by good luck, the necessary correction remained small throughout. In order to avoid a direct action of the current upon the galvanometer needle, the coil was placed at a considerable distance, at the same level, and with its plane horizontal. Any outstanding effect of the kind would, however, be eliminated from the final result by the reversals practised.

The induction throws were always taken by reversal of the battery current. A reversal has two advantages over a simple make or break. In the first place the effect is doubled and is therefore more easily measured; and in the second the battery is more likely to work in a uniform manner, the circuit being always closed except for a fraction of a second at the moment of reversal. The key was of the usual rocker and mercury cup pattern.

The galvanometer was one belonging to the laboratory of about 80 ohms resistance.

It was set up by Mr. GLAZEBROOK for his experiments by an allied method, and with its appurtenances was ready for use at the time that this determination of self-induction was undertaken. The scale was divided into millimetres, and was placed at a distance of 218 centims. from the galvanometer mirror. The instrument was adapted for ballistic work, as the vibrations were subject to a logarithmic decrement of only about ·0142.

The electric balance was provided for by a resistance box from Messrs. ELLIOTTS. The battery current after leaving the reversing key divides itself on entering the box, each part traversing 10 ohms. At the ends of these resistances come the galvanometer electrodes. The first part of the current now traverses the copper coil, and the second part other resistances, after which the two parts reunite and pass back to the battery. In the use of the "other resistances," a special arrangement was adopted which I must now explain. The resistance of the copper coil being somewhat under 24 ohms, the most obvious way to obtain a balance was to add to it a piece of adjustable wire until the whole would balance 24 ohms from the box. The objection to this plan is that the smallest known disturbance which we can then introduce, *i.e.*, by the addition or subtraction of a single unit, is much too great for the purpose.

The difficulty thus arising is completely met by the use of high resistances, taken from a second box, in multiple arc with the 24 ohms.

In order to balance the copper coil and its leading wires at the actual temperature (about 14°), 753 ohms were required in multiple arc with the 24. To calculate the resultant resistance we have

$$\tfrac{1}{24}+\tfrac{1}{753}=\cdot041666667+\cdot001328021=\cdot042994688=\tfrac{1}{23\cdot25869},$$

so that the resistance of the copper coil in terms of the units of the box is 23·25869. A suitable deflection θ was obtained by the substitution of 853 for 753 in the auxiliary box. In this case

$$\tfrac{1}{24}+\tfrac{1}{853}=\cdot041666667+\cdot001172333=\cdot042839000=\tfrac{1}{23\cdot34322};$$

so that the additional resistance was

$$\delta Q=\cdot08453 \text{ unit.}$$

It may be remarked that if the copper coil had been about 1° warmer, its resistance would have been greater by $\frac{1}{300}$th part, and the balance would have required 853 instead of 753 in multiple arc with the 24.

On account of the progressive changes already mentioned, it was advisable to alternate the observations of α and θ as rapidly as possible, and to occupy no more time than was really necessary in taking the readings. A good deal of time may be saved by working the key suitably, and by opening and closing the galvanometer branch (at a mercury cup provided for the purpose) so as to avoid producing unnecessary swings, and to stop those due to induction when done with; but it is unnecessary to go into detail in this part of the subject. After a little practice two induction throws, starting with opposite directions of the current, and two observations of steady deflection, one in each position of the reversing key, could be made in about seven minutes. The vibrations of the galvanometer needle were damped by the operation of a current in a neighbouring coil, the current being excited by a LECLANCHÉ cell and controlled by a key within reach of the observer at the telescope. The readings were taken by Mrs. SIDGWICK, while I reversed the battery current, shifted the resistances, and recorded the results.

In the simple theory of the method the induction throw is supposed to be taken when the needle is at rest and when the resistance balance is perfect. Instead of waiting to reduce the free swing to insignificance, it was much better to observe its actual amount and to allow for it. The first step is, therefore, to read two successive elongations, and this should be taken as soon as the needle is fairly quiet. The battery current is then reversed, to a signal, as the needle passes the position of equilibrium, and a note made whether the free swing is in the same or in the opposite direction to the induction throw. We have also to bear in mind that the zero about which the vibrations take place is different after reversal from what it was before reversal, in consequence of imperfection in the resistance balance. At the moment after reversal we are therefore to regard the needle as displaced from its position of equilibrium, and as affected with a velocity due jointly to the induction impulse and to the free swing previously existing. If the arc of vibration (*i.e.*, the difference of successive elongations) be a_0 before reversal, the arc due to induction be a, and if b be the difference of zeros, the subsequent vibration is expressed by

$$\tfrac{1}{2}(a \pm a_0) \sin nt + b \cos nt,$$

in which t is measured from the moment of reversal, and the damping is for the present neglected. The actually observed arc of vibration is therefore

$$2\sqrt{\{\tfrac{1}{4}(a \pm a_0)^2 + b^2\}}$$

or with sufficient approximation

$$a \pm a_0 + \frac{2b^2}{a}$$

so that

$$a = \text{observed arc} \mp a_0 - \frac{2b^2}{a}$$

In most cases the correction depending upon b was very small, if not insensible. The "observed arc" was the difference of the readings at the two elongations immediately following the reversal. As a check against mistakes the two next elongations also were observed, but were not used further in the reductions. The needle was then brought nearly to rest, and two elongations observed in the now reversed position of the key, giving with the former ones the data for determining the imperfection of the resistance balance. As the needle next passed the position of equilibrium, it was acted upon by the induction impulse (in the opposite direction to that observed before), and the four following elongations were read.

These observations of the throw were followed as quickly as possible by observations of the effect of substituting 853 for 753 units in the auxiliary arc. As soon as the vibrations could be reduced to a moderate amplitude, readings of three or four consecutive elongations were taken. The galvanometer contact was then broken, and the battery key reversed. When the needle had swung over to the other side, the galvanometer contact was renewed, and four elongations were observed. The difference between the two positions of equilibrium represented the disturbance of the resistance balance.

The whole of this disturbance, however, was not due to the additional 100 introduced, but required correction for the corresponding effect observed even with 753 units in the auxiliary arc. For this purpose it was only necessary to add or subtract the difference between the equilibrium positions of the needle with the key in the two positions, as deduced from the observations immediately preceding the induction throws; and in order to eliminate the influence of the progressive change, the mean of these differences as found before and *after* the insertion of the extra 100 units was employed. This result was compared with the mean of the four induction throws contiguous to it, two preceding and two following, and in this way a ratio obtained which was independent of the gradual but unavoidable changes in the battery current and in the copper resistance. After about half the readings had been taken the galvanometer connexions were reversed.

A specimen set of observations will now be given.

$3^h\ 36^m$	[753]	L	264·4
		Induction	246·6
$3^h\ 38^m$		R	262·5
$3^h\ 38^m$		Induction	245·9
$3^h\ 40^m$	[853]	R	182·3
$3^h\ 41^m$		L	344·7
$3^h\ 44^m$	[753]	L	264·4
$3^h\ 44^m$		Induction	245·7
$3^h\ 45^m$		R	263·1
		Induction	245·6

At $3^h\ 36^m$ with 753 units in the auxiliary arc and with battery key to the left, the position of equilibrium, as deduced from two elongations, was 264·4 on the galvanometer scale. The arc of vibration due to induction consequent on shifting the key from left to right, corrected for the free swing, but uncorrected for damping, was 246·6. In like manner with key to the right, the equilibrium position at $3^h\ 38^m$ was

262·5 and the arc due to induction was 245·9. The difference 1·9 between 264·4 and 262·5 represented the defect of balance. In the second set of induction throws the corresponding difference is 1·3, showing that the changes of temperature in progress were (at this stage) improving the balance of resistances. The difference between the readings R and L with 853 units is 162·4, the reading L being the higher. Since the reading L is also higher with 753 units, we have to *subtract* from 162·4 the mean of 1·9 and 1·3, *i.e.*, 1·6. The corrected value is thus 160·8. With this we have to compare the mean of 246·6, 245·9, 245·7, 245·6, *i.e.*, 245·9, and we thus obtain as the ratio of the two effects

$$\frac{245 \cdot 9}{160 \cdot 8} = 1 \cdot 529$$

The numbers obtained in this way were 1·535, 1·532, 1·529, 1·528, mean 1·5310; and with galvanometer reversed 1·534, 1·529, 1·530, 1·530, 1·532, mean 1·5310. The reversal of the galvanometer appears to have made no difference, and we have as the mean of all 1·5310. The comparison of the partial results shows that during the hour and a half over which the readings extended the battery current fell slowly about one part in 120, and that the resistance of the copper gradually increased, until the balance was perfect, and afterwards became too great, the whole change being about one part in 6000, which would correspond to about one-twentieth of a degree centigrade.

A small correction is required in identifying the above determined ratio with $2 \sin \frac{1}{2}\alpha / \tan \theta$. If A be the induction arc and B be difference of equilibrium positions with 853 units when the commutator is reversed,

$$\tan 2\alpha = \frac{\frac{1}{2}A}{D}, \qquad \tan 2\theta = \frac{\frac{1}{2}B}{D}$$

where

D= distance of mirror from scale=218 centims.

From these we get

$$\frac{2 \sin \frac{1}{2}\alpha}{\tan \theta} = \frac{A}{B} \, \frac{1 - \frac{11}{32} \frac{A^2}{4D^2}}{1 - \frac{1}{4} \frac{B^2}{4D^2}}$$

or in the present case with A=24·5, B=16·0,

$$\frac{2 \sin \frac{1}{2}\alpha}{\tan \theta} = \frac{A}{B} (\cdot 99925)$$

and

$$\frac{A}{B} = 1 \cdot 5310$$

So far we have omitted to consider the effect of damping, which must necessarily cause the observed value of A to be too small. If λ be the logarithmic decrement, the correcting factor is $(1+\lambda)$. The throw from zero to the first elongation is diminished by the fraction $\frac{1}{2}\lambda$, and the distance from zero to the second elongation is too small by the fraction $\frac{3}{2}\lambda$. Observations made in the usual manner after the other readings were concluded gave with considerable accuracy

$$\lambda = \cdot 0142$$

The time of vibration was taken simultaneously. It appeared that

$$T = 11\cdot 693 \text{ seconds}$$

A sufficient approximation to the ratio of currents $x' : x$ can be obtained by neglecting in both cases the current through the galvanometer, whose resistance (80 units) was considerable in comparison with the other resistances. On account of the small resistance of the battery, the difference of potentials at the battery electrodes may be regarded as given. On these suppositions we get at once

$$\frac{x'}{x} = \frac{10 + 23\cdot 25869}{10 + 23\cdot 34322}$$

whence

$$\log \frac{x'}{x} = \bar{1}\cdot 99891$$

A more elaborate calculation, in which the finite conductivity of the galvanometer was taken into account, gave a practically identical result

$$\log \frac{x'}{x} = \bar{1}\cdot 99886$$

We may now enter the numbers in the formula

$$L = \delta Q \frac{x'}{x} \cdot \frac{T}{2\pi} \cdot \frac{A}{B} (\cdot 99925)(1 + \lambda)$$

in which we must remember that δQ is to be expressed in absolute measure. Now the value given before, viz.: $\delta Q = \cdot 08453$, is expressed in B.A. units. What this would be in absolute units involves the entire question to whose solution this paper is directed. We will suppose that

1 B.A. unit $= \cdot 987$ ohm

δQ	log $\cdot 08453 \times 10^9$	$= 7\cdot 92701$
Correction to absolute units . . .	log $\cdot 987$	$= \bar{1}\cdot 99432$
A : B	log $1\cdot 5310$	$= \cdot 18498$
Correction for finite arcs	log $\cdot 99925$	$= \bar{1}\cdot 99967$
Correction for damping	log $1\cdot 0142$	$= \cdot 00612$
Time of vibration	log $11\cdot 693$	$= 1\cdot 06793$
Ratio of currents	log (x'/x)	$= \bar{1}\cdot 99886$
		$9\cdot 17889$
	log 2π	$= \cdot 79818$
	log L	$= 8\cdot 38071$

whence

$$L=2\cdot4023\times10^8 \text{ centims.}$$

The value by *à priori* calculation is

$$L=2\cdot400\times10^8 \text{ centims.}$$

about one part in a thousand lower.*

Correction for level.

If the axis of rotation deviate from the vertical in the plane of the meridian a corresponding correction is required. If I be the angle of dip, and β the deviation of the axis from the vertical towards the north, the electromotive forces are increased in the ratio $(1+\tan I\,\beta):1$, in which proportion we must suppose GK increased. (See pp. 106, 124 of former paper.) The angle of dip at Greenwich for 1881 is about 67° 30′, so that

$$\tan I=2\cdot414$$

The correction for an error in level is thus of the first order, and is magnified by the largeness of the angle of dip in these latitudes. If the experiments were made at the magnetic equator, we should not only reduce the correction for level to the second order, but also obtain the advantage of a nearly doubled horizontal force.

Observations on the level were made by Dr. SCHUSTER on June 1, by myself on August 30, and by Mrs. SIDGWICK on October 13, and on November 11 and 23. The August observations gave $\beta=\cdot26'$; the October observations gave $\beta=\cdot30'$; and the November observations gave $\beta=\cdot25'$. The position of the axis is necessarily to a slight extent indefinite, and the differences are probably accidental. The same level was used throughout, and the value of its graduations was tested. We may take

$$\beta=+\cdot27'=+\cdot000079 \text{ circular measure}$$

and

$$1+\tan I\,\beta=1\cdot00019$$

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* A further small correction is called for by the fact that at actual temperature of the room (about 14°) the resistances given by the boxes were not exactly multiples of the B.A. unit. The difference in the case of the principal box, which is marked as correct at 14°·2, may be neglected, but the resistances taken from the auxiliary box (marked 18°·3) must have been smaller than their nominal value, to the extent of a little over one part in a thousand. By the same fraction δQ, and consequently L, must be *greater* than is supposed in the above calculation. The corrected value of L will be

$$L=2\cdot4052\times10^8$$

It is about *two* parts in a thousand greater than that found from the measured dimensions, and is, in my opinion, quite as likely to be correct.

Correction for torsion.

To determine τ, about five complete turns in either direction were given to the upper end of the fibre. The difference of reading for one turn was found to be in June 2·58, and in August 2·45. If we take as the mean 2·51, we get

$$\tau=\frac{2{\cdot}51}{4\pi\times 2670}={\cdot}000075$$

Value of GK *corrected for level and torsion.*

Calling the corrected value $\mathfrak{GK}$, we have

$$\mathfrak{GK}=\frac{\mathrm{GK}(1+\tan \mathrm{I}\beta)}{1+\tau}=\frac{1{\cdot}00019}{1{\cdot}000075}\,\mathrm{GK}$$

so that

$$\log \mathfrak{GK}=8{\cdot}17686$$

The corrections are in fact almost insensible.

Calculation of U.

In this we take for $\mathfrak{GK}$ the value just found. For L we take the mean of the values found by *à priori* calculation and by direct experiment, *i.e.*,

$$\mathrm{L}=2{\cdot}4026\times 10^{8}.$$

Thus

$$\log\frac{2\mathrm{L}}{\mathfrak{GK}}={\cdot}50485,\qquad \frac{2\mathrm{L}}{\mathfrak{GK}}=3{\cdot}1978$$

For the values of $\tan\phi$ and $\tan\phi_0$ we must anticipate a little. The ratio is itself in some degree a function of the speed, but it will suffice to take the values applicable to the highest speed, for which $\tan\phi_0 : \tan\phi = 7{\cdot}81 : 439{\cdot}41$.

Thus

$$\frac{\tan\phi}{\tan\phi-\tan\phi_0}=1{\cdot}0181$$

and

$$\log \mathrm{U}={\cdot}84325$$

Measurement of tan μ.

This is the tangent of the angle through which a suspended magnetic needle would be turned when the principal magnet is presented to it at a distance $\sqrt{(a^2+b'^2)}$ to the east or west, the axis of the principal magnet lying east and west. Actual measurements with the aid of the auxiliary magnetometer were made in April, June, and November; and as a check upon the constancy of the magnetic moment frequent observations were taken of the time of vibration.

To explain the procedure it will be sufficient to take the data of the November measurement. Two positions were chosen for the principal magnet, nearly equidistant from the suspended magnet, to the east and west. The length of the line joining the two positions was 695 millims., and it passed horizontally about 36 millims. below the suspended magnet. In each position the magnet was reversed backwards and forwards several times and readings taken. When the principal magnet was to the east, the mean difference of readings due to reversal was 13·55 divisions on the millimetre scale. When the principal magnet was in the westerly position, the corresponding difference of readings was 14·61. We are to take the mean of these, *i.e.*, 14·08, as the difference of readings due to reversal at a distance of 347·5 millims. The half of this, or 7·04, corresponds to the simple presentation or removal of the magnet. The distance from mirror to scale was 2670 millims, so that the tangent of the angle of deflection was $\frac{7\cdot04}{2\times2670}$. This result has to be adjusted to correspond with the distance $\sqrt{(a^2+b'^2)}$, in place of 347·5. Hence

$$\tan\mu=\frac{(347\cdot5)^3\times7\cdot04}{(238\cdot5)^3\times2\times2670}=\cdot00408$$

In this calculation the error due to the principal magnet having been necessarily placed at a different level from that of the suspended magnet is ignored. As a matter of fact a relatively considerable correction is required. If θ be the altitude of one magnet as seen from the other, the observed effect is too small in the ratio $(1-3\theta^2):1$. The above written value of tan μ requires to be increased about 3 per cent.; so that we take

$$\tan\mu=\cdot00420.$$

Measurement of D.

For the first and second series of spinnings the distance from mirror to scale was measured exactly as described by Dr. SCHUSTER (p. 126 of former paper). The value adopted for the second series, after correction for the thickness of the glass window in the magnet box, was

$$D=2669\cdot0 \text{ millims.}$$

The same method of measurement was applied at the beginning of the third set, and a watch was kept by means of the measuring rods already spoken of (p. 667). Slight movements were in fact observed, principally of the nature of a recovery of the telescope stand from the rather violent treatment to which it had been subjected as a test. Minute corrections are accordingly introduced into the tabular statement (p. 693), so as to make the results of different days comparable. At the close of the spinnings the direct measurement was repeated, when there appeared a slight discrepancy between the results obtained by Mrs. SIDGWICK and myself. It is in fact rather a difficult matter to say exactly when the pointer has advanced to the equilibrium position of the centre of a suspended mirror, which cannot be prevented from swinging. Although the amount in question was not important, I thought it might be more satisfactory to check the result by another method, and therefore arranged a travelling microscope focussed alternately upon the centre of the mirror and upon a scratch on the window of the magnet box, by which the distance between these two points was determined. The remaining distance between the scratch and the scale was easily measured with the rod. The result tended to confirm the smaller value previously found. The value adopted for the spinnings of the third series before November 5 is

$$2668{\cdot}8 \text{ millims.}$$

and for November 5 and subsequent nights

$$2669{\cdot}4 \text{ millims.}$$

From these numbers we have to subtract 1·1 millim., as a correction for the thickness of the glass window; so that

$$D \text{ before November 5} \quad = 2667{\cdot}7 \text{ millims.}$$
$$D \text{ November 5 and after} = 2668{\cdot}3 \text{ millims.}$$

These distances are expressed in terms of the divisions of the scale, whose exact agreement with millimetres is of no consequence.

Reduction of results.

In order to give a clear idea of the results and of the manner in which they have been reduced, it will be advisable to quote from the note book the details of one set of spinnings. I have chosen at random one of the third series made on October 31, 1881, with a speed of "45 teeth."

TABLE I.

		Time* P.M.	Readings of auxiliary magnetometer in millims.	Readings of principal magnetometer in millims.	Readings corrected by auxiliary magnetometer.	Sum.	Difference.	Mean deflection.	Standard minus copper, in bridge-wire divisions.	Temperatures.			Beats per minute.	Correction to middle of bridge.	Correction to 18° of standard.	Correction to 50 beats.	Correction to 17° of fork.	Corrected deflection
										Standard.	Air.	Fork.						
Contact open.	I.	8.16	81·3	593·38	593·38	1197·24	10·48											
	II.	8.18	81·6	604·16	603·86	1197·27	10·45											
	III.	8.20	82·2	594·31	593·41	1197·51	10·69											
	IV.	..	83·0	605·80	604·10	1197·55	10·65	5·29										
	V.	8.23	83·8	595·95	593·45	1197·50	10·60											
	VI.	8.25	84·2	606·95	604·05													
		8.36	..	..	..	..	..	..	+212	9·95	12·2	12·5	56					
Contact closed.	VII.	8.45	83·1	903·38	901·58	1197·69	605·47											
	VIII.	8.47	82·5	297·31	296·11	1197·65	605·43											
	IX.	8.50	81·4	901·64	901·54	1197·96	605·12											
	X.	8.52	81·0	296·12	296·42	1197·75	604·91	302·56	Mean −52	Mean 10·02	..	Mean 13·05	Mean 56·5	+·03	−·27	−·10	−·12	302·10
	XI.	8.55	81·0	901·03	901·33	1197·89	604·77											
	XII.	8.58	81·1	296·36	296·56													
		9.3	..	..	..	..	..	..	−316·5	10·1	12·0	13·6	57					

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* To prevent misapprehension it may be mentioned that the times given in this column are approximate only and were not relied upon to secure simultaneous readings of the two magnetometers. A signal was passed from one observer to the other at the beginning and end of the readings.

The first column gives the number of the spinning, the first six being made with wire circuit open, and the last six with the wire circuit closed. In spinnings I., III., V., VIII., X., XII., the rotation was in the direction reckoned negative, and in the remaining ones positive. The second column gives the time, the third the reading of the auxiliary magnetometer, the fourth the reading of the principal magnetometer, the fifth the result of correcting the latter by the former, the sixth and seventh the approximately constant sums and differences of consecutive pairs of numbers in the fifth column, and the eighth gives the mean deflection from zero, *i.e.*, 5·29 for the open contacts, and 302·56 for the closed contacts.

The ninth column shows the results of the resistance comparisons between the platinum-silver standard and the revolving copper coil before and after the closed contact spinnings. The first number (+212) means that at $8^{h}\ 36^{m}$ the resistance of the standard exceeded that of the copper by 212 bridge-wire divisions, each of which represents $\frac{1}{20000}$ of an ohm. It will be seen that during the spinnings the resistance of the copper increased, which accounts for the gradual fall observable in the seventh column. The mean of the comparisons before and after spinning is taken to correspond with the mean deflection 302·56. The three following columns show respectively the temperatures of the water in which the standard was immersed, of the air in the neighbourhood of the copper coil, and of the standard tuning-fork, while the thirteenth column gives the number of beats per minute between the electrically maintained and the standard fork.

For the sake of more convenient comparison of the results obtained at the same speed on different nights, small corrections are calculated to reduce the actually observed deflections in the eighth column to what they would have been in a standard condition of the resistance and of the speed. Under each of these heads we have two corrections to consider. In the first place the copper circuit differed in resistance from the standard coil by the outstanding (−52) divisions of the bridge wire. The resistance of the whole being about 24 ohms, each division of the wire corresponds to one part in 480,000, so that in the present case the correction is additive and equal to 52 parts in 480,000, *i.e.*, is equal to +·03 division of the scale. This is given in the fourteenth column. Secondly, the resistance of the standard itself depends upon a variable temperature. The mean temperature of the standard in this series was about 13°, to which all the observations are reduced. In the present case the temperature was below the normal, so that the resistances were too small and the deflections too large. Accordingly the correction is negative. To estimate its amount the change of resistance with temperature is taken at three parts in 10,000 per degree; so that in the present case we are to subtract 2·8 parts in 3000 of the whole deflection, *i.e.*, ·27, as entered in the fifteenth column. With use of these corrections we obtain the deflection as it would have been observed had the resistance of the revolving circuit (together with the long copper bars by which it was connected with the bridge) been on every occasion exactly that of the standard at 13°.

In like manner two other very small corrections have to be introduced to make the results correspond exactly to a normal speed of rotation. The standard number of beats is taken at 59, and the standard temperature of the fork at 17°. In the specimen set the number of beats is $2\frac{1}{2}$ per *minute* too small, which means that the octave of the electrically maintained fork made (relatively to the other fork) $2\frac{1}{2}$ complete vibrations per minute *too many*. The whole number of vibrations per minute being 60×127, the speed was too great by $2\frac{1}{2}$ parts in 60×127, by which fraction the observed deflection must be reduced. The correction is thus $-\cdot 10$. But besides this the standard fork at 13·05° vibrated faster than its normal rate at 17°, by about one part in 10,000 for each degree of difference. The correction for this is $-\cdot 12$.

In addition to the corrections already mentioned the observations of November 5 and after were subjected to another small correction for the observed change in D.

The accompanying Table (II.) exhibits the results of the second series in a manner which after what has been said will not require much explanation. Column VIII. gives in each case what the deflection would have been if the revolving circuit and the copper connecting bars had exactly balanced the platinum-silver standard at 16°, the electric fork vibrating at such a speed as to give 59 beats per minute with the standard fork at 17°, and thus allows us to test the agreement or otherwise of the results obtained on various occasions at the same speed. From this point onwards the means only need be considered; but as there is reason (as already explained) to distrust the observations of August 29, I have added a second mean from which the distrusted elements are excluded. The deflection (d) thus arrived at is equal to D tan 2ϕ, whereas what we require is 2D tan ϕ. The connexion between the two quantities is obtained in a moment from the formula

$$2\tan\phi=\tan 2\phi\,(1-\tan^2\phi)$$

by successive approximation. Thus

$$2\tan\phi=\tan 2\phi\,\{1-\tfrac{1}{4}\tan^2 2\phi+\tfrac{1}{8}\tan^4 2\phi\},$$

or

$$2\mathrm{D}\tan\phi=d-\tfrac{1}{4}\frac{d^3}{\mathrm{D}^2}+\tfrac{1}{8}\frac{d^5}{\mathrm{D}^4}.$$

Column X. gives the value of 2D tan ϕ, XI. that of 2D $(\tan\phi-\tan\phi_0)$ in the notation of p. 675, and XIII. that of log $(\tan\phi-\tan\phi_0)$.

For the further calculation we require the value of ω. If f be the frequency of vibration of the electrically maintained fork, F that of the standard at 17°, N the number of teeth,

$$\omega=\frac{4\pi f}{\mathrm{N}},$$

and when the number of beats is 59 per minute,

$$f=\tfrac{1}{2}(\mathrm{F}-\tfrac{59}{60}).$$

TABLE II.—Second series.

Date, August, 1881.	Open contact deflections.	Closed contact deflections.	Correction to middle of bridge wire.	Correction to 16° of standard.	Correction to 59 beats.	Correction to 17° of fork.	Closed contact deflections, corrected to standard resistance and speed.	Correction for scale reading $-d^2/4D^2$ $+d^4/8D^4$	Closed contact deflections corrected for scale reading.	X. *minus* II.		Logarithm of $\tan\phi - \tan\phi_0$.
	II.	III.	IV.	V.	VI.	VII.	VIII.	IX.	X.	XI.	XII.	XIII.
						60 TEETH.						
15	3·97	228·60	−·15	+·01	·00	·00	228·46	[− ·42			D=2669·0 millims.	
23	3·87	228·68	−·33	−·01	·00	·00	228·34	+ ·00				
26	3·95	228·41	+·04	+·01	−·03	·00	228·43	=]				
Mean	3·93	..	..	..	..	..	228·41	− ·42	227·99	224·06		$\bar{2}$·62298
						45 TEETH.						
19	5·25	302·22	−·33	·00	·00	−·01	301·88	[− ·97			D=2669·0 millims.	
24	5·27	302·31	−·34	+·01	−·02	−·01	301·95	+ ·01				
27	5·21	302·48	−·50	·00	−·04	−·01	301·93	=]				
Mean	5·24	..	..	..	..	..	301·92	− ·96	300·96	295·72		$\bar{2}$·74350
						35 TEETH.						
15	6·68	383·41	+·22	+·03	·00	·00	383·66	[−1·98			D=2669·0 millims.	
24	6·82	383·90	−·18	+·02	+·02	−·01	383·75	+ ·02				
29	6·81	383·53	−·18	−·01	−·05	−·02	[383·27]	=]				
Mean	6·77	..	..	..	..	..	383·56 or 383·70	−1·96 −1·96	381·60 or 381·74	374·83 or 374·97		$\bar{2}$·84645 or $\bar{2}$·84662
						30 TEETH.						
19	7·86	442·79	−·01	+·02	·00	·00	442·80	[−3·04			D=2669·0 millims.	
26	7·80	442·73	−·18	+·01	−·03	·00	442·53	+ ·04				
29	7·81	441·85	+·20	+·02	−·03	−·01	[442·03]	=]				
Mean	7·82	..	..	..	..	..	442·45 or 442·66	−3·00 −3·00	439·45 or 439·66	431·63 or 431·84		$\bar{2}$·90773 or $\bar{2}$·90794

For F at 17° we take 128·130 (see p. 137 of former paper), so that

$$f=63\cdot574.$$

Thus

$$\log(2\pi f.\mathfrak{G}\mathfrak{K})=10\cdot77832=\log(10^{10}\times6\cdot0023),$$

and

$$R=\frac{10^{10}\times6\cdot0023}{N(\tan\phi-\tan\phi_0)}\left[\tfrac{1}{2}(1+\cdot00422\sec\phi)+\surd\{\tfrac{1}{4}(1+\cdot00422\sec\phi)^2-U(\tan\phi-\tan\phi_0)^2\}\right]$$

in which

$$\log U=\cdot84325$$

TABLE III.—Second series.

	Number of teeth.			
	60.	45.	35.	30.
R by preceding formula in ohms. .	23·639	23·655	23·660 23·651	23·670 23·659
Resistance of standard at 16°. . .	23·642	23·658	23·663 23·654	23·673 23·662
Resistance of standard at 13°. . .	23·621	23·637	23·642 23·633	23·652 23·641

The immediate result of the formula is the resistance in absolute measure of the revolving circuit, on the supposition that with the connecting bars it exactly balances the standard at 16°. The resistance of the standard itself is therefore given by addition of the resistance of the bars, *i.e.*, ·003 ohm. In the last line the results are reduced to the temperature of 13° for comparison with the third series.

TABLE IV.—Third series.

Date, 1881.	Open contact deflections.	Closed contact deflections.	Correction to middle of bridge wire.	Correction to 13° of standard.	Correction to 59 beats.	Correction to 17° of fork.	Correction to D=2667·7.	Closed contact deflections corrected.	Correction for scale reading as in Series II.	Correction for curvature of scale.	Closed contact deflections corrected for scale.	XII. *minus* II.		Logarithm of tan ϕ − tan ϕ_0.
I.	II.	III.	IV.	V.	VI.	VII.	VIII.	IX.	X.	XI.	XII.	XIII.	XIV.	XV.
60 TEETH.														
Oct. 22	3·92	228·47	+·19	−·06	−·06	−·07	..	228·47						
Oct. 29	3·95	228·74	−·08	−·10	−·06	−·08	..	228·42						
Nov. 7	3·92	228·21	+·27	+·13	−·06	−·01	−·05	228·49						
Mean	3·93	..	..	..	..	..	..	228·46	−·42	+·05	228·09	224·16	D=2667·7 millims.	$\bar{2}$·62339
45 TEETH.														
Oct. 29	5·20	302·93	−·37	−·20	−·08	−·11	..	302·17						
Oct. 31	5·29	302·56	+·03	−·27	−·10	−·12	..	302·10						
Nov. 5	5·22	301·84	+·31	+·17	−·11	−·02	−·07	302·12						
Mean	5·24	..	..	..	..	..	..	302·13	−·96	+·08	301·25	296·01	D=2667·7 millims.	$\bar{2}$·74414
35 TEETH.														
Oct. 22	6·77	383·49	+·64	−·16	−·05	−·12	..	383·80						
Oct. 31	6·77	383·92	+·39	−·33	−·10	−·15	..	383·73						
Nov. 7	6·73	383·48	+·29	+·18	−·10	−·03	−·09	383·73						
Mean	6·76	..	..	..	..	..	..	383·75	−1·96	+·11	381·90	375·14	D=2667·7 millims.	$\bar{2}$·84702
30 TEETH.														
Oct. 29	7·92	443·64	−·25	−·26	−·12	−·17	..	442·84						
Nov. 5	..	442·31	+·54	+·22	−·17	−·03	−·10	442·77						
Nov. 7	7·90	..	..	..	..	..	..	..						
Nov. 10	7·94	442·74	+·16	+·19	−·11	−·04	−·10	442·84						
Mean	7·92	..	..	..	..	..	..	442·82	−3·00	+·11	439·93	432·01	D=2667·7 millims.	$\bar{2}$·90830

TABLE V.—Third series.

	Number of teeth.			
	60	45	35	30
R by formula.	23·616	23·618	23·627	23·635
Resistance of standard at 13° . .	23·619	23·621	23·630	23·638

If we compare the results of the second and third series at the same speed, we find the agreement satisfactory (with a partial exception at the speed corresponding to 45 teeth), especially if we take the means from which the observations of August 29 in the second series are excluded. Adding together all the results of each series we should obtain from the second series 23·638, or with exclusion of August 29, 23·633, and from the third series 23·627, between which the extreme difference is less than one part in 2000. When, however, we compare the values obtained from observations at different speeds, we see from both series, but more especially from the third, evident signs of a tendency to rise with the speed, as if the self-induction of the revolving circuit had been underestimated. In view of the remarkable concordance of the results obtained at the same speed on different nights, it is impossible to attribute these discrepancies to errors of observation, and it is important to consider what cause of systematic disturbance can have remained unallowed for. The first question which presents itself is whether it is possible to admit an error in the adopted value of L sufficient to explain the progression. The proportional correction for self-induction is approximately $-\mathrm{U}\tan^2\phi$, or for the speed of 30 teeth ·0457. For the speed of 60 teeth the correction will be only one-fourth of this. To bring the results for the two speeds into agreement it would be necessary to increase the value of U by nearly 3 per cent., which would correspond to an increase of about one per cent. in L. It is difficult to believe that the value of L adopted for the wire circuit can be in error to this extent.

Another direction in which an explanation might be looked for would be the influence of air disturbance, or from tremor. The accompanying table, however, shows such an extraordinary agreement of the open contact deflections, both among themselves and with numbers proportional to the speeds of rotation, as to prevent us from supposing that this cause of disturbance can have operated.

TABLE VI.—Deflections with wire circuit open.

	Number of teeth.			
	60	45	35	30
Mean of first series	3·89	5·22	6·74	7·85
„ second series . . .	3·93	5·24	6·77	7·82
„ third series	3·93	5·24	6·76	7·92
After the wire had been removed, December 7 . . .	..	5·23	6·76	..
Numbers proportional to speed	3·94	5·25	6·76	7·88

On the whole, it would appear to be the most probable explanation that there were currents in the ring flowing in circuits not conjugate to the wire circuit, and therefore influencing the induction phenomena. But whatever view we may take on this matter, there is no reason to doubt that the true value will be obtained by introducing such a correction proportional to the square of the speed as will harmonise the several results, a course equivalent to determining the coefficient of self-induction from the spinnings themselves. In this way the numbers corresponding to any two speeds may be made arbitrarily to agree, but the numbers for the two remaining speeds will afford a test of the admissibility of this procedure. The only hypothesis upon which the simple mean of the numbers already obtained for the various speeds should be taken as final would appear to be one that would attribute to the discrepancies an accidental character, and seems quite out of the question.

The simplest way to carry out the correction will be to determine the amount of the coefficient from the two extreme speeds. The squares of the speeds are as

$$1 : \tfrac{16}{9} : \tfrac{144}{49} : 4 ;$$

so that the difference of the numbers for the two extreme speeds, 23·638—23·619, *i.e.*, ·019, is three times the quantity by which the lowest is to be reduced. We are accordingly to subtract respectively

$$\cdot 0063,\ \tfrac{16}{9}\times\cdot 0063,\ \tfrac{144}{49}\times\cdot 0063,\ 4\times\cdot 0063$$

with the following results.

TABLE VII.—Third series.

	Number of teeth.				Mean.
	60.	45.	35.	30.	
Resistance of standard at 13°, uncorrected	23·619	23·621	23·630	23·638	23·627
Correction proportional to square of speed	·006	·011	·018	·025	..
Resistance of standard at 13°, corrected .	23·613	23·610	23·612	23·613	23·612

It will be seen that the agreement is practically perfect, the coefficient given by the extreme speeds suiting also the requirements of the intermediate speeds. The maximum difference corresponds to about $\frac{3}{100}$ths of a millimetre only in the deflections of the principal magnetometer. The number $23·612 \times 10^9$ is therefore to be regarded as the resistance in absolute C.G.S. measure of the platinum-silver standard at 13°. If, however, the correction be rejected, the result will be different by decidedly less than one part in a thousand.

Although the experiments of the second series will not bear comparison with those of the third, it may be well to mention that they lead to substantially the same conclusion. The simple mean (taken with exclusion of August 29) of all the values is 23·633, and after introduction of the correction proportional to the square of the speed, 23·618.

The results of the first series of spinnings are given in Table VIII. They have been reduced by Dr. SCHUSTER, so as to show the value of the platinum-silver standard in absolute measure from the observations of each night at each speed. The mean radius of the coil was taken from the first measurements, and a somewhat higher value of U was employed than the subsequent calculation of the ring currents seemed to justify.

TABLE VIII.—First series.

Teeth.	Resistance in absolute measure of standard at 13°.				
80	23·651,	23·632,	23·628	..	Mean 23·637
60	23·646,	23·629,	23·601	..	Mean 23·625
45	23·678,	23·691,	23·686	..	Mean 23·685
35	23·608,	23·615,	23·632,	23·665	Mean 23·630
30	23·644,	23·639,	23·628	..	Mean 23·637
					23·643

Comparison with the standard B.A. units.

Four distinct sets of comparisons between the platinum-silver standard and the ultimate B.A. units have been effected in the course of these investigations, and two distinct methods have been followed. In the first method two coils of about five units, called for brevity [5]'s, were compared separately with five standard units combined in series with mercury cups. Secondly, the two [5]'s in series were compared with a [10]. Thirdly, the [10], the two [5]'s, and four singles were combined in series and compared with the platinum-silver standard [24]. The differences in every case were expressed in divisions of the wire of FLEMING'S bridge, whose value in terms of the B.A. unit is known. This method is simple enough in principle, but the arrangement of so many mercury connexions is troublesome, and the calculation of the innumerable temperature corrections necessary is tedious. The labour would have been greater still had we not been able to avail ourselves of the previous work of Professor FLEMING, who had carefully compared the various standard units, and had drawn up a chart on which is exhibited the comparative resistances of the coils over a considerable range of temperature. The mean B.A. unit, in terms of which our results are expressed, was defined by him, but the differences between the single standards is scarcely of importance for our purpose. In calculating the temperature corrections for the two [5]'s, the [10], and the [24], which were all of platinum-silver wire, the co-efficient ·0003 per degree has been used. The temperatures were those of the water in which the coils were immersed. They never differed much from the temperature of the room, and were referred to a Kew standard. The results of three comparisons, executed by Mrs. SIDGWICK, are as follows:—

Resistance in mean B.A. units of platinum-silver standard at 13°.

July, 1881	23·9326
September, 1881	23·9341
November, 1881	23·9348

In February, 1882, a fourth determination was executed by myself, in which a different method was employed. Five coils approximately equal to each other and to five units were arranged in a closed case upon a tube of brass. The ten copper terminals emerged below from the ebonite bottom of the case, and rested in mercury cups upon an ebonite base-board, which was so arranged that by a single motion the terminals could be transferred from one set of cups which combined the coils in series to another set which combined them in multiple arc.* In this way resistances are obtained in the proportion of 25 : 1, independently of any exact equality between the single coils; for it is obvious that if the resistance in series is given, the resistance in multiple arc is a maximum in the case of equality, and therefore varies little, even if the equality be not exact. By the aid of this apparatus the [24] was compared with a standard unit, without the assistance of other coils. In the first place [24]+[1] was compared with the five coils in series, and in the second place the [1] was compared with the five coils in multiple arc. The only precaution necessary is to effect the second comparison so quickly after the first that the five coils have no time to change their temperature. Two determinations by this method on different days gave as the resistance of [24] at 13°

23·9350, 23·9358—mean, 23·9354.

It would seem not impossible that the resistance of [24] has gradually increased, but the changes are unimportant. We will take as the resistance with which the absolute measurement is to be combined, that found in November 23·9348; so that

$$23\cdot9348 \text{ B.A. units} = 23\cdot612\times10^9 \text{ C.G.S.} = 23\cdot612\,\frac{\text{earth quadrant}}{\text{second}}$$

Hence, as the result of the investigation, we conclude that

$$1 \text{ B.A. unit} = \cdot98651\,\frac{\text{earth quadrant}}{\text{second.}}$$

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* I believe that Professor ROWLAND has used a contrivance of this sort.

XIV. *On the Comparative Structure of the Brain in Rodents.*

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[PLATES 49, 50.]

IN a former memoir published in the Philosophical Transactions* I have detailed the result of researches into the minute anatomy of the cortex cerebri in the Pig as contrasted with that of the Sheep, the Cat, and other animals. In continuation of the same series of researches it is proposed to consider here the results obtained by a still more extended enquiry into the structure of the brain in the Rodentia, and with this object in view, I shall divide my subject into the two lines of investigation followed out:—first, *the histology of the entire cortical envelope*, and secondly, *the central projections of the olfactory organ.* It is necessary that such an account be preceded by a short sketch of the external conformation of the brain in Rodents, and that its topography should be mapped out under the more recent and accurate nomenclature introduced by the late Professor BROCA, whose work upon the comparative anatomy of the convolutions in Mammalian brains† is by far the most philosophical and trustworthy treatise which has appeared on this subject since the publications of TURNER,‡ BISCHOFF,§ and ECKER.‖ Since the cerebrum of the Rabbit affords the type for the brain of the Rodent, and since this animal, together with the Rat, was chiefly utilised in these investigations, my sketch will be limited to the brains of these two animals.

EXTERNAL CONFORMATION OF BRAIN IN THE RAT AND RABBIT.

When investigating the structure of the brain in the Pig and Sheep I had to deal with a highly developed olfactory apparatus, associated with a richly convoluted cortex. In the brain of the small Rodents now under consideration there is also a notable development of the olfactory organ and its central connexions, but on the other hand the great extra limbic or parietal mass is wholly devoid of gyri. The Mammals possessing such smooth, non-convoluted brains have been grouped in an artificial class by OWEN, who terms them *Lissencéphales;* the animals possessing convoluted brains are termed *Gyrencéphales.* BROCA, on the other hand, has adopted the terms Osmatic* and Anosmatic Mammals,† as indicative of a highly-developed olfactory apparatus in the one, as contrasted with the defective, rudimentary, or wholly absent olfactory lobes of the other class. Both these terms are extremely useful in such investigations as we at present are concerned with, and I shall therefore freely adopt them, together with the nomenclature of the convolutions advocated by BROCA. From what has been stated it is obvious that Osmatic Mammals may have smooth or convoluted brains, but the Rat and Rabbit present us with typical brains of the Osmatic Lissencéphales. The cerebrum in these animals is mapped out into two grand divisions :—

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* "Researches on the Comparative Structure of the Cortex Cerebri," Phil. Trans., Part I., 1880.

† "Anatomie Comparée des Circonvolutions," par M. PAUL BROCA. Revue d'Anthropologie.

‡ "Convolutions of the Human Cerebrum Topographically considered." W. TURNER, Edin. Med. Journ., 1866.

§ "Die Grosshirnwindungen des Menschen." Dr. Th. L. W. BISCHOFF. München, 1868.

‖ "The Convolutions of the Human Brain." Dr. ALEX. ECKER. Translated by J. C. GALTON.

1. A median portion, encircling the basal ganglia and peduncular mass as it leaves the cerebrum, forming the great limbic lobe.
2. The more exposed hemispheric mass forming the outer and upper aspect of the brain corresponding to the parietal lobe of higher animals, and which I shall term, after BROCA, the extra-limbic mass.

The former, or great limbic lobe, is divisible into three portions—

a. An upper arc corresponding to what has been called the convolution of the corpus callosum (gyrus fornicatus).
b. A lower arc, also called the gyrus hippocampi.
c. An anterior arc, formed by the olfactory lobe, which unites the upper and lower limbic arcs.

Looked at from above, the brain of the Rat is heart-shaped, with the apex directed forwards. The hemispheres diverging behind reveal a minute, semi-translucent, pale-grey body, ovoid in form, projecting betwixt them, the pineal body, posterior to which the corpora quadrigemina appear. The surface of the extra-limbic mass is almost perfectly smooth, being marked only by a few extremely delicate venules. There is no distinct frontal, occipital, or temporal lobe differentiated from this extra-limbic mass, which must be regarded as entirely constituting a parietal lobe. We therefore find no trace of a fissure of ROLANDO, as is distinctly seen in Osmatic Gyrencéphales (*e.g.*, Pig and Sheep).‡ We shall, however, retain the terms frontal and occipital poles, as indicative of the anterior and posterior extremity of the hemisphere, it being understood that these terms do not imply the existence of a frontal or occipital lobe. The inner margin of the hemispheres bordering upon the great longitudinal fissure may also be conveniently spoken of as the sagittal border. Along this border, in the hinder half of the hemisphere, there is seen a strip of cortex, much paler than the remaining portion of the hemisphere, about 2 mm. wide behind, but becoming gradually narrower as it extends forwards. This pale strip is not marked off by any linear depression, yet it forms an important region characterised by a peculiar type of cortex. In the Rabbit, on the other hand, the same sagittal region is mapped off from the extra-limbic mass outside it by a constant though shallow depression (Plate 49, fig. 1, K K), the representative of the primary parietal sulcus of higher animals, and which, as in the Pig and Sheep, separates the sagittal from the Sylvian gyri of the parietal lobe. Moreover, the upper aspect of the brain in the Rabbit is of more pyriform contour, and remarkably attenuated in front, so that the pointed frontal poles contrast strangely with the broader frontal extremities of certain animals included in the same category (fig. 8, Beaver, *op. cit.*, BROCA.). The hemispheres of a four-months' old Rabbit measured 28 mm. long, and weighed (together with corpora quadrigemina) just six grammes. The extra-limbic lobe is traversed by three or four delicate channels for blood-vessels, which course upwards and backwards from the limbic fissure, and very slightly indent its surface. At the widest portion the cerebrum has a diameter of 30 mm., whilst in the Rat it is but 15 mm. across. The brain of the Rat displays the same cordate outline at its base, the hemispheres widely separated behind by the descent of the cerebral peduncles. The lower limbic arc forms so prominent a feature as *wholly to conceal from view* the lateral portions of the extra-limbic lobe (Plate 49, fig. 2, N). The base of this cordate area is therefore formed by the gyrus hippocampi, in front of which extends the olfactory lobe. Along the course of each olfactory lobe the superficial olfactory medulla (external root) runs back, rapidly narrowing as it approaches the gyrus hippocampi, in which it loses itself (Plate 49, fig. 2, M). Betwixt the two superficial olfactory bands, and in front of the optic commissure, lie two pyriform grey areas, the olfactory field of GRATIOLET (Plate 49, fig. 2, P). They are bounded externally by the fasciculus just named, and lie in contact with one another on the inner side, the optic nerves being here placed superficial to them. The optic nerves terminate in a well-marked optic commissure (Plate 49, fig. 2, R), from which the optic tracts diverge, and are lost beneath the gyrus hippocampi. The tuber cinerum and infundibulum forms a prominent mass of grey matter behind the optic commissure (Plate 49, fig. 2, L), and are crossed behind the optic tracts by a distinct white band of fibres extending across the angular interval left betwixt the diverging tracts. Stellate pigment cells of a jet-black hue accumulate more or less thickly along the whole lower limbic arc and olfactory lobe. They are especially frequent over the basal aspect of the olfactory lobe and along the limbic fissure, back even as far as the posterior extremity of the latter.

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* Includes the large majority of Mammalian brains.

† The Cetacea, Amphibious Carnivora, and Primates.

‡ Vide Plate 9, fig. 2, *op. cit.*

Upon separating the olfactory lobe from the frontal end of the hemisphere and reflecting it backwards, we become aware of the fact that the frontal extremity is obliquely truncated, sweeping forwards and upwards to the frontal pole, and thus forming an oblique depression in which lies the opposed olfactory lobe. Into this part of the hemisphere runs the upper or fourth olfactory fasciculus. If we direct our attention to the base of the brain in the Rabbit, on the other hand, we observe that the great limbic lobe does not, as in the Rat, conceal by its extension outwards the extra-limbic portion of the hemisphere, a large portion of which projects on either side of the well-marked limbic fissure. The olfactory lobe runs directly forwards beneath the frontal extremity of the hemisphere, its external or superficial fasciculus coursing backwards parallel to, and 2 mm. distant from, the limbic fissure (Plate 49, fig. 1, M), until at the gyrus hippocampi it curves inwards and is lost to view. On the median aspect of the Rabbit's brain the upper limbic arc is deep anteriorly, but narrowed behind, and curving down behind the corpus callosum, forms a deep fossa, here overlapped by the occipital pole in which lies the quadrigeminal bodies. Below this it becomes continuous with the gyrus hippocampi or lower limbic arc (Plate 49, fig. 1, B). Along the upper arc two delicate furrows are seen, the representatives of the sub-frontal and sub-parietal segments of the limbic fissure (Plate 49, fig. 1, A). In the Rat, however, there is no trace of any such furrows, the upper arc being here directly continuous with the sagittal portion of the parietal lobe. In both animals the Sylvian fissure is nearly absent, being indicated by a very slight depression betwixt the gyrus hippocampi and the olfactory region (Plate 49, figs. 1, 2, S).

Section I.—THE CEREBRAL CORTEX.

The complete cortical envelope of the cerebral hemispheres in the animals under consideration (and this with special application to the Rabbit and the Rat) may be readily mapped out into six distinct regions characterised by a notable diversity in their laminar constitution (Plate 49, fig. 1). These regions, abruptly marked off from each other by their topographical relationships as well as their intimate structure, are the upper limbic arc, the lower and anterior limbic arcs, the extra-limbic lobe, the olfactory bulb, and the cornu ammonis; but whereas the two former of these regions are subject to further modifications of their textural laminations—modifications which are not simply those of *degree*, but of *kind*—it results that I must extend my classification of the varied cortical realms beyond the limits mapped out by former authorities. Thus MEYNERT distinguishes five *types* of cortical lamination as follows:—

1. A common type.
2. Occipital type.
3. Sylvian type.
4. Type of cornu ammonis.
5. Type of olfactory bulb.

He also casually refers to a peculiar constitution of the olfactory area, but enters into no detailed account of the same, stating that it still awaits a monographical exposition.*

Since, in accordance with the rule which I find it essential to adopt as to what is to be understood by a divergence in the *laminar type* of the cortex, I should exclude from his list the third or Sylvian, the diversities of structural type embraced in MEYNERT'S category will be four only. This, however, but very inadequately expresses the truth as regards the cortex of the Mammalian brain, for I find at least *eight characteristic cortical areas* distinguished by their laminar structure. These we may enumerate as follows :—

1. Type of upper limbic arc.
2. Modified upper limbic type.
3. Outer olfactory type.
4. Inner olfactory type. } Comprised within the limits of the lower and
5. Modified olfactory type. } anterior limbic arcs.
6. Extra-limbic type.
7. Type of cornu ammonis.
8. Type of olfactory bulb.

In enumerating these types I have distinguished them by terms having special reference to their regional distribution, not that I believe this to be the better method ultimately to adopt, but the simplest in our present state of defective knowledge upon these points. With advancing knowledge of the intimate structure and peculiarities of the various regions of the cerebral cortex it will be found advisable to term these varied types after some important and essential structural peculiarity. The same remark applies to the enumeration of the individual cortical layers which are usually denominated by their relative positions as first, second, or third layers, &c. This method cannot fail to mislead and introduce much confusion, since we are constantly meeting with intercalation of fresh layers or the absence of others, and as a result of this method the same denomination becomes applied to layers which are wholly different in constitution. It is, therefore, on this account necessary to term each layer after some characteristic structural feature, more especially one which finds its counterpart in all the various members of the Mammalian series, and to subordinate the numbering of layers to the expression of their relative depth.

Upper Limbic Arc (Plate 49, fig. 1).

Adopting the above considerations, I find it necessary to regard the cortex typical of the upper limbic arc as consisting of four well differentiated layers, which may be termed respectively—

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* "The Brain of Mammals," STRICKER'S 'Human and Comp. Histology,' Sydenham Soc. Trans., vol. 2, p. 414.

1. Peripheral cortical zone.
2. Small pyramidal layer.
3. Ganglionic layer.
4. Spindle cell layer.

1. *Peripheral cortical zone.*—As seen by the naked eye this is a uniformly light grey belt sharply defined from the darker layers beneath. Its structure, revealed by microscopic aid, does not differ from that of higher animals, being constituted of a fine neuroglia matrix derived from the delicate prolongations of connective cells supporting the meshwork formed by the repeated sub-divisions of the apical processes of nerve-cells in the underlying strata. We meet here, as in the Sheep, Pig, and other animals, with the DEITER cell. These cells are found immediately beneath the pia-mater, their processes extending downwards into this layer and *a larger process invariably being connected* with a nucleus along the sheath of a blood-vessel. As regards depth there is a very evident diminution of this layer upwards and outwards over the exposed aspect of the hemisphere as well as backwards towards the occipital pole. Thus in regions of this upper limbic arc anterior to the corpus callosum the depth is ·511 mm., whilst posterior to this commissure it scarcely attains to ·377 mm. A still more notable diminution in depth occurs outwards, viz.: in extra-limbic regions where this outer cortical zone measures but ·279 mm. in depth. In the cortex of the Pig's brain a similar fact was noted and is recorded in the table of depths of the cortical layers given in my former paper.*

2. *Small pyramidal layer.*—In the upper limbic arc of the brain of Man and of some of the higher Mammalia I have described the typical formation as that of a five-laminated cortex. In the same region, however, of the Rabbit's brain the cortex is four-laminated, and this arises from the absence of the small oval and angular elements which in Man constitute the second layer. In certain localities, more especially extra-limbic in position, a shadowing forth of this stratum is represented by the closer aggregation of elements superficially which become also appressed in small clumps at intervals and may possibly represent a rudimentary form of this layer. So little marked is this in the upper limbic arc that it may virtually be considered devoid of this layer. The second layer, then, is in this region constituted by a stratum of small pyramidal nerve-cells as well as a few scattered angular cells. In the size of these elements, in their mode of distribution, in the depth of the stratum and its relationship to subjacent layers a very close resemblance is borne to the upper half of the third layer of higher animals. In the pyramidal cells of the human cortex a notable increase in their dimensions occurs with their depth so that at the lowest level of this series in Man I find cells whose proportions, though not usually so great as the cells of the still lower ganglionic layer, still render them giants beside their pigmy representatives higher up. This increase in size is *gradually* attained.† Now this feature is wholly absent in the small

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* *Op. cit.*, p. 62.

† "The Cortical Lamination of the Motor Area of the Brain." Proceedings Roy. Soc., No. 185, 1878.

pyramidal layer of the Rabbit—the cells remain small to their lowest level. In form, these cells are usually pyramidal or pyriform, whilst the small irregular, angular cell is met with chiefly on the outskirts of this layer where it joins the outer cortical zone. The apex process of these cells divides and subdivides into a delicate network of fibrils, those which are deeper seated undergoing the primary division at some distance from the cell, those nearer the cortical zone undergoing instant division, so that instead of a single elongated apex process the apex of the cell itself is bifid or horned. A similar feature pertains to the subjacent layer, the apex process of which may be traced often into the outer cortical zone ere primary division occurs. It therefore appears probable that whilst the greater bulk of nerve-fibre element of the lower four or five layers is formed by the numerous secondary processes of the nerve-cell,* the outer cortical zone derives its nervous element chiefly from the apex process of cells of underlying strata. The fact that the bifurcate or horned cell is met with at the commencement of this layer in sparse detached clumps has been alluded to above as significant probably of a rudimentary form of the second layer of higher animals. Now it has been shown that this latter stratum is also extremely defective and in great part entirely absent in the cortex of the Sheep; in the Pig it is also but a shallow belt averaging ·093 mm. deep, increasing in certain regions to ·139 mm., or even ·186 mm., whilst in Man its greatest development is fully ·279 mm. in depth. It appears clear, therefore, that the development of this belt bears an important relation to the position of the animal in the scale of organisation. The average size of these small pyramids in the Rabbit is 13μ long by 9μ broad, with a nucleus of 8μ in length—the largest do not exceed $17\mu \times 12\mu$. This closely approximates to the upper series of the third layer in Man, which in the ascending frontal gyrus measure $12\mu \times 8\mu$, and in the ascending parietal gyrus $15\mu \times 11\mu$.

This second layer attains its greatest depth in that portion of the upper limbic arc lying in front of the corpus callosum, gradually diminishing in depth backwards, so that behind the corpus callosum it is but ·372 mm. as contrasted with ·883 mm., the depth of the anterior region. In the direction of the extra-limbic mass, *i.e.*, over the exposed aspect of the hemisphere, the depth of this layer is often increased to 1·023 mm., being double the depth in the limbic lobe on a corresponding plane.

3. *Ganglionic layer.*—Immediately beneath the small pyramids of the second layer at the frontal end of this upper arc we find the far larger cells of the ganglionic layer. These cells, whilst differing widely from those of the ganglionic layer in Man, approximate to them sufficiently in many features to justify us in regarding them as representatives of the same series. Thus they are the largest and most characteristic cells in the cortex; they lie immediately upon the spindle layer, with a slight pale poorly-celled zone extending above and below it as in Man; their processes are peculiarly elongated, extending right through the pyramidal series into the outer

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* By secondary processes are meant all extensions from the cell exclusive of the apex and basal or axis cylinder process.

cortical zone; and lastly, with the increase from a four to a five-laminated cortex by the addition of a small angular layer, the latter lies immediately between the pyramidal layer and this stratum of large cells, which I therefore propose to consider as identical with the ganglionic series in Man. This intercalation of an angular layer between them and the pyramids suffices to distinguish them from the large pyramids of the lower half of the third layer in human brain. Another character common to this series and that of Man and higher animals is their distribution in confluent clusters, as a deep stratum on the one hand, or on the other, as the solitary or linear arrangement.* The confluent and linear arrangement of these elements is as marked a feature here as it is in the Pig and Sheep. It has been noted that the great development of the ganglionic series in Man was found associated with the five-laminated, and not the six-laminated, cortex, and was peculiarly distributed over the upper limbic arc in its anterior half—so in the Rabbit these cells vastly predominate in the four-laminated cortex, *i.e.*, before the intercalation of an angular layer occurs, and is richly distributed over the upper limbic arc and its immediate neighbourhood. To sum up the special features whereby I identify this layer with the common ganglionic series:—

a. The large size and characteristic form of the cells.
b. The histological structure of the stratum.
c. Peculiar distribution of apex processes.
d. Relationship to the angular layer.
e. Arrangement in dense clusters or solitary cells.
f. Special development in certain areas.

It now remains to consider how they differ histologically from the same series in higher animals, and this object will be best furthered by considering in detail the general structure of the ganglionic layer as regards—1, depth; 2, form of cell; 3, size of cell.

1. *Depth of ganglionic layer.*—Great variations in depth occur throughout this layer, corresponding to a more or less rich development of cells. Upon examining sections taken vertically through the hemisphere, as at B (Plate 49, fig. 3), one cannot fail to have the attention arrested by the rich development of these cells in the upper limbic arc, and their rapid diminution and thinning out in the outer regions of the extra limbic mass. Referring to our figure of the upper limbic arc in the Rabbit (Plate 49, fig. 3), let us briefly examine the depth of this layer along the length of this arc, as also along the upper angle where it unites with the extra limbic portion. The depth attained is as follows:—

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* "The Comparative Structure of the Cortex Cerebri." BEVAN LEWIS. Phil. Trans., Part I., 1880, page 38.

	Limbic lobe. mm.	Union with extra-limbic. mm.
At A. (Plate 49, fig. 3) . .	·376–·418	·743
B. „ . .	·511	·743
C. „ . .	·232	·511
D. „ . .	·232	·372
E. „ . .	·279	·376

Hence its depth is greatest at A and B, increasing upwards and outwards towards the extra-limbic portion, and thus forming a specially rich formation in FERRIER'S region (7).* When these measurements are contrased with those of the same stratum in the Pig,† a great similarity is at once perceived. As before shown, the depth varies with the two kinds of arrangement observed by these cells, viz.: the clustered or the solitary. Thus in the Pig the clustered cells attained an average depth of ·595 mm., whilst the dense confluent series in the Rabbit average ·506 mm. The depth, however, affords but an imperfect guide to the relative richness of the stratum in cells—a better notion is gained by the fact that the quarter-inch field will reveal in the confluent series as many as 90 to 100 cells, whilst the laminar series will give to the same power not over 6 or 8 cells.

2. *Form of cell.*—The cell of this series is usually an elongate pyramid, although the oval and large fusiform cell is occasionally met with, as is also a bifurcate or horned cell; the latter, however, far less common than in the Sheep and Pig. It was stated as regards the form of these cells in the Pig, a striking uniformity in their contour was observable, "by far the greater proportion taking the form of an elongate pyramid, the few exceptions occurring being usually gigantic spindles." ‡ In this respect the ganglionic cells in the Rabbit's brain agree with those of the Pig and Sheep in not exhibiting the great irregularity in marginal conformation, which is a peculiar character of the same series of cells in Man. The group of ganglionic cells from the cortex of the Pig, figured in my last memoir, represents the typical form here.§

3. *Size of cell.*—The dimensions attained by these cells along this region from before backwards have a fair uniformity. The largest cells are found at C and D (Plate 49, fig. 3), where they measure 32μ long by 18μ broad, having a nucleus 13μ in length. Anterior to this they measure $24\mu \times 17\mu$, whilst posterior to it they diminish gradually in size to $26\mu \times 16\mu$ behind the corpus callosum. With the exception of these slight variations I may state that throughout the upper limbic arc *great uniformity in the size of the structural elements of this layer is maintained.* When examining the structure of the parietal or extra limbic portion of the hemisphere we shall find still larger cells,

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* Vide fig. 36, area 7, in brain of Rabbit. "The Functions of the Brain."

† *Op. cit.*, p. 62.

‡ *Op. cit.*, plate 7.

§ *Op. cit.*, page 43.

although they do not approach the dimensions of the same elements in the Pig. Thus in the upper limbic arc of the Pig the larger cells averaged $48\mu \times 17\mu$, the largest in the Rabbit being $32\mu \times 18\mu$, measurements suggestive not only of the greater size of these cells in the Pig, but of their far more *elongate* contour. The typically elongate cell however occurred in the Sheep, as exhibited by their measurements of $46\mu \times 11\mu$.

The above considerations lead us to conclude that whilst these cells undoubtedly represent the ganglionic series of Man and the higher Mammalia, yet in size, contour, and structural type these cells in the Rabbit lose the strong distinctive features which characterise the ganglion cells of Man, whilst on the other hand they acquire structural affinities to the *larger pyramids* in the human cortex approximating as closely to them as was the case in the Sheep and Pig; in short, the structural type of the ganglionic cell is modified, being less complex.

Regional distribution (Plate 49, fig. 3).—In describing the cortex of the Pig and Sheep it was shown that this ganglionic layer assumed the form of a nested or clustered and a laminar or linear arrangement of its cells, but that in the former case the clusters were either far apart or discrete, or tended to become confluent. In the Rabbit the true nested arrangement is not to be found, as the clusters all tend to become confluent. The characteristic formation therefore over the upper limbic arc is that of a deep and dense layer, the cells uniformly distributed throughout, and in this plan again they more closely resemble the large pyramidal cells of Man than the subjacent layer. In the portion of the upper limbic arc which bends down in front of the corpus callosum (Plate 49, fig. 3, A and B), there is a most notable development of this layer. The ganglionic belt is here deep, and consists of large cells densely congregated and commencing abruptly beneath the small pyramidal layer. Its depth and richness in cells increases upwards towards the *exposed aspect* of the hemisphere, the quarter-inch field usually showing 60 to 80 cells, so that the margin of the longitudinal fissure here has a most exceptionally rich development of this layer. This rich formation is continued outwards *over the whole of the areas numbered* 7 *and* 9 *by* FERRIER,[*] and terminates only at the limbic fissure. As in the corresponding series in higher animals, the deepest portion of this layer is pale, having very few small cells, although at wide intervals apart are found some of the largest size ganglionic cells. Again referring to the diagram, we find from C to D the decreasing depth of the layer shows a corresponding diminution in the richness of its cells, which, however, are still numerous as far as D, and increase in number toward the outer or exposed aspect of the hemisphere. Beyond the sagittal margin, however, we find that from being closely congregated, the poorest development of cells yet met with occurs, as they rapidly thin out into an almost linear series down the outer aspect of the hemisphere as far as the great limbic fissure. Betwixt them and the small pyramidal layer I met with a belt of small angular cells, so that the cortex becomes five-laminated. Beyond D, the layer becomes rapidly shallower and the cells distant and sparse, 18 to 20 cells alone being seen in the quarter-inch field in striking contrast to their abundance in the areas A and B.

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[*] "Functions of the Brain," fig. 36.

Continuing to diminish in depth and in the number of its cells, this layer becomes a very insignificant tract beyond E, being, however, always most richly developed along the sagittal angle of the hemisphere. The whole exposed or extra-limbic aspect of the hemisphere, from D to E, is distinctly five-laminated, and a typical linear arrangement of cells is maintained in the ganglionic layer—the cells being placed at wide distances apart, and all on the same plane or at a uniform depth from the surface of the cortex.

Modified Upper Limbic Type (Plate 50, fig. 4).*

It has been already inferred that the structural type of the *posterior extremity* of both upper and lower limbic arcs exhibit very important modifications. The change, as far as it affects the upper limbic arc, is as follows:—On the outer aspect of the hemisphere a shallow sulcus is seen (Plate 49, fig. 1, K) disposed parallel to the great longitudinal fissure and mapping off from the extra-limbic mass outside it, a portion of the hemisphere which in the convoluted brains of higher animals corresponds to the superior parietal or sagittal convolutions.

Now this sulcus forms an *abrupt boundary* betwixt two forms of lamination; external to it lies the cortex typical of the larger mass of the extra-limbic lobe, internal to it is a cortex distinguished by a remarkable granule formation. It has been noted that as we examine the upper limbic cortex towards the posterior extremity of the callosal commissure an intercalated series of granule and angular elements becomes interposed betwixt the small pyramidal and ganglionic layers (vide Plate 50, fig. 4). Now, these small angular cells increase rapidly in number, and approaching the surface wholly replace the second or small pyramidal layer. In lieu of the latter, therefore, we eventually find here a deep belt of densely congregated small granule-like bodies, the granule-like character being due to the large size of their nucleus compared with their investing protoplasm. They are very closely crowded at the upper limits of the layer, but less so at deeper levels, taking a position in horizontal serried rank which is an especial feature of this formation—a peculiarity due to their separation into rows by the passage betwixt them of arciform fasciculi or bundles of medullated fibres running parallel to the surface of the cortex. In this region, therefore, the cortex is still four-laminated, consisting of—

1. Outer cortical zone (Plate 50, fig. 4, A).
2. Deep belt of granule-like cells (Plate 50, fig. 4, B).
3. Ganglionic belt (Plate 50, fig. 4, C).
4. Spindle-cell layer (Plate 50, fig. 4, D).

This formation is not limited to the upper limbic arc, but stretching over on to the exposed aspect of the hemisphere, extends to the shallow parietal sulcus above described (Plate 49, fig. 1, K). If we follow this formation backwards it is found to occupy the whole remaining median aspect of the hemisphere as far back as the limbic sulcus.* From this point downwards its distribution is more and more restricted to the inner moiety of the limbic lobe until it finally dwindles into an insignificant tract to disappear at the posterior extremity of the gyrus hippocampi. The gradually narrowing tract as it comes downwards and inwards leaves an angular portion of cortex betwixt it and the limbic sulcus, whose apex corresponds to the extremity of the latter sulcus (Plate 49, fig. 1, C). In this region is included the formation alluded to as the modified lamination of the inferior limbic arc. Vertical sections through the hemisphere behind the callosal commissure when stained by aniline black exhibit to the naked eye a remarkably defined dark belt corresponding to the granule belt just described (Plate 50, fig. 4, B). This belt can be traced outwards through the whole limbic arc and is seen to *terminate abruptly at the parietal sulcus.* On the other hand, fresh unstained vertical sections exhibit clearly to hand-magnifiers two delicate white stripes, one lying in the lower half of the first layer and the other lying within the granule belt of the second layer. These are found invariably in this region of the upper limbic arc (modified limbic type), but disappear as they bend over the marginal or sagittal angle to reach the exposed aspect of the hemisphere. They are constituted by two stripes of medullated fibres and will receive further attention when we come to describe the central projections of the olfactory organ. The depth of the individual layers of this modified upper limbic type immediately behind the corpus callosum was as follows :—

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* The area covered by this formation is shaded by dots in Plate 49, figs. 1 and 3.

1. Peripheral cortical zone, ·325 mm. to ·372 mm.
2. Granule layer, ·372 mm.
3. Ganglionic layer, ·604 mm.
4. Spindle layer, ·604 mm. to ·744 mm.

The granule layer where deepest does not exceed ·418 mm., nor does this formation in any region attain the depth of the small pyramidal layer which it displaces and which latter usually measures ·604 to ·833 mm. deep. The substitution of this densely crowded, though shallow, layer for the deeper pyramidal series, together with the *horizontal* connexions rather than vertical, established by means of the various parallel arciform medullated belts, accounts for the notable shallowness of the whole cortex posteriorly as compared with anterior realms. Thus near the occipital pole the depth of the cortex is 1·737 mm., whilst in the neighbourhood of the frontal pole it is 2·577 mm. The granule cell is very small and averages $10\mu \times 8\mu$, having a nucleus measuring 4μ to 5μ. A large number, however, scarcely reach the dimensions of $9\mu \times 9\mu$. In form they are conical, the nucleus constituting their chief bulk, whilst the very limited and delicate protoplasm extends irregularly into several processes, the most notable being usually, although by no means universally, apical

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* Represented by dotted area on medium aspect of brain. Plate 49, fig. 1.

For a short distance internal to the primary parietal sulcus the upper stratum still contains a limited number of small pyramidal cells freely interspersed with these granule-like bodies, and giving this upper boundary a more densely packed appearance; but the lower portions of this second layer are constituted purely of granule cells. The latter resemble in every respect the intercalated belt of angular cells of the five-laminated realms of this animal, and, except in size, the smaller granules of the inner olfactory type and those distributed in the deep medullated structures of the olfactory area.

To sum up the chief features of the modified upper limbic cortex—

1. It is a four-laminated cortex with a dense belt of granules for its second layer.
2. The granule cell formation ends abruptly at the parietal sulcus.
3. The upper layer possesses a deep arciform medullated band.
4. The granule cells are disposed in serried rank by a similar arciform stripe of medulla.

The lower and anterior limbic arcs which next claim our attention comprise three distinct types of cortex—the outer olfactory, the inner olfactory, and the posterior or modified lower limbic.

Outer Olfactory Type (Plate 50, fig. 5).*

Vertical sections carried through the hemisphere, which pass through the septum lucidum, present us with four well-marked because structurally differentiated regions. These are, on the inner side or median aspect, the upper limbic arc; on the upper and outer aspect of the hemisphere the extra-limbic cortex, extending down as far as the limbic sulcus; beyond the sulcus, and occupying the basal aspect of the hemisphere, the outer olfactory portion of the lower limbic arc extends, and internal to this is a region continuous with the basal portion or head of the corpus striatum—the inner olfactory cortex.† The five-laminated cortex extending over the greater portion of the extra-limbic segment terminates abruptly at the limbic sulcus (Plate 49, fig. 1, C), immediately beneath which a wholly distinct formation commences, that of the outer olfactory cortex. This cortex is constituted by three layers, possessing but two belts of nerve-cells:—

1. A peripheral cortical zone (Plate 50, fig. 5, A).
2. A densely-compressed but shallow layer of small and very irregular pyramidal cells (Plate 50, fig. 5, B).
3. A scanty series of large pyramidal cells (Plate 50, fig. 5, C).

The outer peripheral zone has superimposed upon it, and also imbedded in its structure, the medullated fibres of the superficial olfactory fasciculus, which is seen in vertical sections as an oval or oblong belt, its fibres cut across transversely, and forming a notable white prominence upon the exposed surface, whilst it is also deeply imbedded so as apparently to rest in a slight sulcus of this cortical layer. The second layer of nerve-cells bends inwards in an arched form beneath it. The fibres of this superficial olfactory fasciculus (Plate 49, fig. 2, M) may be traced not only backwards towards the gyrus hippocampi, but spreading downwards into the olfactory cortex and arching outwards over its whole extent. The apex processes of the nerve-cells of the second and third layers blend with this fasciculus, whose exact connexions will be considered further on. The depth attained by this outer cortical zone varies between ·186 mm. and ·232 mm. In intimate structure it is fundamentally a supporting matrix of neuroglia, as in other regions, the nervous elements imbedded in it consisting of a dense meshwork formed by the branching of the apex processes of the nerve-cells beneath (Plate 50, fig. 5, B), and the ramifications of medullated fibres from the superficial olfactory fasciculus which are found at all depths of this layer beneath the surface. Immediately beneath the peripheral zone lies the layer of small pyramidal cells, forming but a shallow belt of densely-crowded elements, very irregular in contour, but usually pyramidal, oval, or fusiform, their apex processes bifurcating early and approaching in character very closely the small pyramidal cells of the second layer in other regions. These cells have a peculiar arrangement in clumps or appressed groups, especially in close proximity to the superficial olfactory tract. The nerve-cells vary between $18\mu \times 10\mu$ and $19\mu \times 12\mu$ in size, and possess a nucleus whose diameter is 9μ. Scattered sparsely among these smaller cells are pyramidal cells of greater dimensions, which become more frequent below the first belt of nerve-cells, forming here the third layer, through which a few really large pyramids are scattered, often measuring $31\mu \times 13\mu$, with a nucleus 13μ in diameter. The depth of the second layer is ·139 mm. to ·186 mm., that of the third or large pyramidal layer, varying from ·106 mm. (olfactory region) to ·568 mm. (gyri hippocampi). Tracing the passage of the extra-limbic formation into that of the olfactory region at the limbic sulcus, we find the layers of nerve-cells assume the laminated aspect peculiar to a sulcus. Thus the cortex becomes much shallower by the rapidly decreasing depth of the second or small pyramidal layer, and the close approach towards the latter of the ganglionic series. The second layer at the inner side of the sulcus becomes shallower and more compressed; the larger pyramidal cells become sparse, form a less distinct layer, and are wholly absent from the innermost portion of the olfactory region, so that a clear space here intervenes betwixt the small pyramidal belt and the claustral formation beneath. Thus the second layer of the lower limbic arc is directly continuous with the small pyramidal layer of the cortex of the vault, whilst the large pyramids of its third layer are continuous with the ganglionic series of cells. The claustrum is seen beneath these layers descending from the parietal region as a deep stratum of spindle cells from ·511 mm. to 1·116 mm. in depth (Plate 50, fig. 5, D). Arching beneath the limbic sulcus, and separated by medullated fasciculi from the corpus striatum, they appear to terminate superficially near the inner border of the superficial olfactory band. Here, as will be mentioned further on, they seem to communicate with a superficial arciform system. The axis of these cells is arranged parallel to the surface of the cortex.

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* Area covered by this type of cortex is shown in Plate 49, fig. 1, by cross-hatching at base.

† The two lower of these regions are shown in diagram (Plate 49, fig. 8).

Gyrus hippocampi (Plate 49, fig. 1, B). The structure of the cortex of this region is in most respects identical with that of the anterior segment of the limbic arc just described. The layers are three in number and alike in constitution, but the development of the third or large pyramidal layer is a more marked feature here. Whereas the second or small pyramidal layer was the only fully developed formation in the outer olfactory cortex, in the gyrus hippocampi the third or large pyramidal cells form a well differentiated layer, increasing in size and number towards the posterior extremity of this gyrus. Here they are identical in appearance with the pyramidal cells of the third cortical layer of human brain, showing in vertical sections six or seven branches and a long apical process. The depth of this layer also increases from before backwards.

*Inner Olfactory Type.**

Near the inner margin of the superficial olfactory fasciculus we find the commencement of a thin and peculiar cortex, which spreads over the anterior perforated space to the inner margin of the hemisphere, where, with a slight bend upon itself, it becomes continuous with the septum lucidum (Plate 49, figs. 1, 2, P). This cortex covers an area the relationship of which will be more particularly described further on, and which we shall denominate, after GRATIOLET, the olfactory area, or in contradistinction to the remaining cortex extending from it to the limbic sulcus—the inner olfactory area. The coarse or naked-eye appearances of this cortex are peculiar, as is also its histological constitution. Vertical sections carried through the hemisphere in this region will exhibit a wavy disposition of the second layer, which assumes a folded or duplicate arrangement wholly peculiar to this realm of the brain. In these V or W-shaped duplicatures the superficial layer of the cortex does not participate, so that at one time the second layer is close to the surface and at other times quite remote. The layers constituting the cortex here are three :—

1. Superficial or peripheral zone.
2. Layer of granule cells.
3. Layer of spindle cells.

1. *Peripheral zone.*—The first layer has an average depth of ·232 mm., but from the duplicatures of the subjacent layer its depth will, of course, be subject to great variation. In structure it is similar to the same layer in other regions of the cortex, differing, however, in being perforated by numerous large vessels, which pass through

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* This region is seen in diagram (Plate 49, fig. 8), also by black area at base, Plate 49, fig. 1.

the cortex in straight or arched direction, within perivascular channels, which in the fresh state attain a diameter of ·110 mm. In the deeper region of this peripheral zone I find a shallow belt of arciform medullated fibres, which originating near the inner margin of the peripheral olfactory fasciculus (apparently continuous with the claustrum) is here continued inwards over the whole of the inner olfactory area.

2. *Granule cell layer.*—These cells forming the involuted layer characterising this region attain the average dimensions of 9μ long by 6μ broad, and contain a large oval or spheroidal nucleus 6μ in longest diameter. Amongst the cells of this layer we also find an abundance of very minute granules scarcely attaining a diameter of 5μ. These latter resemble the granules of the modified upper limbic cortex, but are smaller in size than these. They are common to the cortex here and to the peculiar medullated region which extends as far as the central olfactory fasciculus and anterior commissure, and may be regarded as specially the olfactory region of the nucleus caudatus. The depth of this region amounts to 3·27 mm., whilst the granule cell formation or second layer is ·116 mm. deep.

3. *Layer of spindle cells.*—Immediately beneath the wavy layer of small angular and granule cells lies a formation of fusiform cells, the elements of which attain a notable magnitude. Whilst the majority of these cells measure $32\mu \times 13\mu$, we frequently meet with far larger elements attaining the dimensions of $60\mu \times 9\mu$, with an oval nucleus 13μ in longest diameter. These spindle cells have their long axes parallel to the surface of the cortex, and lie along the course of similarly disposed medullated fibres which bend inwards to continue beneath the same layer of the septum lucidum. This cortical stratum is therefore essentially an arcuate fasciculus of medullated fibres connected with unusually large spindle cells, which bring the cortex of the inner olfactory area in connexion with that of the septum lucidum and upper limbic arc. Beneath the cortical layers above described are the densely aggregated olfactory fasciculi, which traverse the deep structure of the olfactory area, and from which vertical prolongations descend to the deeper layers of the cortex, probably fusing with the arciform medulla.*

Modified Lower Limbic Type (Plate 50, fig. 6).†

In the anterior portions of the lower limbic arc and the olfactory lobe, it was stated that the cortex was chiefly characterised by the development of the second layer as closely appressed clusters of small pyramidal cells, and with the exception of a scanty formation of large pyramids, no other nerve-cells are present. The further back we examine the lower limbic arc the greater is the development of the larger pyramidal cells, until eventually we reach a region where the large cells are much more densely grouped. Here, however, a remarkable change has occurred in the second layer, the cells of which are no longer pyramidal but swollen, inflated, globose or flask-shaped (Plate 50, fig. 6, B). They are also greatly increased in size, measuring $37\mu \times 32\mu$, or $37\mu \times 23\mu$; whilst others are more elongated, measuring $46\mu \times 27\mu$. Their nucleus has a diameter of 13μ. Their dimensions are thus *more than double* those attained by the elements of the second layer in other regions of the brain, whilst their irregular mode of branching as a bifurcate cell gives this formation the aspect of an unusually rich development of the ordinary second layer. Scattered amongst these larger cells, and descending as a distinct layer, are small pyramidal cells measuring but $23\mu \times 13\mu$, so that in this region *the series of larger cells is placed superficial to the smaller elements*. As regards either pyramidal series in other regions of the cortex, this latter feature is never observed—the upper series being invariably composed of smaller cells. Immediately beneath this third layer of pyramidal cells is a clear white belt devoid of nerve cells, and to this succeeds a belt of spindle cells (Plate 50, fig. 6, D, E). The cortex of this modified olfactory type is therefore composed of five layers :—

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* Diagramatically represented in Plate 49, fig. 8, by *f*.

† Area covered by this form of cortex is the light shaded portion T to W in figs. 1 and 3.

1. Peripheral cortical zone (depth ·418 mm.).
2. Layer of large inflated cells (depth ·279 mm.).
3. Small pyramidal cells (depth ·372 mm.).
4. Pale belt devoid of nerve cells (depth ·184 mm.).
5. Spindle series (depth ·372 mm.).

The region embraced by this formation is that portion of cortex at the extreme posterior extremity of the lower limbic arc where the upper limbic arc unites with the gyrus hippocampi (light-shaded area from T–W, Plate 49, fig. 1). It extends as far as the upper extremity of the limbic fissure seen on the median posterior aspect of the hemisphere, being bounded externally by the limbic sulcus (C) and internally by the gradually vanishing granule formation of the modified upper limbic cortex (dotted area, Plate 49, fig. 1). Sections carried *horizontally* through the occiput at this site will therefore exhibit from without inwards four types of lamination ;—

1. Five-laminated type of extra-limbic cortex,
2. Modified olfactory type.
3. Modified upper limbic type.
4. Type of cornu ammonis.

Independent of the peculiar arrangement of cells and disposition of the layers in this region, a characteristic feature is acquired in the presence of two arciform stripes of medullated fibres, disposed as in the modified upper limbic cortex, one above the second layer and the other immediately beneath the same. They will be more minutely described in the section on the central projections of the olfactory lobe. For the present it will suffice to bear in mind the important fact that both these modified occipital regions of the great limbic lobe are characterised by the *double stripe of intracortical arciform medulla*, and by the *dense granule belt* of the one, as contrasted with the *great inflated cell formation* of the second layer in the other.

Extra-limbic mass.—Having now completed the examination of the great limbic lobe, I have next to consider the greater portion of the hemisphere which lies external to the constricting limbic arc, or that portion which in higher animals enters into the formation of a frontal, parietal, and tempo-spheroidal lobe. As in the detailed account of the various layers of the cortex in the upper four-laminated limbic arc I have anticipated many points bearing upon the structure of this extra-limbic region, it appears only necessary here to allude to the general points of divergence or similarity borne to other regions, and not to consider each individual layer minutely. The cortex, then, of this region* is five-laminated, consisting of similar layers of nerve-cells to those found in the upper limbic arc along with the interposition of a layer of small angular cells. The relative position of layers is as follows :—

1. Peripheral cortical zone.
2. Small pyramidal layer.
3. Belt of granule or angular cells.
4. Ganglionic series.
5. Spindle cells.

As regards diversity of cell formations, therefore, this is by far the richest region examined, and but a cursory glance at sections from near the occipital and frontal poles, as well as midway betwixt these points, will satisfy us that though this richly-laminated cortex has a most extensive distribution the varied layers differ much in their relative development in these different regions. Thus the small pyramidal layer is uniformly rich and deep, whilst the ganglionic series is especially developed in the anterior and median regions, and the granule and angular cells predominate in the posterior realms. Directing attention to the variations which occur in *mid-cortical regions* from within outwards, *i.e.*, from the great longitudinal fissure outwards and downwards to the lower boundary of this region, the limbic fissure, I find first, that the outer cortical zone, or first layer, *diminishes in depth*, attaining its greatest shallowness near the limbic fissure; next, the small pyramidal layer *increases in depth and richness of cells* in the same direction; thirdly, the ganglionic series tend to become distributed, after the laminar or solitary type, as we approach towards this region ;† whilst in the upper realms, near the median fissure, they retain the dense confluent aspect and depth of stratum which is likewise seen in the upper limbic arc. It will likewise be observed that the intercalated layer of angular elements commencing in the posterior half of the upper limbic arc extends over the far greater area of the extra-limbic mass, and progressively increases in richness of cells and distinctness of lamination outwards and especially backwards towards the occipital pole. As already stated, this type of lamination terminates abruptly at the limbic sulcus, the small pyramidal and ganglionic series alone being continued beyond into the lower limbic arc.

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* The five-laminated cortex of this region is the unshaded area at summit and sides, Plate 49, fig. 1.

† The solitary arrangement is seen well in section from sulcus of limbic lobe of Pig, *op. cit.*, plate 7.

Again, if our attention be directed to the variations observed from before backwards, *i.e.*, from frontal to occipital pole, over the extra-limbic realm, we first observe a progressive decrease in the depth of the outer cortical zone in the latter direction, with a decrease in size of the small pyramidal cells of the second layer. With this there is also associated a progressive increase of the granule or angular series which attains its greatest development posteriorly. On the other hand, the ganglionic series, from being rich in cells and deep as a stratum, at the frontal pole undergoes a progressive impoverishment along the regions midway betwixt the great longitudinal fissure and the limbic fissure, thinning out gradually to a laminar formation, whilst towards the extreme occipital cortex the ganglionic belt again becomes deep, with a layer of granules superimposed. From these particulars it may be gleaned that—

a. The cortex near the frontal pole in the extra-limbic realm approaches the type of the four-laminated upper limbic cortex.

b. The cortex midway betwixt frontal and occipital pole is a typical five-laminated formation, and represents, as does the six-laminated cortex of Man, the *common type*, because more extensively distributed.

c. The cortex of the extra-limbic realm at the occipital pole is a still further modified form of the five-laminated, but is by no means a distinct *kind* of cortex.

A notable diminution in size of the small pyramidal cells of the second layer occurs from the frontal to the occipital pole of the extra-limbic realm. Thus in front of the corpus callosum they average $22\mu \times 12\mu$, with a nucleus of 9μ, and progressively decrease in size until, on a level with the primary parietal sulcus, they measure $13\mu \times 11\mu$ (nucleus 8μ). Towards the occipital pole they again increase to $20\mu \times 11\mu$. The same features as regards size of nerve-cell were observed along the upper limbic arc ($17\mu \times 12\mu$ to $13\mu \times 9\mu$). On the other hand, the second layer in the *anterior and lower limbic arcs*, it will be remembered, increase in size towards the occipital pole, wholly lose their pyramidal contour, and from measuring $19\mu \times 12\mu$ become globose, and measure $37\mu \times 32\mu$.

It must be borne in mind that whilst the extra-limbic cortex appears to be developed out of the four-laminated cortex by the intercalation of a belt of granule cells, the modified upper limbic cortex is characterised by the granule cell-formation actually replacing the second layer of small pyramids, which is here either very defective or wholly absent.

Distribution of the Ganglionic Series.

Proceeding upon the same line of inquiry which was adopted in my former article upon the brain of the Pig and Sheep, it remains for me now to map out the area over which the ganglionic series is spread, and more especially that division of the series characterised by a *rich confluent and deep belt of cells* co-extensive with a four-laminated cortex. Such a formation extends over the anterior half of the upper limbic arc, and stretches upwards and outwards over the exposed aspect of the hemisphere. By far the richest formation lies in the latter position—viz.: on the exposed aspect of the vault, in a lengthened strip of territory lying parallel with the great longitudinal fissure, and represented by the shaded area in the diagram.* It extends forwards well into the frontal extremity, covers the whole outer aspect of the hemisphere here, and becoming much poorer in cells, spreads along the lower confines of the extra-limbic mass, skirting the limbic fissure as far as, and even behind, the slight depression which in this animal represents the Sylvian fissure. Between these two tracts the enclosed area is spanned by a *five-laminated cortex* whose ganglionic series is poor, and distributed in the linear or solitary arrangement. The region which may therefore be considered, *par excellence,* as that of a rich ganglionic formation, is the narrow strip of cortex along the sagittal margin of the hemisphere from A to D (Plate 49, fig. 3), extending over the outer aspect of the hemisphere betwixt A and B as far down as the limbic fissure, and next to this the upper limbic arc. The far less characteristic formation adjacent to the limbic fissure, however, is interesting chiefly from the fact that it foreshadows a richer structure, which appears along this course in higher animals. It will be convenient to name these formations the sagittal and Sylvian formations of large ganglionic cells. Thus it was shown that the five-laminated cortex, with its clustered cells, in the Pig not only extends over the anterior limbic arc and frontal lobe, but bends back over the first and second parietal gyri "from their union with the ascending parietal (post-Rolandique, BROCA) towards and around the Sylvian fissure."† In the Pig, therefore, as also in the Sheep, we have demonstrated the distribution of the rich ganglionic layer over the regions bordering upon the limbic fissure. Just as in these and other allied animals we have betwixt the two regions of the clustered ganglionic cells an intermediate district characterised by a laminar arrangement of this series and an intercalated series of angular elements, so in the Rabbit a similar cortex intervenes betwixt the inner and outer series of rich ganglionic areas. We thus conclude that the fundamental type of the structure of the cortex over the extra-limbic and upper limbic realms is identical in these various animals. The dense layer of ganglionic cells bordering upon the great longitudinal fissure maintain a fair uniformity of size throughout. A large number of measurements give the following average dimensions :—

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* Area shaded by small crosses, Plate 49, fig. 1.

† See fig. 3, p. 44, "On the Comparative Structure of the Cortex Cerebri." Phil. Trans., Part. I., 1880.

Plane of Section.	Length.	Width.	Nucleus.
	μ	μ	μ
Anterior to corpus callosum	32	16	13
On a plane with Sylvian depression . . .	34	16	13
Posterior to Sylvian depression	34	17	13
Posterior border of corpus callosum	34	16	12
Through superior parietal sulcus	24	18	12
Posterior to latter	31	19	13

These nerve-cells are therefore larger than those of the corresponding series in the upper limbic arc (vide table of measurements), also larger than those of the lower limbic arc, which average for the olfactory gyrus 25μ × 13μ; for the gyrus hippocampi 31μ × 13μ; and for the modified olfactory type near the occipital pole 23μ × 13μ. Larger cells occur occasionally, scattered widely apart, the largest registered attaining the dimensions of 60μ × 18μ. If these average dimensions be contrasted with those of the *second cortical layer*, in the region of the modified olfactory type (Plate 50, fig. 6, B), the important fact is disclosed that this peculiar formation consists of cells which attain a much higher average than that of the ganglionic cells. These globose inflated cells usually average 37μ × 32μ, and frequently reach the dimensions of 46μ × 27μ. These are therefore the largest nervous elements to be found throughout the cortex cerebri. The lower limb of the rich ganglionic tract extending from the frontal pole along the limbic fissure beyond the Sylvian depression, possesses cells of a smaller size than those of the corresponding upper tract. Thus in front of the corpus callosum the nerve-cells near the limbic fissure measure 27μ × 17μ, as contrasted with 32μ × 16μ for those of the upper tract; behind the Sylvian depression those of the lower tract measure 27μ × 17μ, whilst those of the upper tract measure 34μ × 17μ.

The rich aggregation of cells in the ganglionic layer along the median exposed portion of the vault results in a deep stratum, which near the frontal extremity measures ·558 mm., and posteriorly and internal to the superior parietal sulcus ·604 mm. If this be contrasted with the pale, solitary belt in five-laminated realms, measuring but ·186 mm., its wealth of structure will be apparent. Another fact to be noted is that this layer of ganglionic cells is both deep (·604 mm.) and rich in cells in that area of the granule formation bounded externally by the primary parietal sulcus, and covers the median surface of this posterior region of the hemisphere as far back as the summit of the occipital pole. We must therefore conclude that although the ganglionic cells (next to those near the occipital pole) are the largest throughout the cortex cerebri, the distinctive feature relative to this formation is not the large size attained by the cells, as is the case in the highest of the Mammalian series—for they exhibit in the Rabbit, as in the Pig and Sheep, a remarkable uniformity throughout; but, on the other hand, this formation is especially notable for its richness in nerve-cells—a wealth demonstrated both in depth of stratum and close aggregation of its constituents.

The Cornu Ammonis (Plate 49, fig. 7).

Peripheral cortical zone.—In the cornu ammonis this stratum may be readily seen to consist of a superficial and deeper region, which by MEYNERT and others have been regarded (in the human brain) as two distinct laminæ, representative of the first and second cortical layers elsewhere. But whereas the second layer of superficial angular cells is not developed in the Rabbit, the lower portion of this zone can scarcely be regarded as representative of a series of cells which do not appear in the most complex regions of the cortex in this animal. These two divisions have been named the nuclear and lacunar layers—the latter also termed by KUPFFER the *stratum reticulare.* On close examination of this inner or reticulated structure it becomes apparent that it clearly corresponds to the deeper portion of the peripheral zone of other cortical regions. In diverse realms of the cerebral cortex it was stated that this peripheral zone in various animals exhibited two clearly distinct portions—the outer or more superficial being often distinguished by a layer of horizontally disposed medullated fibres following the various involutions of the cortex and parallel to the surface, and extending often to a depth of one-third or even one-half that of the first layer. Now this peripheral stratum appears as the representative of the nuclear layer of the cornu ammonis. The inner or deeper portion of this zone elsewhere was seen to be composed of a dense network, formed by the apex processes of the subjacent cells dividing and subdividing in their course upwards. This was found to be the case in the Cat, Sheep, Pig, and Rabbit. In the cornu ammonis the same network is apparent, but being here entirely constituted by the apex processes of much larger cells—the ganglionic cells—it is a coarser and far more prominent formation. The whole depth of this peripheral zone is on an average ·836 mm., the outer medullated layer being about ·511 mm. The nuclear lamina, or as I prefer to term it, the medullated division of the peripheral zone, includes the following constituent elements:—

a. Band of medullated fibres derived from re-union of meshwork of the divided apical processes of ganglionic cells.

b. Numerous large blood-vessels.

c. A neuroglia matrix, containing a profusion of the ordinary nucleated connective cell; also the DEITER'S corpuscle and perivascular nuclei.

d. Spindle-form cells.

Of these constituent elements the blood-vessels are large, and dip through the medullated portion from the investing pia mater into the deeper division of the peripheral zone. Here they divide into a series of anastomosing branches, which, with the meshwork of nerve processes, gives this region its characteristic reticulated aspect.

The vessels throughout this course exhibit wide perivascular channeling, and are bordered by the usual perivascular nuclei. The connective element is similar in every respect to that found in this layer in other parts of the brain—the larger cells, measuring 9μ across, are usually nearly globular, and have a nucleus 6μ in diameter. They are very numerous, and throw off on all sides delicate fibrillar processes to constitute the neuroglia framework. The DEITER's cell is found immediately beneath the vascular investment of the cortex. Scattered through the medullated division of this cortical zone, and *especially near the confines of the reticular stratum*, are the nerve cells alluded to. They are elongate spindles, measuring 34μ × 12μ, with an oval nucleus 12μ in diameter. These cells lie parallel to the medullated fibres, throwing off branches from either pole in the direction of these fibres, but also almost invariably exhibiting a lateral process, which dips down into the cortex in the direction of the reticulated layer. These spindle cells are not numerous, and, as stated by MEYNERT, may readily be overlooked. In connexion with their presence here, it is interesting to recall similar spindle cells of great dimensions which have been pointed out as existing in certain portions of this peripheral zone in the olfactory cortex.

Striate layer.—Immediately beneath the vascular and nervous plexus constituting the reticulated portion of the peripheral zone lies the striate layer, so named after the striated aspect given by the numerous apex processes of the ganglionic layer, which radiate towards their site of common union—the nerve plexus. The depth attained by this layer averages ·558 mm., but near the commencement of the infolding of the true cornu ammonis it is much shallower, the branching of the apex processes occurring nearer to the cell. Radiating in the same direction outwards, and lying side by side with these nerve processes are numerous vessels, which rise from the ependyma of the lateral ventricles, and after supplying the medulla of the cornu, unite with the vessels of the peripheral zone (from the pia mater) through the medium of its vascular plexus. The region of these apical radiations undoubtedly corresponds to the small pyramidal layer in other realms of the cortex of the Rabbit.

Ganglionic layer.—This layer is formed by cells quite characteristic in their distribution of the cornu ammonis formation. It forms a shallow stratum varying in depth from ·093μ to ·139μ, the average depth being ·112μ. The cells vary in contour, being frequently oval, elongated, or fusiform, rarely pyramidal, and much more commonly of a swollen pegtop-like contour. At the commencement of the inrolling of the cortex to form the cornu the ganglionic cells may be clearly traced into this formation, first, as a scattered series or deep belt, in which the cells approximate closer and closer until they form a belt two to five deep, in which the cells are so closely applied to each other as to be in actual contact, and form an almost unbroken file through the greater portion of the true cornu ammonis; beyond the second turn of this sigmoid fold, however, they again become more widely separated and scattered so as to form a deeper belt. In hardened sections the shrinking separates the cells widely apart, and they may be observed surrounded by a clearly defined pericellular space. In fresh sections, on the other hand, their close approximation renders their outlines obscure except in extremely fine and carefully prepared sections. Near the commencement of the first involution where the cells are more widely scattered, these elements are observed to be encircled usually by a number of nucleated protoplasts, as are also the spindle cells, which latter have frequently such numbers heaped upon their surface as quite to conceal their real contour. These pericellular elements also tend to render the ganglionic cells of the true cornu ammonis obscure in outline. The more elongated or fusiform cell is prevalent in the lower end of the cornu ammonis or that portion bounded externally by the lower limbic arc. That portion, however, lying internal to the upper limbic arc has the characteristic pegtop-shaped cell throughout its whole extent. Each ganglionic cell besides its apex process, gives off several secondary processes from its basal extremity, and especially a primary branch from the base, which becomes an axis cylinder process. The secondary branches are directed towards the medulla and rapidly divide and subdivide within the subjacent formation (stratum moleculare). The axis cylinder process passes into the medulla of the alveus. The involuted cornu ammonis is most fully developed just posterior to the corpus callosum, and sections taken across the hemisphere at this site will therefore present us with the most extensive tracts of the ganglionic and other layers, the reticulated portion of the peripheral zone being also best studied here. The nearer we approach the inferior extremity of the cornu the more contracted is the area of section, and consequently the less extended are its nerve-cell layers, whilst the individual cells of the ganglionic series not only become more elongated in contour, but exhibit less and less of the striate formation from early subdivision of their apex processes. In the latter site, therefore, in lieu of the straight, elongated apex process of the cells becoming more and more attenuated and thus forming a forest-like structure of delicate twigs (striate layer) they commence as coarse contorted branches, which at the distance of ·046 mm. from the cell sub-divide into brush-like groups of filaments. The dimensions attained by these cells are an average length of 23μ and a width of 18μ, the nucleus measuring 13μ. At the commencement of its involution, however, where the cells are larger, more scattered, and elongated in contour, the dimensions are $37\mu \times 18\mu$ (nucleus 13μ), the apex process becoming bifurcate at from 30μ to 60μ from the cell. A few spindle cells or even pyramidal cells are found occasionally here and there at wide distances apart within the striate layer separated from the main body of cells.

Spindle-cell layer.—Beneath the ganglionic layer we come to a belt in which the neuroglia basis supports the fine fibrillar network formed by the secondary processes of the super-imposed cells. The presence of this nervous network accounts for the uniform faint staining which is here seen in aniline black preparations—a staining common to all parts of the grey cortex possessed of a network of *non-medullated* nerve fibrils and contrasted strongly with the perfectly white unstained layer of *medullated* fibres beneath this belt. Passing up from the ventricular ependyma covering the medulla of the cornu through this stratum numerous large blood-vessels are seen on their way to the capillary plexus beyond the striate region. These vessels in hardened preparations show to a marked degree the wide perivascular channels around them. The stratum which we above described has from its fine granular aspect been termed by KUPFFER *stratum moleculare*. It is wholly absent in Man, where, according to MEYNERT, the axis processes of the ganglionic cells unite immediately with the medulla. In the Rabbit, however, the spindle cell is well represented here, not only scattered widely apart through the *stratum moleculare*, but arranged in some numbers along line of union of this stratum with the deeper medulla. Numbers of spindle cells of large size are also met with at all depths of the deep medulla of the cornu lying parallel with the direction of the medullated fibres. This spindle cell medulla communicates directly with the spindle series of the limbic lobe at the commencement of the inrolling of the cornu, and lies parallel throughout its course with the involutions of this organ. Occasionally an unusually large fusiform cell will be met with immediately beneath the ganglionic series measuring as much as $60\mu \times 11\mu$, and thus rival in size the similarly large spindle cells met with beneath the cortex of the inner olfactory region along the course of the arciform fasciculi. These fusiform elements lie embedded in and continuous with the medulla of the fornix, and usually exhibit three or more processes, one from each pole of the cell following the course of the medulla and the others lateral in origin and smaller in diameter. One of the larger processes may often be traced directly up into the layer of ganglionic cells.

Significance of the Fissures and Sulci of the Brain.

The foregoing considerations as regards the intimate structure of the cerebral cortex in the Rat and Rabbit, taken together with the results of my former examination of the cortex in the Pig, Sheep, and the Cat, lead to important conclusions relative to the significance of the fissures and sulci of the brain.

Hitherto it has been customary to regard the convolutionary arrangement as dependent in a great measure upon the progressive growth and development of the cortex, restricted and modified by the encircling cranial bones, the varied folds and complex fissures being really in this sense mere accidents of development, their uniformity of distribution in different classes of animals being due to the uniform action of the forces thus brought into play. Our investigations would lead us to a far different conclusion, nor can we, judging from our histological data, regard the divisions mapped off from each other by fissures and primary sulci as other than structurally distinct organs. In no case is this so palpable as in the various regions of the great limbic lobe, for here both a superficial survey, *i.e.*, of the structure of the cortex and an investigation of the deep medullary connexions of these regions conclusively confirm the statement now advanced. In the former memoir* I insisted strongly upon the fact that certain sulci or fissures formed sharply-defined boundary lines betwixt regions differing in the structure of their cortex: these were the crucial and infra-parietal sulci, together with the fissure of ROLANDO. My later investigations enable me still further to extend this list by others, viz.: the limbic fissure, olfactory sulcus, and primary parietal sulcus. Thus in the Sheep and Pig we find that the crucial and sub-parietal sulci separates the five-laminated cortex, with its clustered ganglionic series, from the six-laminated cortex, with its solitary ganglion cells, the former lying in front and the latter behind these fissures. In like manner the inner extremity of the crucial sulcus formed the boundary betwixt the five-laminated cortex of the upper limbic arc anteriorly, and the peculiar granule formation of this arc behind the crucial sulcus, which extends as far as the retro-limbic annectant. In the Rat and Rabbit it is again observed that the limbic fissure, in separating the lower limbic arc from the parietal or extra-limbic mass, sharply defines two entirely distinct types of cortical lamination. Above the fissure lies the *five-laminated cortex* of the extra-limbic mass, with its intercalated series of granule cells; beneath it lies the *three-laminated cortex* of the outer olfactory region and gyrus hippocampi, whilst still further back there is found internal to it the remarkable modified olfactory cortex which has been described near the occipital pole. Within the great limbic lobe we find a sulcus, dividing two important and morphologically distinct regions. These regions are the outer and inner olfactory areas whose boundary line is the olfactory sulcus, wherein lies imbedded the superficial olfactory fasciculus. It was moreover shown that the superior parietal sulcus forms a sharp line of demarcation betwixt the five-laminated type of the parietal mass and the peculiar dense granule formation of the posterior or modified upper limbic cortex. In the higher animals, as was indicated in the former paper, the fissure of ROLANDO, here lying at the frontal pole, separates a five from a six-laminated cortex. We surely have here a most important fact, and one which, if followed up in further researches, will enable us to map out the convolutionary surface of the brain after a less arbitrary plan than the one usually adopted. The more fully I investigate the minute structure of the cortex and its deep connexions, the more forcibly am I impressed by the belief that the various fissures and sulci are not mere accidental productions, but have a deep significance of their own, dividing off the cortical superficies into *morphologically* if not *physiologically distinct organs*. Hitherto the fissures and sulci which I have found to be the boundary lines of distinct cortical realms are the following:—

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* *Op. cit.*, p. 45.

1. The limbic fissure.
2. The infra-parietal sulcus.
3. The crucial sulcus.
4. The superior parietal sulcus.
5. The inter-parietal sulcus.
6. The olfactory sulcus.
7. The fissure of ROLANDO.

In the lowest of the animals examined, the Rat and Rabbit, we find the *anterior lower limbic arc* already well developed, distinct from surrounding regions, and forming the far greater bulk of the great limbic lobe. We find the limbic fissure deep and complete as far back as the region where in higher animals the retro-limbic annectant unites the limbic to the occipital lobe. Within this fissure lie the three distinct cortical regions of the outer, inner, and modified olfactory types. On the other hand the *upper* limbic arc in these animals is far from being so definitely separated from the neighbouring regions of the extra-limbic mass. The anterior or four-laminated segment of this arc is, in fact, directly continuous with the exposed cortex of the vault, no fissure, sulcus, or depression however small intervening. In connexion with this fact it is, however, important to bear in mind the identical nature of the laminated type of this portion of the limbic arc and the exposed margin of the hemisphere which bounds the longitudinal fissure (sagittal). The modified limbic type of the posterior segment of this arc, however, is sharply defined from the five-laminated cortex of the extra-limbic mass by the primary parietal sulcus. In the Pig and Sheep, on the other hand, whilst the bulk and differentiation of the lower limbic arc is maintained, there is a decided advance in this respect evident in the upper limbic arc also. The anterior portion of this arc, from its size, actually overlaps so as to appear upon the upper surface of the hemisphere, whilst a most characteristic and deep sulcus, the *crucial*, separates it sharply from the complex convoluted surface of the parietal lobe.* The oblique course of this sulcus carries it inwards to the median aspect of the hemisphere, midway betwixt frontal and occipital pole, where it meets the subparietal sulcus, regarded by BROCA as the continuation upwards of the limbic fissure. This latter sulcus, sweeping backwards, does not join the limbic fissure posteriorly, being separated by the retro-limbic annectant, but as the latter annectant corresponds to the region which in the Rat and Rabbit is characterised by the modified olfactory type of cortex, this sub-parietal sulcus forms a line of demarcation betwixt the latter type of cortex and the granule formation of the upper arc. The mapping out of distinct morphological areas exhibited in the lower limbic arc of the smooth brain of the Rat and Rabbit, is a feature which in the convoluted brain of the Pig and Sheep becomes extended to the upper limbic arc also through the medium of the crucial and sub-parietal sulci. Moreover in these convoluted brains the deep inter-parietal sulcus also divides two highly characteristic formations. Still higher, as in the Carnivora, the fissure of ROLANDO constitutes a similar limitary zone, the cortex of the ascending parietal (post-Rolandique of BROCA) being six-laminated; that of the ascending frontal five-laminated.

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* *Op. cit.*

Section 2.—CENTRAL PROJECTIONS OF THE OLFACTORY ORGAN.

The Olfactory Lobe and Bulb.

The olfactory lobe is constituted by a tubular prolongation of the cerebral cortex given off from the inferior aspect of the anterior pole of the hemisphere. Within the confines of the limbic fissure its cortex is directly continuous with that of both extremities of the limbic lobe along the lower and lateral aspects of the brain, whilst above and outside the boundaries of the limbic fissure it is overlapped by the projecting frontal extremity of the hemisphere, with the cortex of which its surface is also continuous. Its continuity with the anterior extremity of the upper and lower limbic arcs thus completes the circuit made by the great limbic lobe.

The summit of the olfactory lobe is capped by the olfactory bulb, which completely ensheaths its extremity and receives the olfactory nerve fibres.

The connexions of the olfactory cortex, inclusive of the bulb, are in this animal both complicated and extensive—complicated in that its medulla brings it into relation with most diverse regions of the brain, and extensive in so far that its fibres are projected back into the occipital pole of the hemisphere, and embrace in their circuit the whole extent of the limbic arc, thus associating the most distant realms of the brain in direct organic connexion. For convenience of description as well as upon anatomical grounds, we may consider the olfactory medulla as consisting of the following distinct systems :—

1. A central decussating and commissural fasciculus.
2. A connecting system with the striate body.
3. An arciform series.
4. A superficial medullated band.

1. *Central fasciculus.*—A rapid convergence of fibres from the bulbus olfactorius constitutes the origin of a central medullated core for the olfactory lobe, which soon becoming almost cylindrical in contour courses backwards through this lobe, and arching slightly outwards plunges into the anterior extremity of the caudate nucleus. Owing to its outward curve, vertical sections through the frontal extremity exhibit an oval contour of this fasciculus, such oblique sections measuring 1·3 mm. in its greater diameter, by ·78 mm. in its shorter diameter. It is, however, a uniformly cylindrical fasciculus, its usual diameter being ·87 mm. Receiving no fresh addition of fibres, the central olfactory fasciculi of both hemispheres attain their greatest distance apart within the corpus striatum (4·76 to 5·046 mm.), and thence converge towards the descending pillar of the fornix, passing in this course slightly upwards in relation to the cortex at the base, and lying betwixt the corpus striatum on the outer side and the thickened folia of the septum lucidum internally, in close relationship to the floor of the lateral ventricles. In the Rabbit, the central canal is seen lined with endothelium,

and surrounded by the central medullary fasciculus, forming a direct communication between the olfactory lobe and the lateral ventricle. Still converging, these two fasciculi decussate in the median line immediately in front of the pillars of the fornix, constituting by this convergence the anterior commissure of the cerebrum, being in this latter course imbedded in the septum lucidum. The resultant fibres of this decussation pass out from the commissure as a compact fasciculus, and are not projected directly backwards, but arch *suddenly outwards*, passing through the substance of the corpus striatum. In this course the fasciculi of either side diverge so rapidly that they lie very nearly in one and the same line, a single vertical section through the hemispheres presenting us with the commissural band and both posterior fasciculi as far as the outer border of the striate body. To follow the further course of these posterior projections from the olfactory bulb, horizontal sections of the cerebrum of the Rat should be examined, and it will then be found that each fasciculus passes through the striate body, and upon reaching its most external border each olfactory band sub-divides into several secondary fasciculi similar in appearance to those which result from the union of the fibres of the first link of the projection system at their connexion with the receiving surface of the corpus striatum (Plate 49, fig. 9, A).

These sub-divisions of the posterior olfactory fasciculus curve directly back into the occipital pole (Plate 49, fig. 9, B). If *vertical* sections of the hemisphere at this site be examined, the connexions of the cortex with the corpus striatum by the projection system of fibres will be found to be chiefly affected by the upper surface of the ganglion, with which a large bulk of coarse fasciculi are connected (Plate 49, fig. 10, A); yet the *whole outer aspect* of the ganglion in like manner receives numerous fasciculi, which form a complete medullated investment over its outer surface. This medullated investment being augmented by fewer fibres, and yet fewer at lower levels, eventually thins off into an insignificant tract (Plate 49, fig. 10, B). Now it is by no means improbable that a certain number of minute fasciculi in the Rat pass from the sub-divisions of the posterior olfactory fasciculus upwards along this medullated tract. I have repeatedly imagined that I could trace such a connexion, and if this turns out to be correct then we have here a distribution similar to although far more restricted than what I have assured myself is the case with the Rabbit's brain. The position of this posterior olfactory fasciculus from the anterior commissure in its course through the corpus striatum is parallel to the base of the lenticular wedge, lying in fact betwixt its two main divisions, as may be clearly seen in horizontal sections taken at the proper plane, in which will also be very evident the fact, that whilst these fibres run directly backwards, the majority of the projection fibres of the corpus striatum pass directly outwards or forwards to the cortex (Plate 49, fig. 9, A–C). The olfactory fasciculus, prior to the formation of the anterior commissure, measures in its shortest diameter ·78 mm. to ·87 mm.; the fasciculus emerging from the commissure, as it leaves the septum lucidum to enter the corpus striatum, measures but ·52 mm. This reduction in bulk is due to the absence of a series of fibres which run from one olfactory bulb to that of the opposite side, as a purely commissural system. This of course is in accordance with the fact indicated by GRATIOLET, MEYNERT, and others, that in the Rodentia and Carnivora, where the olfactory apparatus is largely developed, the great bulk of fibres proceeding from the anterior commissure communicate with the olfactory lobe; whilst in Man and Apes, where its development is small, the larger portion of the commissure is distributed backwards to the occipital and to the temporo-sphenoidal lobes. The central olfactory fasciculus behind the anterior commissure exhibits a notable difference in its distribution in the Rabbit and the Rat, a divergence which has an important bearing upon the functional relationships of the olfactory lobes. In the Rat, as has just been stated, the fasciculus passes outwards through the corpus striatum, and *immediately* upon reaching its outer border divides into a brush-like head of nerve bundles, some of which probably unite with the projection fasciculi passing through them up to the vault, but by far the most extensive portion turns back, and runs towards the occipital pole of the hemisphere (Plate 49, fig. 9). In the Rabbit, on the other hand, when the posterior olfactory fasciculus reaches the outer surface of the corpus striatum it does not, as in the Rat, break up into numerous diverging bundles, but bends upwards and ascends as a more or less compact fasciculus in contact with the outer surface of the ganglion (Plate 49, fig. 11, A). Upon reaching the level of the upper pole of the striate ganglion it meets with the accumulating mass of the projection and callosal systems (C, D), and at this spot it suddenly breaks up into an extensive brush-like head of fibres (B), which pierce through the mass of the projection system, and are thence directed towards the cortex of the inner margin of the hemisphere, *i.e.*, the margin bounding the great longitudinal fissure (E). In vertical sections which have been carried successfully through the plane of the posterior commissural fasciculus, we may readily trace it—first in its horizontal course through the striate ganglion, next curving round and up the outer aspect of that ganglion, and lastly on its arrival at the upper pole, grasping, as it were, by a mass of tentacular-like extensions, the base of the projection system, decussating freely with the callosal fibres, meeting them from an opposite direction, and with the projection fibres betwixt striate body and cortex, which in part traverse the same course.* What is the destination of this important system of fibres? It appears indeed to form that extensive arcuate system which lies beneath the cortex of the summit of the hemisphere at these planes. This arcuate system may be readily seen lying betwixt the cortex of the vault and the great mass of the projection and callosal systems above the corpus striatum (Plate 49, fig. 11, F). They therefore are directed nearly at right angles to the radiating callosal fibres for the cortex of the vertex, and running parallel to the surface of the cortex, terminate in the region of the large ganglionic cells which have been referred to as seen in the cortex along the inner margin of the vault. In short, then, the olfactory bulbs of the Rabbit are thus brought into direct and decussating connexion through an extensive course with the specialised cortex of the inner

* The whole of this course is represented in Plate 49, fig. 11.

marginal aspect of the brain in its anterior regions—a region proved by **Ferrier** to have motor endowments. In the Rat, on the other hand, this fasciculus is brought into relationship not so much with the motor cortex as with the cortex of the occipital pole, with which it has most extensive connexions. No doubt need be entertained upon these facts after a careful examination of horizontal and vertical sections of the frozen brain of these animals, and by dissection and teasing out of these structures. In vertical sections I have repeatedly succeeded in tracing the posterior commissural fasciculus upwards towards the projection system above, and have teased out, by means of a dissecting needle, the whole of its course from the anterior commissure to the brush-like head, leaving it a compact band of fibres entirely separated from the surrounding structures.

The olfactory lyre.—This is a structure which I have ventured to name by what appears a very appropriate term when its configuration is thoroughly understood—it has up to the present been entirely overlooked in descriptions of the olfactory apparatus. **Meynert** distinctly states, when describing the posterior extensions from the anterior commissure in his classical work on the brain of Mammals,* that the fasciculus passes "uninterruptedly through the corpus striatum." This statement is, however, undoubtedly an error, and it requires but a careful examination of vertical sections through this commissural tract in the Rabbit or a dissection of the fresh brain of the Rat or Rabbit to show not only that such is not the case, but that **Meynert** and others must have entirely overlooked here a structure which is most peculiar and important in its bearings. Vertical sections through the fresh brain of the Rabbit show very distinctly oval fasciculi of no mean size separating the strands of the posterior olfactory band, largest and most numerous near the anterior commissure (Plate 49, fig. 11, G). They consist of divided medullated fibres, which distinctly traverse the structure of the olfactory fasciculus from the portion of the striate body lying in front to that lying behind the commissure. In passing through the olfactory fasciculus a very apparent increase in the size of this tract is occasioned by this separation of its strands, so that a great oval enlargement characterises the earlier portion of this fasciculus in the Rabbit. A dissection of the brain of the Rat will indicate still more readily the nature of the structures here exhibited. Such a dissection will be described in detail further on—let it suffice here to state that by this method we expose a series of delicate fasciculi of medullated fibres passing from the posterior olfactory band forwards parallel to the central olfactory fasciculus. In part, these fibres traverse the structure of the olfactory band at right angles to its course, passing thus into the corpus striatum behind it, whilst others attached to the anterior margin of the same band curve inwards to accompany the rest of its fibres across the commissure. In passing forwards these fibres form a delicate floor or dissepiment betwixt the structure of the corpus striatum proper and that portion of the same ganglion which forms the olfactory area. This delicate sheet of medullated strands occupies a triangular space bounded internally by the central olfactory fasciculus, behind by the posterior olfactory fasciculus, and externally and deeply by the superficial olfactory medulla, and betwixt these boundaries the fibres pass forwards like the strands of a lyre in relation to its several sides—hence the term by which I have denominated this structure.

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* **Stricker's** 'Handbook,' Sydenham Soc. Trans.

Tænia semicircularis.—This long arciform band of medullated fibres arising from the summit of the gyrus hippocampi follows the curved inner surface of the caudate nucleus through the whole of its course, and consists of superficial and deeper-seated fibres, the latter connected with the ganglionic structure of the caudate nucleus. It has been affirmed by numerous authorities that this arciform band terminates in the descending pillar of the fornix.* That the more superficial fibres so terminate I will not deny; but I have hitherto failed in *these animals* to assure myself of the fact. Of its deeper fibres I can speak with confidence. They not only *do not terminate* in the fornix, but end in two distinct directions, the deepest curving downwards to enter the anterior perforated space (Plate 49, fig. 12, 2); the fibres above this series arch inwards and forwards to enter the posterior commissure (Plate 49, fig. 12, 1). The latter connexion is a most important one to recognise, and of its existence I have repeatedly assured myself by horizontal and vertical sections. The fibres enter the commissure from behind in such a direction as to ensure their decussation within it. The cortex of the gyrus hippocampi and caudate nucleus are thus brought into crossed relationship with the olfactory bulbs.

2. *Connexions with striate ganglion.*—The anterior extremity or head of the caudate nucleus bends downwards to the base, and approaching the median line is separated from the ganglion of the opposite side by the mass of the septum lucidum. At the base the ganglion becomes almost superficial, being merely covered by a very thin layer of cortex, which is continuous with the general surface of the olfactory lobe. Medullated fibres from the cortex of the olfactory lobe and the granule layers of the bulb pass from before backwards into this basal extension of the caudate nucleus in numerous bundles, which in vertical sections appear mapped out into triangular wedge-shaped spaces by the large vessels traversing this region (pars perforata antica). This basal region of the corpus striatum, the olfactory field of GRATIOLET, is best studied in fresh preparations by horizontal sections carried across the hemispheres. The brain is placed on the section plate of a microtome with its base downwards, frozen to the height of a few millimetres, the unfrozen portion removed, and then successive sections should be made down as far as the cortex of the base. All these sections examined fresh will prove most instructive. A section carried in this way through the cortex shows this basal or olfactory realm of the brain to be clearly mapped out into two very distinct areas. One, the most internal, is of pyriform contour, its narrow end continuous with the olfactory lobe, its larger end directed backwards. This area constitutes the base of the corpus striatum and has a peculiarly characteristic structure. The other, or outermost area, is constituted by a continuation of the olfactory cortex with the lower

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* MECKEL, ARNOLD, JUNG, and LUYS.

limbic arc, and is sharply separated from the former area by the superficial olfactory fasciculus, whilst on its outer side it is of course bounded by the limbic sulcus. Following exactly the outer border of the pyriform area, the superficial olfactory band is naturally at first curved outwards, and then inwards towards the Sylvian depression. In sections taken across a slightly higher level the same regions are distinctly seen and their structure rendered clear. The innermost area is now seen to consist of numerous oblong and elliptic masses of grey matter enclosing nerve corpuscles; these grey nuclei vary much in size, but usually measure ·348 mm. to ·870 mm. in length by ·204 mm. to 300 mm. in width. They are closely embraced by bundles of medullated fibres, which form dense fasciculi betwixt them and are all directed from before backwards. They originate in the cortex of the olfactory lobe and pass through this ganglionic region. Mingled with these is a separate system of fibres which arise from the granule layers of the olfactory bulb, and coursing chiefly along the inner basal aspect of the olfactory lobe also enter this olfactory area, but do not form fasciculi. Their peculiarity consists in their running as separate fibres very irregularly, as bending, decussating and crossing each other in various directions so as to give the impression of their forming an open meshwork. These nerve fibres measure ·002 mm. to ·004 mm. in diameter. They penetrate the oval nuclei, and are found throughout their substance. As they do not extend beyond this region they terminate almost certainly in the nerve-cells of its grey masses. The outer surface of this olfactory region is covered, as was indicated by MEYNERT, by a thin layer of cortex, whose structure has already been fully described (p. 714). In passing through the olfactory area this deep mass of medulla is disposed in numerous longitudinal divisions or clumps well shown in vertical sections, from which smaller fasciculi are seen to descend to the cortex of the base, where they become apparently continuous with the arciform medulla superficial to and beneath the second cortical layer. Continuous with this deep olfactory medulla is a minor segment given off from the superficial olfactory fasciculus along its inner margin, and which can be best demonstrated by longitudinal sections through the hemisphere. The following scheme represents in a diagram the numerous systems of medullated fibres seen in vertical sections through this olfactory area (Plate 49, fig. 8). In the first place, we trace the cortex from the limbic fissure (*a*) downwards beneath the superficial olfactory fasciculus (*b*) prolonged over the basal aspect of the caudate nucleus. Its second layer is seen disposed in the peculiar duplicatures described, and beneath which lie the clumps of deep olfactory medulla seen in cross section (*m*) forming massive fasciculi contrasted strangely with the more distant fasciculi of the corpus striatum proper. Betwixt the deep medulla and the second layer is the deep arciform belt (*d*) with its great spindle cells apparently continuous with the claustral formation externally (*c*) and with the arciform medulla passing up the septum lucidum internally (*e*). Descending from the deep medulla into this arciform belt are numerous vertical fasciculi (*f*), which may also probably form connexions with the second layer of the cortex. The deep medulla gives off laterally a dense mass of fibres which ascend deeply within the substance of the septum lucidum (*g*), whilst the cortex of the septum is seen to possess a double arciform belt, one superficial to its second layer (*h*), and the other beneath these cells (*k*). These arciform stripes communicate by vertical intermediate fibres with each other in their course upwards to the corpus callosum.

From this olfactory area an important medullated fasciculus descends to enter the motor columns of the cord, hence affording a direct channel for the impulses propagated on excitation of the olfactory bulb. The double connexion with motor columns of the medulla and the cerebral cortex is the usual condition in Mammalia, although, as BROCA has indicated, the connexion with the *cortex* alone is the only constant feature.

3. *Arciform fasciculi.*—If the olfactory lobe be reflected downwards, away from the frontal pole, an upper medullated connexion betwixt the former and the hemisphere, may be traced lying invariably over the central canal of this lobe. But by far the more important arciform fasciculi form two series, which taking origin in the cortex of the olfactory lobe, are destined for the gyrus fornicatus and posterior regions of the brain. In vertical sections through the frontal end of the hemisphere, near the commencement of the caudate nucleus, the central olfactory fasciculus is seen in transverse section, imbedded in the head of this nucleus, which rises dome-like above it. Descending from the region of the central medulla of the hemisphere, are two medullary finger-like processes which appear to grasp the caudate nucleus betwixt them. The outer is constituted by fasciculi arising from the external borders of the caudate nucleus and disappears in the central medulla. The inner lies parallel with the central mass of medulla, but is wholly distinct from it, and constitutes the arciform fasciculi, which originate in the cortex of the olfactory lobe at the base and median aspect, and curving up the inner side of the hemisphere is closely connected with the cortex of the upper limbic arc. At the median aspect, where the upper limbic arc unites with the olfactory cortex, numerous medullated fasciculi arising from the latter follow the bend of the hemisphere, and thence arching strongly inwards come into relationship with the under surface of the corpus callosum, and turning backwards, course along its inferior aspect, whilst other fasciculi pierce its structure and join the arciform system of the gyrus fornicatus, which rests upon the upper surface of the corpus callosum. (Plate 49, fig. 12, ^{7}). At its origin this important fasciculus is seen in vertical sections, ascending in a sigmoid curve, with the convexity first inwards and then outwards towards the under surface of the callosal commissure. A little further back the septum lucidum is found to the inner side of the olfactory area. In this animal it appears as a largely-developed structure, heart-shaped in vertical section, and formed of two lateral folioles uniting centrally, and not enclosing a ventricle betwixt them as in Man. Through the substance of the septum a large number of medullated fibres ascend to the inferior aspect of the corpus callosum (Plate 49, fig. 8, G). The longitudinal band formed by the arciform fibres of the olfactory and upper limbic

cortex is distributed eventually to a peculiar region already described, near the occipital pole of the hemisphere, which is covered by cortex of the modified upper limbic type. This termination will be described when I refer to the central expansions of the olfactory medulla.

4. *Superficial olfactory fasciculus* ("External root").—This fasciculus appears, from a surface view, as a flattened white lamina covering the basal aspect of the olfactory lobe. Widest in front, it encircles the greater surface of this lobe, grasping its lower, outer, and portion of its upper aspect. Thence directed backwards, it sweeps round the olfactory area, forming, as already stated, its outer boundary, and becoming rapidly attenuated, runs as a delicate white streak inwards, in a direction which, if continued, would pass nearly at right angles across the origin of the optic tracts. In this, its latter course, it passes over that slight furrow which here indicates the Sylvian fissure, and is lost in the substance of the gyrus hippocampi. Vertical sections show that this fasciculus is not a mere flattened lamina, but a thickened oval or oblong band, which is sunk to a little depth in the peripheral zone of the cortex, covered externally by a delicate layer of DEITER's cells connected with its pia mater, from which large vessels dip vertically through its substance and enter the subjacent cortex.

In what manner does this medullated band terminate? It has long been known that in Man and the higher Mammals the external olfactory medulla ends in the temporal and occipital lobes, but in these animals it is seen by the naked eye that a rapid attenuation is undergone ere it reaches the Sylvian furrow (Plate 49, figs. 1, 2, M). To comprehend its distribution we must examine closely vertical and horizontal sections through this region anterior to the Sylvian depression. In vertical sections the superficial olfactory fasciculus forms the inner border of what we have termed the external olfactory region, limited outwards by the limbic sulcus. The cortex of this region is composed, as already stated, of the three layers—the peripheral zone; the irregular and angular cells; the large pyramidal layer. Now beneath the lowest layer of this cortex, at variable depths, is found an important formation of spindle-cells, which taking the reclinate position as regards the surface, descends from the extra-limbic portion of the hemisphere. This belt of spindle-cells is the representative of the claustral formation of higher Mammals, and is directed towards the surface of the cortex below, along the inner margin of the superficial olfactory fasciculus. It forms therefore a deep belt, dividing off the olfactory area from the external olfactory regions (Plate 49, fig. 8, *c*). Having noted this fact, a horizontal section should be taken through the hemisphere, close to its basal cortex, as was done for the examination of the connexion of the olfactory lobe with the olfactory area. If such a section pass through the deeper part of the superficial olfactory band we shall note that whilst its outer fibres are long and continued uninterruptedly backwards, the innermost fasciculi constantly tend to bend outwards and pass through and across the longitudinal band, arching in fan-like manner over the whole external olfactory region as far as the limbic sulcus. This passage of the inner marginal fibres of the band into the cortical substrata must eventually lead to its total disappearance. These fan-like radiations are continuous with the cells of the cortical layers; yet it does appear as though a connexion was also established betwixt its innermost fibres and the claustrum, which latter formation, however, is mainly continuous with the arciform spindle-celled medulla of the olfactory area (Plate 49, fig. 8, *d*). Prior to the arrival of this superficial medullated band at the Sylvian furrow a notable diminution in its width has occurred, apparent to the naked eye. This is due to the detachment from its margin of a large proportion of fibres which have already been alluded to as running continuous with the deep olfactory medulla within the olfactory area. It has been usual to speak of the central projections of the olfactory medulla under the terms olfactory roots, an *external, middle, and internal root* being usually those described in higher animals. A fourth also is described as a *superior root,* connecting the upper surface of the olfactory lobe to the hemisphere. By the external root is indicated what I have above described as the superficial olfactory fasciculus; the connecting system with the striate body is identical with the so-called middle root, whilst the arciform series includes the remaining two roots, the internal and superior.

Dissection of Basal Olfactory Regions.

The tracing of the coarser strands of medullated fibres by microscopic examination of sections, should invariably be supplemented, where possible, by frequent dissections which often enable us to fully confirm by this second method results arrived at by the former. As regards the olfactory regions at the base the following points may by this method be satisfactorily demonstrated in the Rabbit and the Rat:—

a. Uninterrupted course of the central olfactory fasciculus into anterior commissure.
b. Projection of posterior olfactory fasciculus into the occipital lobe.
c. Connexion of the latter fasciculus also with the cortex of the vertex.
d. Peculiar connexion of the same fasciculus with the corpus striatum.
e. Relationships of olfactory area at greatest depth below cortex.
f. Descent of the tænia semicircularis into olfactory area.

The above facts may be confirmed by placing the brain of the Rat in its natural position with the vertex upwards, and keeping it during dissection moistened occasionally by a few drops of a 2 per cent. solution of common salt. With a delicate pair of dissecting scissors the membranes stretching across between the hemispheres are divided through their whole length, and the latter sliced off horizontally outwards upon a level with the corpus callosum by a sharp and fine scalpel. This commissure is next divided along the median line from behind forwards, each segment being turned outwards. Such a section exposes the corpus striatum in its extra- and intra-ventricular portions placed in front of the cornu ammonis. In the median line separating the two striate bodies lie the double leaflets of the septum pellucidum. The callosal fibres are seen to meet those ascending from the internal capsule and upper pole of the ganglia. If the extra-ventricular nucleus be pressed gently outwards away from the caudate nucleus, we observe numerous white fasciculi beneath its surface arching upwards parallel to the latter to enter the projection mass at the upper pole, whilst running parallel with the converging tract of the fornix, and closely connected with the whole inner border of the caudate nucleus, is the fairly broad white band—the tænia semicircularis. By means of a fine dissecting needle and camel-hair brush the whole remaining dissection may be completed. First, the septum pellucidum is gently raised from behind and thrown forwards, the hemispheres pressed asunder so as to expose the descending pillar of the fornix, in front of which the anterior commissure crosses from one to the other hemisphere. By means of the needle the continuation of this commissure, with the olfactory lobe of one side, may be most readily traced, and upon cautiously teazing out the substance of the striate body on one side, the continuation from the commissure outwards through its substance may be quite as easily traced. We now see gleaming deeply through the basal structures of this region a pale band which may be regarded as the outer side of a remarkable triangle, the inner side being constituted by the central olfactory fasciculus, and the base by the anterior commissure and its posterior extension. Now this triangular area is the deeper portion of the pyriform region recognised at the base as GRATIOLET'S olfactory area. Closely examined by a hand magnifier we now see the very peculiar structure alluded to as the "olfactory lyre." Gently raise by the needle the posterior olfactory fasciculus and examine also with a lens. From its anterior border arise numerous delicate fibres constituting more or less fine fasciculi, the largest arising near the anterior commissure, whilst a few delicate offsets are given off from the commissure itself. Whilst these fasciculi curve inwards and appear *continuous with the olfactory tract into and through the commissure,* other fasciculi may be seen passing through the structure of the posterior olfactory band directly backwards, separating its strands and *emerging behind to enter this region of the corpus striatum.* Traced forwards these delicate fasciculi lie parallel to and on a plane with the central olfactory fasciculus, forming as it were a floor for this triangular space (the strands of the lyre), and in fact sharply defining the upper limits of the olfactory area from the structure of the caudate nucleus proper. With the scissors divide these fibres along their line of connexion to the posterior olfactory tract, and teaze out with the needle the structures lying immediately beneath. We now see by means of a low power, and even by the naked eye, the coarse fasciculi enclosing betwixt their strands the oval grey nuclei of the olfactory area. Next, by a delicate dissection with needle and moistened brush, the posterior olfactory fasciculus may be followed from the outer margin of the striate nucleus backwards; its brush-like division here sending a small detachment of fibres upwards along the outer aspect of the corpus striatum to reach the vertex. The posterior division of the brush may readily be traced into the inferior arc of the great limbic lobe, its fasciculi never rising above the level of the limbic sulcus. They are ultimately distributed to that peculiar region at the occiput characterised by the modified olfactory cortex.

It now remains for us to follow up the extensions of medullated fasciculi from the olfactory lobe, and to trace the expansions into the distant cortical areas of the central olfactory and arcuate olfactory systems. Prior to doing so, it will be advisable to consider the structure and cortical connexions of the corpus striatum and callosal commissure.

Corpus Striatum and its Cortical Connexions.

Corpus striatum.—This ganglionic body exhibits in these animals, as in higher members of the animal kingdom, a marked division into lenticular and caudate segments, or, in other words, extra- and intra-ventricular nuclei, which assume also a higher level as compared with the thalamus, since the largely developed cornu ammonis and the bulky fibrous fornix commences at higher and anterior planes, and not, as in Man, *behind* the thalamus. These structures therefore intervene betwixt the cortex of the vertex and thalamus, and displace the latter to a lower level, so that sections carried horizontally through the upper part of the hemisphere will exhibit the striate body and cornu ammonis cut through in one and the same plane. Anteriorly the corpus striatum exhibits a large rounded head, which bends downwards to the base of the hemisphere within the confines of the superficial olfactory fasciculus. Posteriorly it becomes rapidly attenuated and is continued as a small cylindrical belt of grey matter—the tail of the caudate nucleus, which curves backwards and downwards within the descending horn of the lateral ventricle to terminate in the inferior limbic arc, near the summit of the gyrus hippocampi (Plate 49, fig. 10, C). To study the form and relationship of the striate body, vertical sections should be examined passing through the septum pellucidum in front of the optic commissure. This ganglion is then seen as a large irregularly pyriform structure, its greater mass directed upwards, its upper and outer border nearly concentric with the surface of the hemisphere (Plate 49, fig. 12, 4),* whilst the inner or intra-ventricular border lies partly adjacent to the thick foliole of the septum lucidum (fig. 8, *b*), and at the angle of union with the latter the transverse section of a compact band of medullated fibres is seen, the central olfactory fasciculus (Plate 49, fig. 8, *h.*). The true structure of the corpus striatum ends upon a level with this olfactory fasciculus, and is mapped out by a succession of fasciculi, which are cut across and lie in a continuous line from the central olfactory fasciculus to the outer margin of the ganglion. These fasciculi are of greater size than those beneath them, and are constituted by the strands of the "lyre" already alluded to. Beneath this line the striate nucleus still extends as far

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* The diagram (fig. 8) exhibits in outline a part of the caudate nucleus and its full basal relationships.

as the base, forming the modified structure referred to as the olfactory area, and which is here covered by a thin layer of cortex.* The outer margin of the striate body is sharply differentiated from the outer olfactory realm (lower limbic arc), and terminates abruptly at the inner margin of the superficial olfactory band, to which point run the fibres of the claustral formation (Plate 49, fig. 8, *c*). On the inner side, at its base, we find this olfactory area of the corpus striatum bounded not only by the central olfactory fasciculus, but by sheaves of fibres arising from the cortex of the olfactory lobe (Plate 49, fig. 8, G, *g*), and running as the arciform fibres up the septum lucidum to reach the under surface of the corpus callosum. In these anterior realms the internal capsule which separates the two main divisions of the striate body is not as yet clearly differentiated, and hence the large oval mass is constituted by the blending together of the caudate and lenticular nuclei, and the fasciculi here being chiefly projections from the cortex of the frontal lobe are seen in transverse or oblique section. To obtain a good view of the fibres radiating from the frontal pole through the structure of the corpus striatum vertical sections must be taken across the hemispheres. At the summit or upper pole of the striate body (Plate 49, fig. 11, H) the fibres of the corpus callosum (D) meet those of the projection system from the cortex to the basal ganglia, and at their line of intersection mark off the intra-ventricular (K) from the extra-ventricular portion (L). We have therefore in such sections a large ganglionic mass formed by the blending of the two striate nuclei, enclosed in a concentric manner by the hemisphere, and exhibiting the following systems of fibres: the callosal fasciculi (D); the striate radiations to cortex (C); the claustral formation (Plate 49, fig. 8, *c*); the external (*b*); median or deep (*m*); internal and central olfactory fasciculi (G and *h*). In vertical sections carried through the hemisphere of the Rabbit on a plane with the anterior commissure the distinction betwixt the two nuclei of the corpus striatum is very obvious. Here the caudate nucleus lies reclinate upon the sloping oblique side of the internal capsule, measuring from 9 mm. to 10 mm. in its longer, and 2·5 mm. in its shorter diameter, its upper pole directed upwards and outwards, receiving the radiating fibres from the cortex; its inferior pole directed downwards and inwards as a thin prolonged margin, exhibiting on its ventricular aspect the transverse section of the stria cornea (Plate 49, fig. 12, 4). Beneath the stria cornea with the structure of this inferior pole are nerve fasciculi, which in part pass downwards into the anterior perforated space (Plate 49, fig. 12, 2) (olfactory area), whilst a few arch inwards and are directly continuous with the anterior commissure (Plate 49, fig. 12, 1). Beneath the sloping roof formed by the internal capsule lies the lenticular nucleus, which is distinctly wedge-shaped in form, its base directed outwards and curved concentrically to the surface of the hemisphere, its apex directed downwards and inwards terminates in the internal capsule (Plate 49, fig. 9, D). The lenticular nucleus appears to consist of outer and inner segment; the inner,

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* Vide Plate 49, fig. 8. All structures below *h*.

forming the peduncular portion and apex of the wedge (Plate 49, fig. 9, D), contains a far greater proportion of fibres than the outer, which are consequently much more closely approximated, consisting, as MEYNERT indicates, of fibres extending from the outer segment, as well as fibres originating afresh in itself. The further our examination of these nuclei by vertical sections extends, the more restricted becomes the dimensions of the lenticular nucleus, the mass of the thalamus intervening until on a plane with the origin of the pineal peduncles; its extent is very insignificant,* whilst the tail of the caudate nucleus (Plate 49, fig. 10, D) and the stria cornea are seen as two small oval bodies, separated from it by the intervening cortical connexions of the ganglia. In these posterior planes, vertical sections passing through the tuber cinereum will exhibit within the substance of the inferior limbic arc two noteworthy structures: first, a compact cylindrical band, consisting of some thirty or more fasciculi of medullated fibres, which results from the convergence of fibres from the cortex of this region, and is really the temporal extension of the stria cornea; and secondly, just external to it lies a somewhat oval nuclear grey body, the tail-like prolongation of the caudate nucleus (Plate 49, fig. 10, C). In sections through the hemispheres at that site where the retiring optic tracts are concealed betwixt the crus cerebri and hemisphere, the stria cornea will appear to a certain extent of their course cut lengthwise and entering the substance of the limbic lobe external to the optic tract (Plate 49, fig. 10, E).

Coronal projections to the motor ganglia and internal capsule.—Traced from the cortex downwards it is found that the whole marginal aspect of the hemisphere at the vertex and for a certain distance outside the median line, constitutes the area whence the coronal radiations arise which converge to the internal capsule and corpus striatum below. Upon arriving at the centric pole of the corpus striatum, the fibres form a dense projection mass, which in the form of small fasciculi spread out and almost completely enfold the upper and outer aspect of this ganglion, the innermost series entering the ganglion direct, the outer fasciculi proceeding along the arched surface of the lenticular nucleus to enter it at its outer aspect, whilst the intermediate fibres terminate at various points along the intervening surface (Plate 49, fig. 10, A). The receiving surface or centric pole of the striate body is therefore a very extensive one, being constituted by the whole of its upper and greater part of its outer aspect. The above description of its cortical connexions holds good only as regards that region where the centric pole of these ganglia attains its greatest height, viz.: anterior to the thalamus where the corpora striata lie in closer contiguity. To study its exact distribution a series of vertical sections of the brain of the Rat should be examined at different planes from before backwards. Thus, in regions of the septum lucidum, in front of the anterior commissure, we find the innermost fasciculi of the projection mass where they leave the ganglia only 2 mm. distant from the marginal angle of the hemisphere.

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* Vide Plate 49, fig. 10, between A and B.

In regions where the thalamus intervenes, as at the origin of the optic tracts, the lenticular nucleus retiring further outwards makes the distance between the points 5½ mm. In these regions the coronal fibres are distributed chiefly to the upper aspect of the hemisphere and not so exclusively to its marginal angle. Still further back on a plane with the optic radiations to the thalamus, the distance betwixt the same points is 6½ mm., and here the great mass of the projection system no longer runs upwards to the vertex, but immediately upon the emergence from the lenticular nucleus they bend directly backwards towards the occiput and are therefore exhibited as transverse sections of fasciculi (Plate 49, fig. 10, F). A limited supply can still be traced upwards to the cortex of the outer and upper aspect of the hemisphere (Plate 49, fig. 10, A). These facts warrant us in asserting—

1st. In regions anterior to the thalamus the coronal connexions betwixt cortex and motor ganglia with the internal capsule, arise *par excellence* from that marginal aspect of the vertex whose cortex is characterised by the great ganglionic cell formation, and as entering into the composition of FERRIER's motor realm.

2nd. That the median cortex (upper limbic arc) has no connexion with the same ganglia or capsule throughout any of its course.

3rd. That as these ganglia retire outwards before the intervening thalamus, their coronal connexions arise chiefly from the upper and outer aspect of the hemisphere and still further back from the occipital cortex at their own level.

From what has already been stated as regards the distribution of the callosal commissure, it is evident that in the anterior regions of the brain the marginal and exposed aspect adjacent to it will receive a much larger supply of medullated fibres than with the more external aspect, or the cortex of the median aspect, which receives callosal fibres only. In accordance with this a glance at vertical sections of fresh brain shows the dense sheaves of fasciculi which radiate to the marginal angle (Plate 49, fig. 11, E), and the comparative paucity of medullated supply to the limbic arc and outer aspect of hemisphere.

Junction of projection fasciculi with motor ganglia.—The uniform medullary radiations unite into delicate fasciculi which pass between the fibres of the callosal commissure, decussating with them at various angles, and just where they arrive at the surface of the striate ganglion unite into coarser fasciculi and pass into the outer segment of the ganglion, often bending inwards at right angles to their original course (Plate 49, fig. 11, C). The fasciculi thus formed by convergence of the coronal radiations have an average diameter of ·05 mm., but towards the upper pole of the lenticular nucleus large fasciculi are found with a diameter of ·093 mm., much more closely aggregated. At the lowest level of this body they form long delicate bundles (·025 mm. diameter), which pass uninterruptedly after a straight or curved course into its inner segment, whilst midway betwixt these points the fasciculi are far more widely scattered apart and are of medium size (Plate 49, fig. 13, A–B). The delicate bundles traversing the callosal fibres which by their convergence form the coarse fasciculi of the outer lenticular nucleus are usually one-third or one-fourth the diameter of the latter. Each coarse fasciculus is formed by the union of two or three, and occasionally a large number of such bundles, and during their passage through the outer grey segment frequently divide into two, and appear in their further course through the inner segment subject to still further subdivision. At the upper pole of the lenticular body fasciculi are observed passing from the first segment directly into the internal capsule without previously traversing the inner segment, whilst coronal fasciculi are here also distinctly seen to pass down the internal capsule without establishing any connexion with the ganglia. Upon a plane passing through the anterior or olfactory commissure in horizontal sections of the brain, other fasciculi of medium size, are directed directly backwards into the inner segment of the striate body from the frontal aspect of the caudate nucleus, and obliquely backwards and inwards from its marginal aspects, each fasciculus frequently subdividing on its way but occasionally passing uninterruptedly through the outer segment. In this course they are directed over, beneath, and through the posterior olfactory fasciculus (Plate 49, fig. 9, D). Within the inner segments we meet with two systems of fibres: the coarse fasciculi already dealt with, and a large proportion of minute medullated fibres running separately, and not in fascicles. These ultimate nerve fibres are peculiarly liable to varicosity—they take curved and spiral directions—cross each other frequently, are often much contorted, but are chiefly distributed longitudinally along the course of the large bundles, or in arches across the inner segments of the lenticular nucleus concentric to its base.

They attain a diameter of ·004 to ·005 mm. At a little lower level we find them still more abundantly, but here the orifices of large divided vessels and the peculiar oval nuclei betwixt the coarser bundles indicate that we are examining the anterior perforated space or olfactory area, and now we immediately identify these minute twisted fibres with those already described as peculiar to this region, and can readily trace their connexion with those surrounding the granule layers of the olfactory bulb. At higher levels passing through the anterior end of callosal commissure, coronal radiations pass betwixt cortex of the tip and inner margin of the frontal lobe to the head of the caudate nucleus—the fibres of the corpus callosum crossing their course in their radiations outwards.

To recapitulate—

1. The anterior end of the frontal lobe is connected chiefly with the head of the caudate nucleus.

2. The cortex of the marginal angle at the vertex in front of the thalamus and of a limited surface outside is connected with the centric pole of the lenticular nucleus.

3. The cortex of the outer aspect of the hemisphere behind the latter, and that of the outer aspect of the occipital lobe is connected with that portion of the wedge-shaped nucleus which bends backwards and downwards towards the base.

4. The cortex of the olfactory bulb and lobe is represented in the olfactory area of the nucleus caudatus.

5. The cortex of the inferior limbic lobe will also be strongly represented in the extension of the lenticular nucleus in this direction.

The callosal commissure.—According to the researches of Foville the fibres of the corpus callosum are distributed to none of the convolutions of the brain, but consist of tegmental fibres which, having passed upwards through the thalamus and corpus striatum, unite over these ganglia as a true commissure of the crus cerebri. All my fresh preparations lead me to regard this conclusion as quite fallacious, and demonstrate in the clearest possible manner the passage of the callosal fibres from the convolutions of one hemisphere to those of the other. As maintained by Arnold, Oellacher, and Meynert, it is *truly and exclusively a commissure of the hemispheres.* In vertical sections of the brain the following facts I have repeatedly and satisfactorily demonstrated. In sections thus taken through the region of the septum lucidem, the callosal commissure upon entering either hemisphere meets the mass of the projection system obliquely, and decussating and interweaving with this system proceeds along the curved centric pole of the striate body, becoming more and more attenuated as it extends along the lateral aspects of the ganglia, owing to the constant distribution of fibres from its mass to the cortex throughout the whole of this course. In the first part of its course, where it first blends with the projection fibres to the ganglia, a very large proportion of its mass (one-third to one-eighth of the whole) takes a curved course upwards and inwards so as to reach the *median and marginal cortex here* (Plate 49, fig. 12, 6). By far the greater part accompanies the projection fibres, being similarly distributed, viz.: to the cortex of the upper marginal border of the hemisphere and the region just external to it. At this curve of the corpus callosum, where it interweaves with the projection fasciculi prior to their dispersion as coronal radiations, an angular projection is formed which, taking the whole length of the commissure into consideration, really is a prolonged ridge astride which rests the longitudinal fasciculus of the olfactory arcuate system (Plate 49, fig. 12, 7). In vertical sections taken through more posterior planes, it is seen that the portion of the callosal commissure which thus supplies the inner marginal regions of the hemisphere is distinctly to be traced through the whole of its extent as separate from the lower segment which supplies the cortex further outwards. So notably is this the case that all prepared sections show, as a rule, a distinct separation between these two callosal segments, the inner as it were peeling away from the outer—a natural condition due to their opposed direction. It is therefore obvious, at a first glance, that these inner regions of the brain which are similar and identical are connected by the same series of fibres, and the same may readily be maintained for the outer regions of the hemispheres. In fact, the regular superposition of fibres above one another, and the great uniformity of their course in either hemisphere, conclusively show that they unite identical regions of both sides. The most inferior fibres of the callosal commissure are the lowest throughout their course, and are placed next to the ganglia in their course to the cortex, which for them is the most external part of the hemisphere, *i.e.*, the undermost portion of the corpus callosum terminates in the cortex just above the limbic sulcus externally. On the other hand, the uppermost series of fibres in this commissure connect the more internal aspects of the brain. A superficial section lengthwise through the corpus callosum would therefore deprive the median and inner regions of the brain of their commissural connexions, whilst the *deeper* the incision was carried the more *external* the regions of the hemispheres which would suffer likewise.

Expansion of olfactory arcuate system into occipital cortex.—An important distribution of fibres from the olfactory cortex has been already traced into the occipital lobe lying like a longitudinal fasciculus (of inverted V-shaped form in vertical section) upon the prominent ridge formed by the apposition of the callosal and projection fasciculi (Plate 49, fig. 11, F). The ultimate destination of these fibres may best be studied in vertical sections through the hemisphere behind the posterior commissure. In such sections we expose the central medulla folded around the cornu ammonis, which in its turn lies concentric to the great central ganglia (Plate 49, fig. 10). This mass of central medulla reveals two important regions, (1) the region of section of the great mass of projection fasciculi which here is lateral in position (F); (2) the region of the callosal and arcuate olfactory fasciculi which is median in position (H). The latter with which we are now concerned is wedge-shaped, in such section the base of the wedge directed towards the median cortex, the two converging sides directed outwards. The central core, so to speak, of this wedge is constituted by the callosal fasciculi subdivided into an upper segment destined for the marginal angle and tip of occipital lobe, and a lower segment which is continuous outwards towards the lateral projection system radiating to the cortex throughout this course and the divided fibres of the fornix. Superficial to it and running in the opposite direction is a small band of projection fasciculi, which has also for its ultimate destination the cortex of the outer and upper surface of the hemisphere as far as the marginal angle. Upon the upper basal angle of the wedge rides the fasciculus of the olfactory arcuate and arcuate system of the upper limbic arc, which here have accumulated into a vast collection of fibres opposed to the whole base and upper side of the wedge. These fibres, however, are no longer found in transverse section, but are seen spreading extensively on all sides to the cortex (1) of the median aspect (upper limbic arc), (2) the marginal angle of the hemisphere, and (3) the cortex of the vault as far outwards as the superior parietal sulcus. Outside the latter sulcus the coronal distribution to the cortex consists of scanty projection and callosal fasciculi; within the sulcus, on the other hand, the cortex is supplied by dense callosal, a few projection fasciculi, and the notably large supply of olfactory and limbic arcuate fibres. This rich medullated region reveals its wealth of medullated fibres compared with the rest of the hemisphere when simply examined by the naked eye, and in osmic acid preparations shows the same in the far deeper staining which it takes. The olfactory arcuate fasciculi therefore are wholly distributed to that portion of the median and superficial cortex of the occipital lobe which is characterised by the granule cell formation which here forms a deep second layer (Plate 49, fig. 1, K, D). This formation has been already fully described under the name of the modified upper limbic type. This region is moreover characterised by a further peculiarity, viz.: the two intra-cortical arcuate bands. The more superficial of these is a broad streak of medullated fibres which arising from the apical processes of the large cells in the cornu ammonis, as its first layer (nuclear—MEYNERT) is continued along the lower half of the first cortical layer of the upper limbic arc, and bending over on to the upper surface of the hemisphere is almost immediately lost here. Beneath it lies a second shallower medullated band a little over half the depth of the former, and separated from it by the second layer of granule cells. This stripe also arises from the cornu ammonis, and likewise running parallel to the cortex terminates even earlier than the more superficial stripe, as it ends abruptly at the upper marginal angle of the hemisphere. In such as we are now dealing with (fresh vertical sections), both these stripes may readily be seen by a hand lens running as delicate white streaks parallel to the surface of the cortex of the upper limbic arc, being lost beyond the upper marginal angle, whence no similar white streak can be detected until we arrive opposite the great mass of the projection system which here takes a lateral position. Here again we observe a delicate medullated stripe in the first cortical layer running towards the limbic sulcus, but lost prior to its arrival at that spot. This stripe is even shallower than the deeper arcuate stripe in the upper limbic arc.

Region of expansion of central olfactory fasciculus.—It will be remembered that the termination posteriorly of the inferior limbic arc is characterised by the large size of the nerve-cells of its second cortical layer—the *modified olfactory type.* If *horizontal* sections of the occipital end of the frozen brain be carried through this region, the following facts may be readily confirmed. Most internally is the curved border of the cornu ammonis; next to it comes the granule region, extended from the upper limbic arc; still further outwards is the peculiar region alluded to; and beyond its boundary (the limbic sulcus) is the curved surface of the extra-limbic portion of the hemisphere.

To the *naked eye* a great dissimilarity is at once observed betwixt the appearance of the extra-limbic and the limbic portion. The former exhibits a medulla which has a slight translucent aspect and is of a pale tint, but markedly contrasted with the *perfectly opaque and brilliant white colour* of the limbic medulla. The line of demarcation, again, betwixt these two is sharp and abrupt, and extends from the depths of the corpus callosum directly outwards to the limbic sulcus. Microscopic examination clearly shows this difference in colour to be due to the *vast preponderance* of medullated nerve-fibres in the limbic region at this site, which radiate outwards to the cortex from the terminal portion of the callosal commissure. In the extra-limbic portion of the hemisphere we likewise observe medullated fibres which radiate outwards to the cortex along its whole extent, and proceeding solely from the corpus callosum do not form such dense aggregations, but light radiating fibres some distance apart. In the limbic portion, however, the same fibres are present, plus a strong reinforcement from the posterior extensions of the anterior commissure, *i.e.*, the central olfactory fasciculus. Here then appears demonstrated an important fact, that the cortex of the modified olfactory type (Plate 49, fig. 1, T to W), receives medullated radiations so extremely rich in fibres that it is mapped out to the naked eye from the neighbouring extra limbic medulla by its notable white brilliant aspect, and that these fibres include the terminal expansion of the central olfactory fasciculus.

Here again we meet with *two intra-cortical arciform stripes* of medulla disposed as in the modified upper limbic arc, viz.:—

(*a*) A superficial streak at the lowest confines of the first layer.

(*b*) A deeper streak beneath the large swollen cells of second layer.*

The superficial stripe is, however, here the shallower of the two, and the deeper stripe attains a width equal to the first band in the upper limbic arc. In *horizontal sections* the medullated fibres of these two streaks will be divided obliquely. In sections such as I have just been considering a rich series of arcuate fibres will also be found extending backwards through the *extra-limbic lobe* from the anterior realms of the brain, which thin out gradually towards the abrupt line of denser medulla, commencing at the limbic sulcus, and wholly disappear here. This is the arcuate system of the extra-limbic medulla.

The above facts enable me to conclude that at the occipital pole of the hemisphere there are two important regions definitely mapped out by coarse external configuration of the brain and also by minute structural divergences.

These regions are histologically distinguished by the possession of a cortex, which is peculiar to each, and which we have named respectively the modified upper and lower limbic types. Each region is associated with that of the opposite hemisphere by the callosal commissure.

Each region is brought into relationship with the olfactory lobe and bulb; the upper limbic by means of the arcuate olfactory, the lower limbic through the medium of the central olfactory fasciculi after the decussation in the posterior commissure.

Each region is moreover supplied with the peculiar double stripe of medullated fibres, forming arcuate systems within the cortex itself.

Hence we have in these regions a reflection backwards of the olfactory sensory surface brought strongly into association with the cornu ammonis by means of the arcuate intra-cortical stripes.

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* The site of these intra-cortical bands of medulla is approximately given in Plate 50, fig. 6, *a* and *b*.

DEPTH of Cortical Layers on a Plane with Genu of Corpus Callosum.

	Gyrus fornicatus.	Sagittal angle.	Extra-limbic.	Near limbic sulcus.
	mm.	mm.	mm.	mm.
First layer . . .	·372	·372	·348	·279
Small pyramids . .	·604	·744	1·023	·558
Granule belt . . .	..	..	·372	..
Ganglionic belt . .	·232	·232	·186	·408
Spindle cells . . .	·511	·883	·883	1·069

DEPTH of Cortical Layers on a Plane with posterior border of Corpus Callosum.

	Gyrus fornicatus.	Sagittal angle.		Extra-limbic.
	mm.	mm.		mm.
Peripheral zone . .	·441	·325		·372
Granule layer . .	·418	·372	Small pyramids	·372
			Granules . .	·372
Ganglionic layer . .	·465	·604		·604
Spindle cells . . .	·372	·604		·744

DEPTH of Cortical Layers of the Modified Lower Limbic Type. (Rabbit.)

	mm.
Peripheral cortical zone	·279
Inflated cell formation	·883
Ganglionic layer	·372
Pale belt.	·279
Spindle cells	·418 to ·655

DEPTH of Cortical Layers in the Cornu Ammonis. (Rabbit.)

	Peripheral zone.	Striate and lacunar layers.	Ganglionic cells.	Molecular layer.	Medullated layer.
	mm.	mm.	mm.	mm.	mm.
Anterior regions	·325	·930	·093	·523	·395
Average at six different sites.	·424	·915	·112	·397	·385

Average depth of whole cortex of ammon's horn =2·233 mm.

DIMENSIONS of Nerve-cells in Brain of Rabbits.

UPPER LIMBIC ARC.

	Second layer of small pyramids.		Ganglionic layer.	
Near frontal pole	17μ×11μ	Nucleus 9μ	24μ×17μ	N=10μ
In front of corpus callosum .	17μ×12μ	N=9μ	24μ×17μ	N=13μ
Anterior regions of „ .	15μ×18μ	N=8μ	32μ×18μ	N=13μ
Posterior regions of „ .	16μ×11μ	N=8μ	27μ×16μ	N=13μ
Behind corpus callosum .	13μ× 9μ	N=8μ	26μ×16μ	N=13μ

Granule cells =10μ×8μ N=5μ.

GYRUS HIPPOCAMPI.

Pyramidal cells.		Fusiform ganglionic.	
19μ×12μ	N=9μ	31μ×13μ	N=13μ

MODIFIED LOWER LIMBIC CORTEX.

	Second layer of inflated cells.		Third or pyramidal cells.	
From different sites	37μ×32μ	N=13μ	23μ×13μ	N=13μ
	37μ×23μ	N=13μ		
	46μ×27μ	N=13μ		

OUTER OLFACTORY CORTEX.

	Second layer.		Ganglionic layer.	
Distant from bulbus olfactorius .	19μ×12μ	N=9μ	31μ×13μ	N=13μ
„ „ „ .	18μ×10μ	N=9μ		
Beneath bulbus olfactorius . . .	17μ×11μ	N=9μ		

EXTRA-LIMBIC OR PARIETAL CORTEX.

	Second layer.			Ganglionic layer.	
In front of corpus callosum .	22μ×12μ	N=9μ		32μ×16μ	N=13μ
Anterior regions of „ .	21μ×13μ	N=9μ		32μ×16μ	N=12μ
Posterior regions of „ .	17μ×10μ	N=8μ	Largest	60μ×18μ	N=13μ
				31μ×16μ	N=12μ
Behind corpus callosum . .	13μ×11μ	N=8μ		24μ×18μ	N=12μ
Close to occipital pole . . .	20μ×11μ	N=9μ		31μ×19μ	N=13μ

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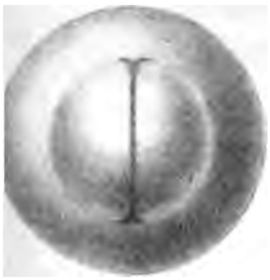
Fig. 1.



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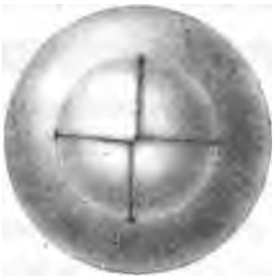

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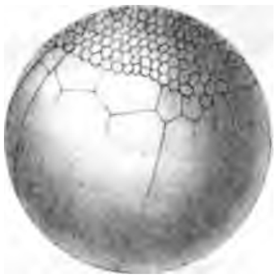

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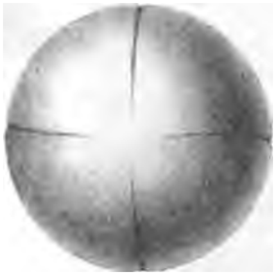

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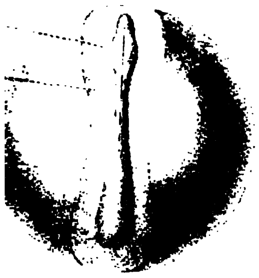

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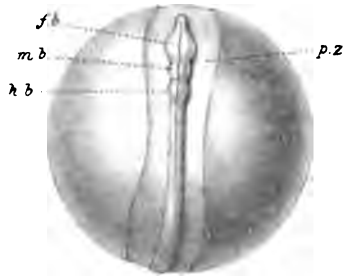

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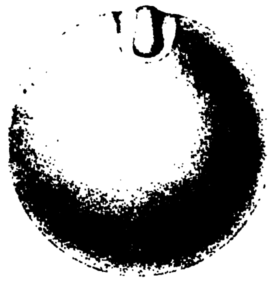

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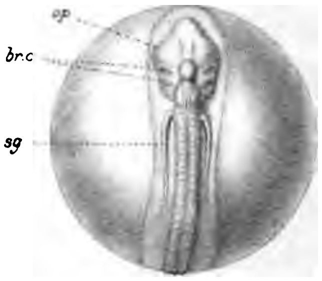

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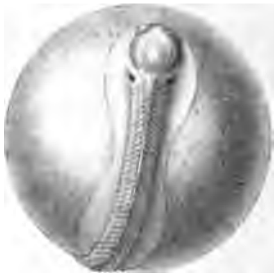

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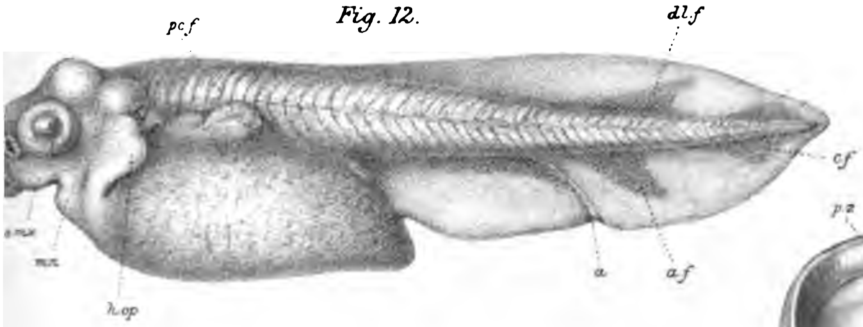

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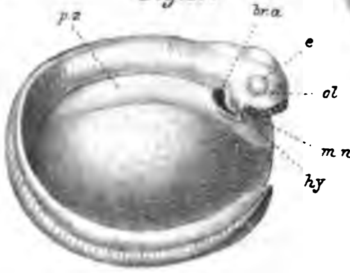

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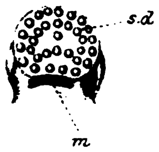

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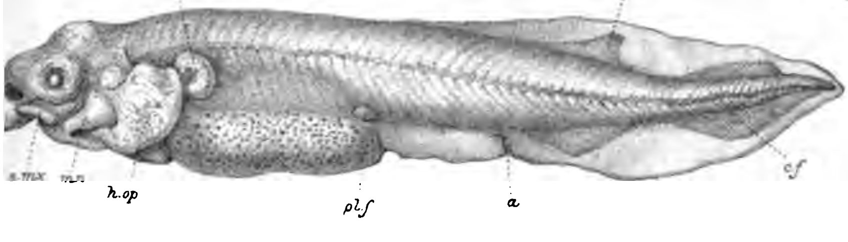

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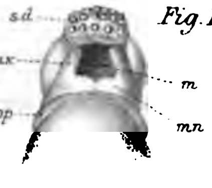

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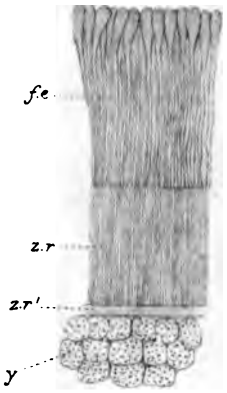

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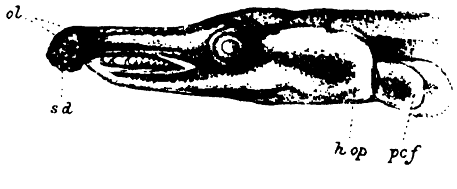

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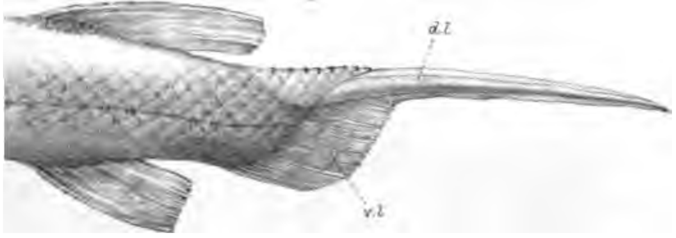

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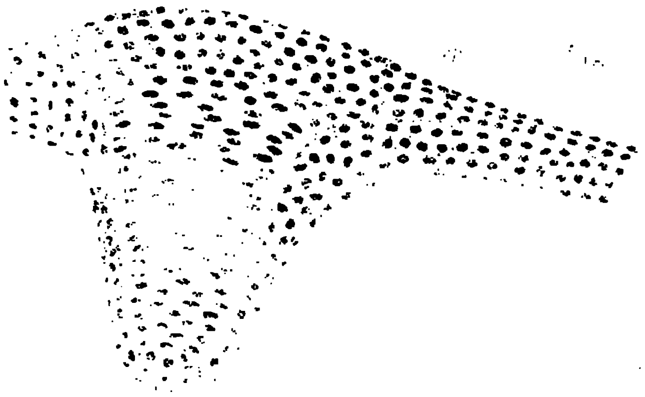

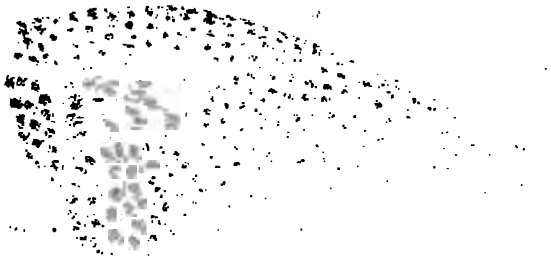

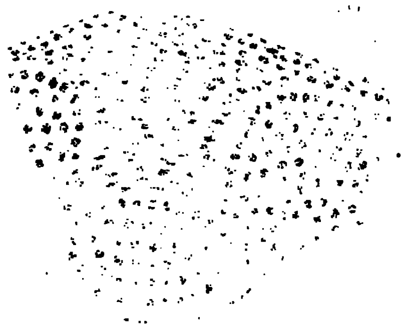

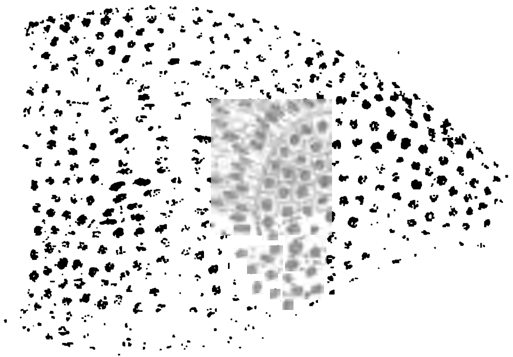

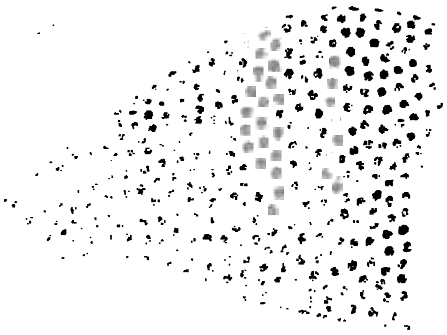

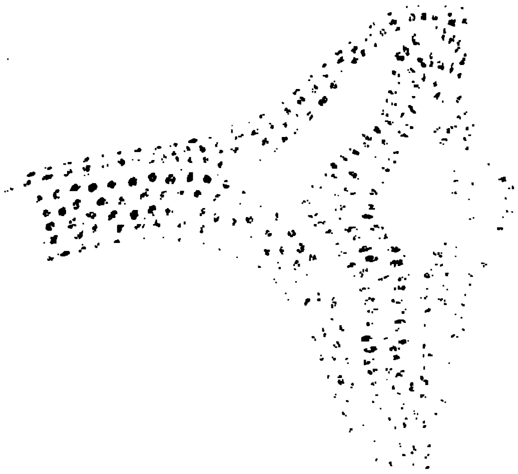

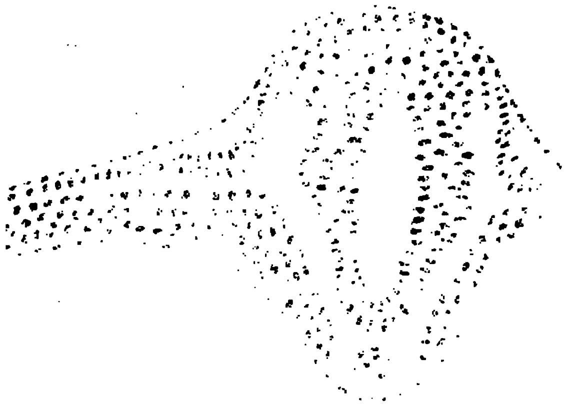

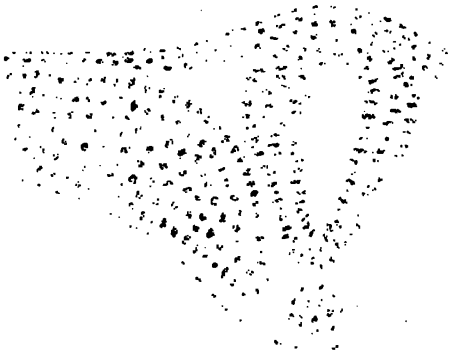

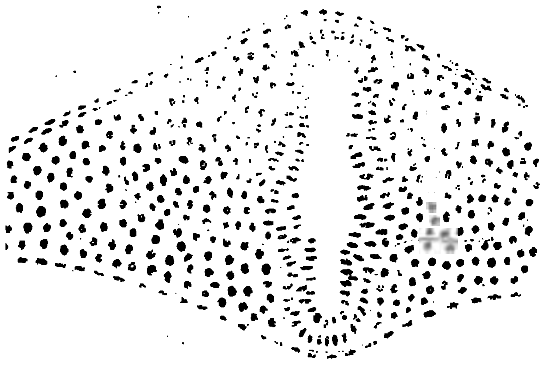

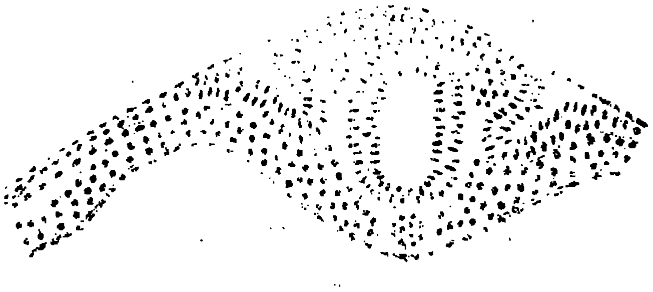

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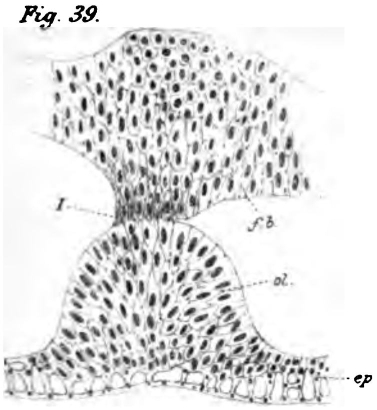

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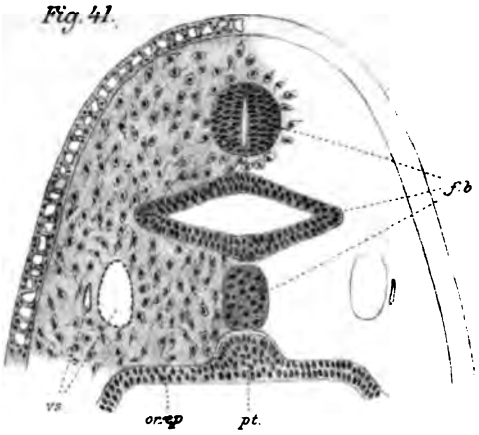

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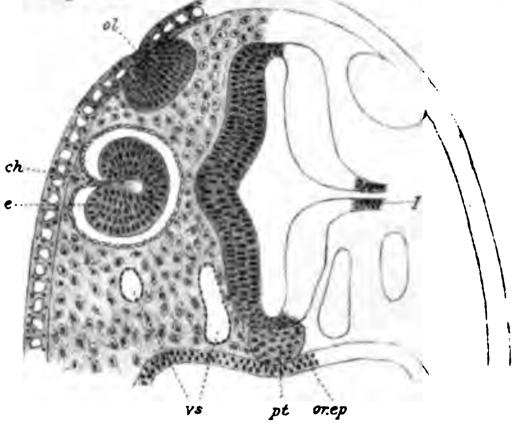

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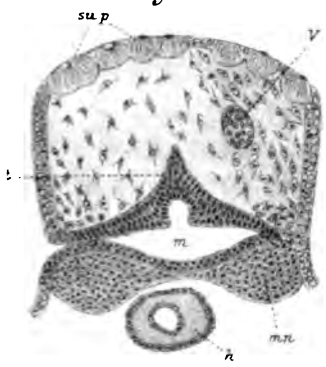

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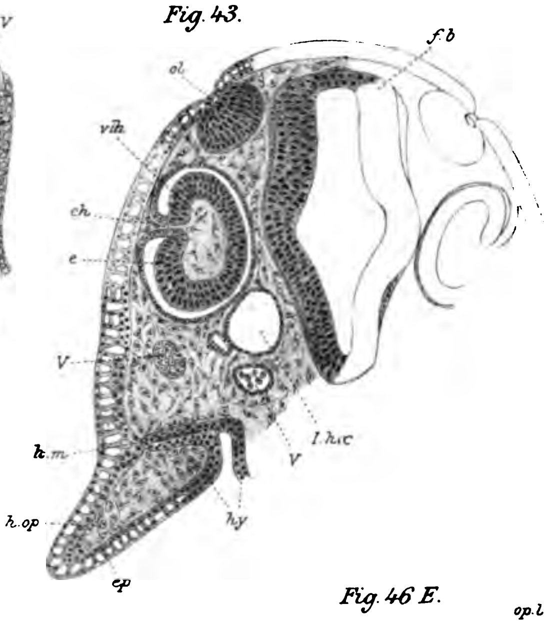

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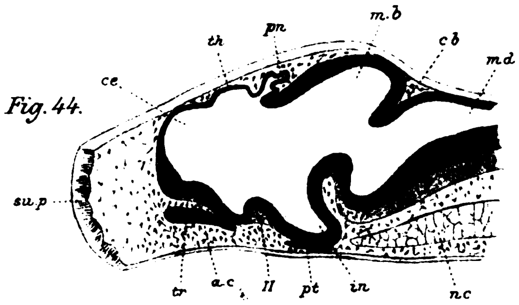

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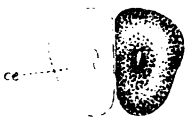


Fig. 46 B.

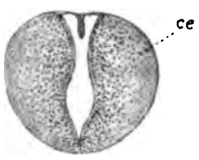


Fig. 46 C.

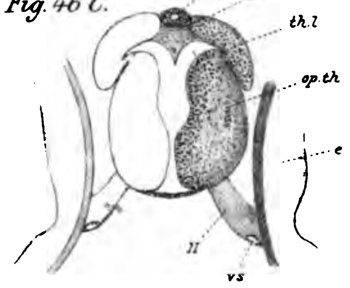


Fig. 46 D.

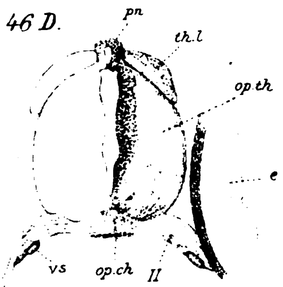


Fig. 46 E.

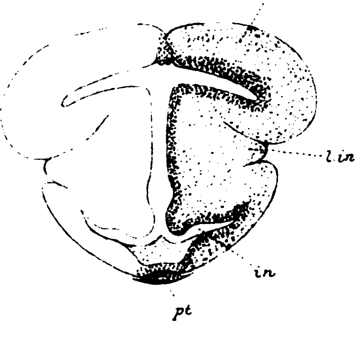


Fig. 46 F.

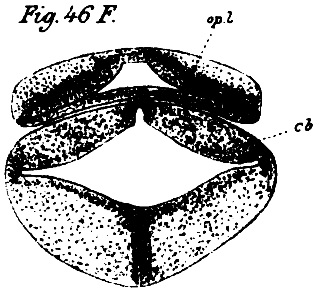


Fig. 46 G.

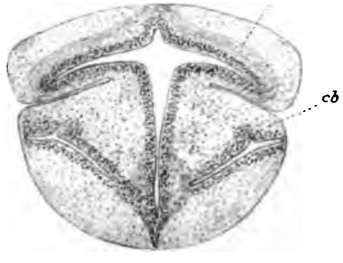
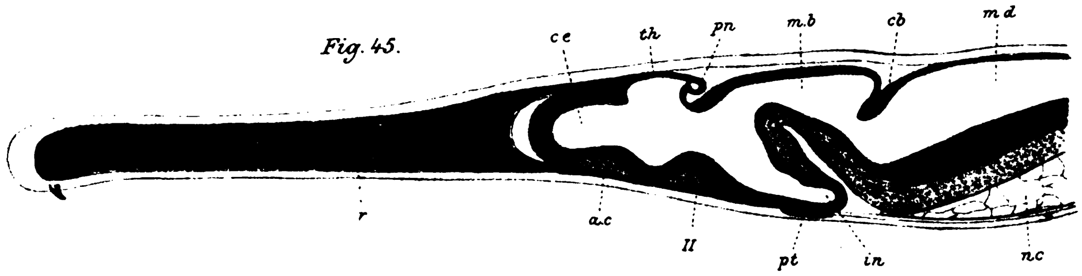
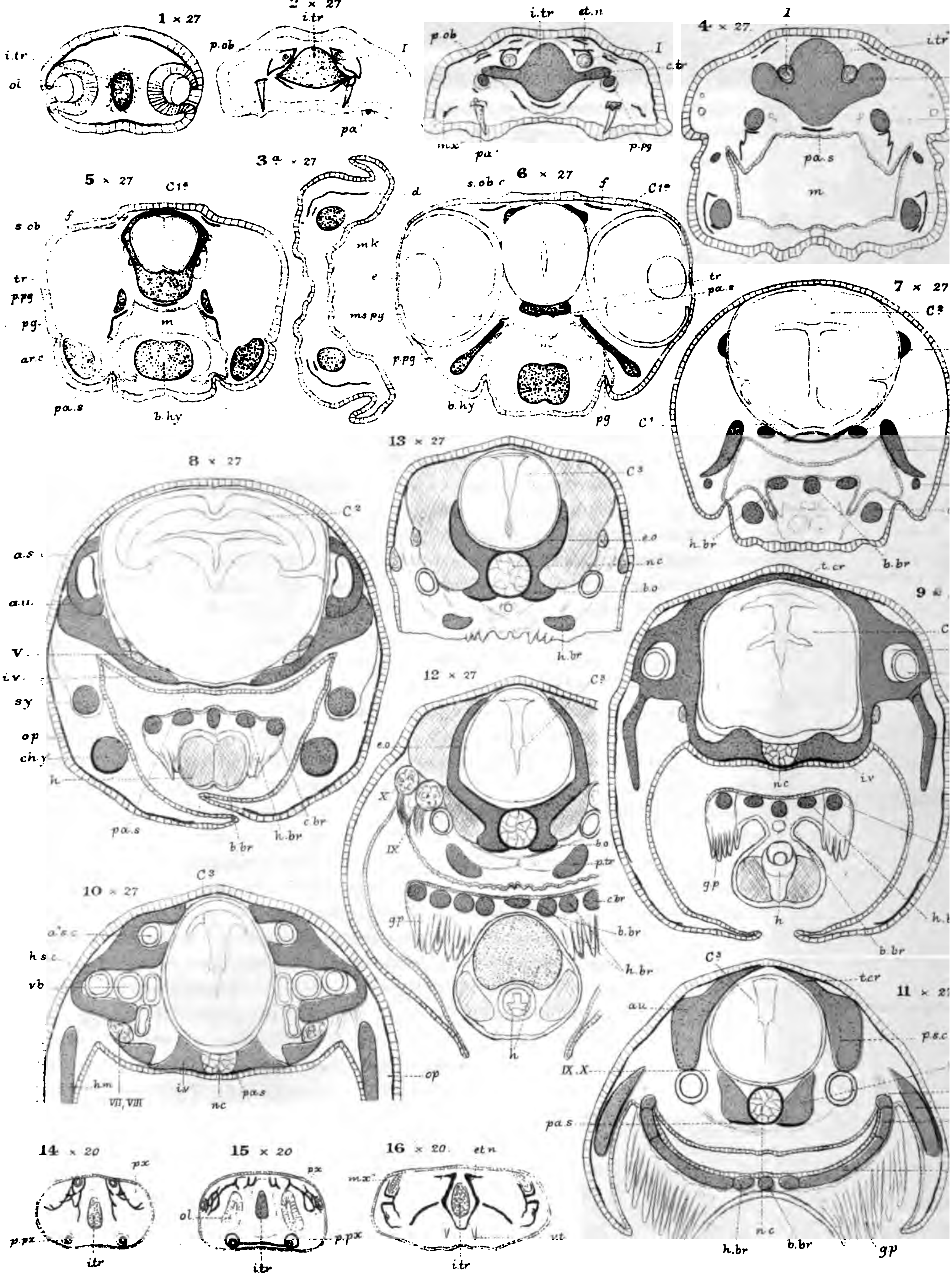


Fig. 45.



W. N. P. del ad nat.

F Huth, Lith^r Edin^r

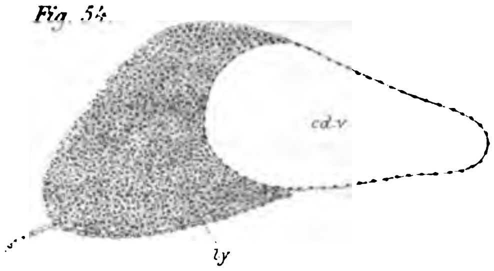


M.P. Parker del. et lith.

LEPIDOSTEUS.

West Newm

Fig. 54.



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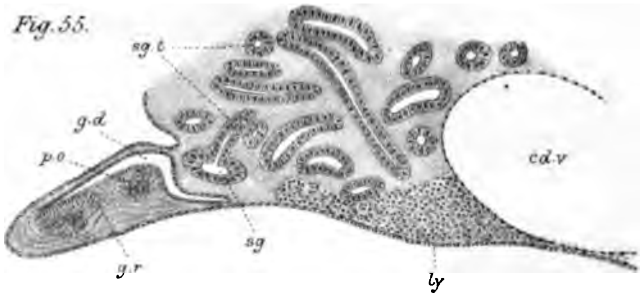

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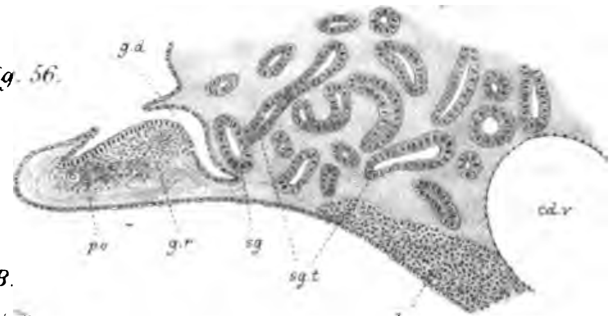

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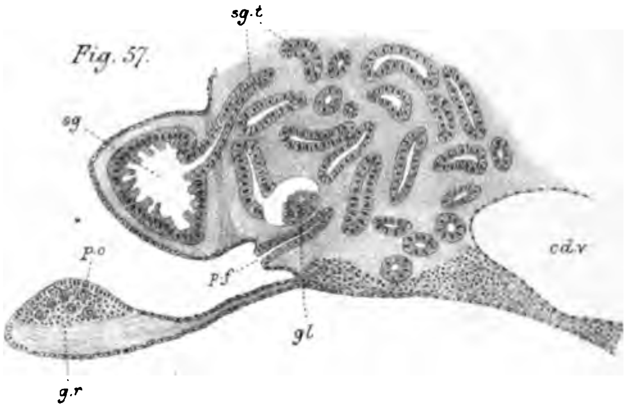
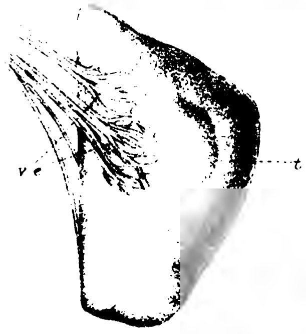


Fig. 58 B.



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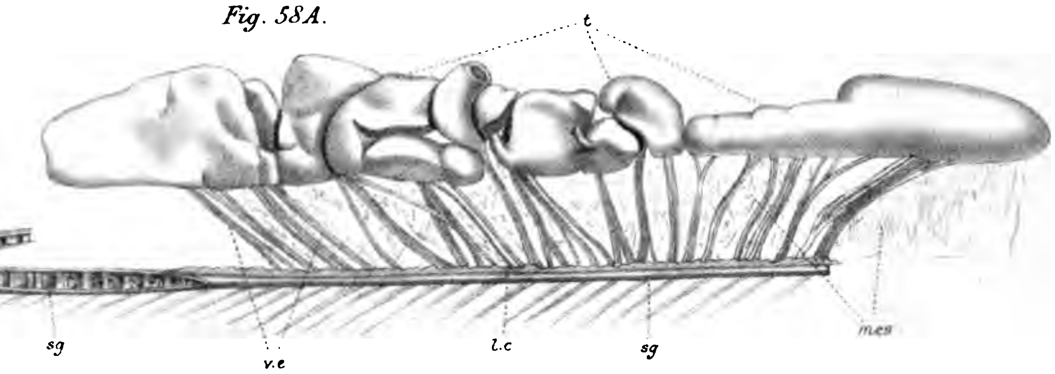

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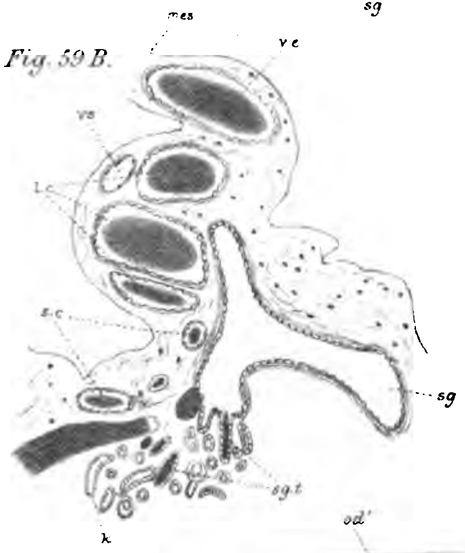

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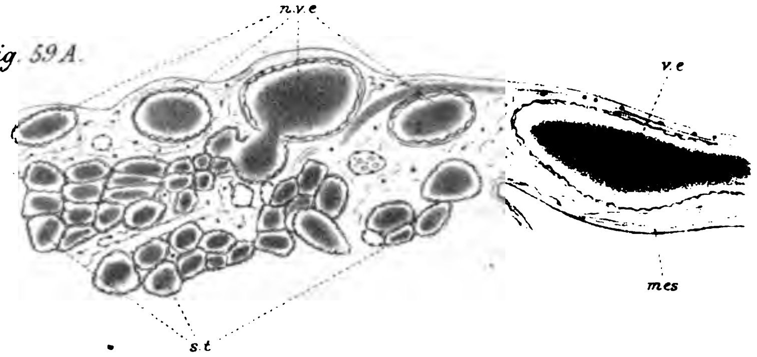

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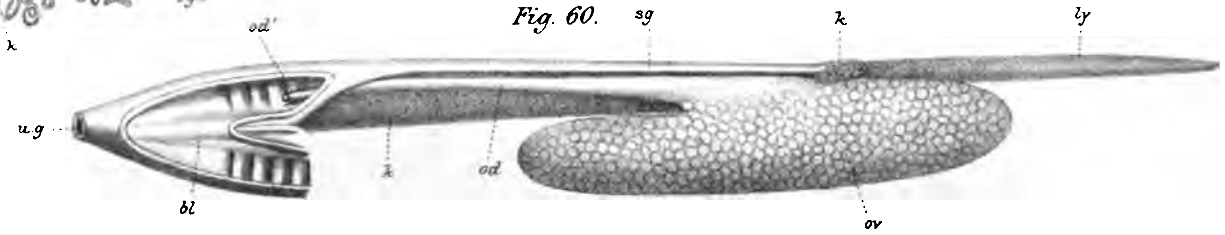
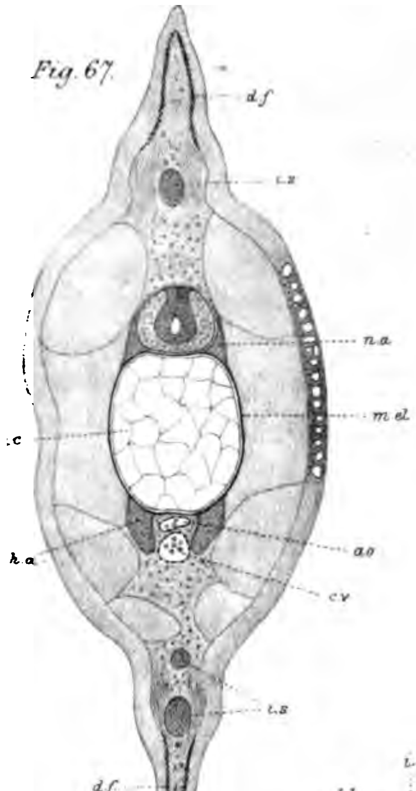


F.M.B., W.N.P. & M.P.P. del. ad nat.

F. Huth, Lith^r Edin^r

Fig. 67.



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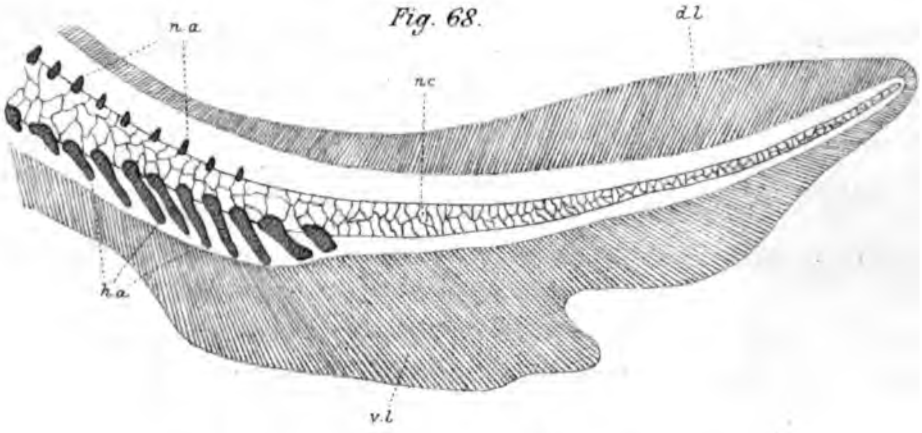

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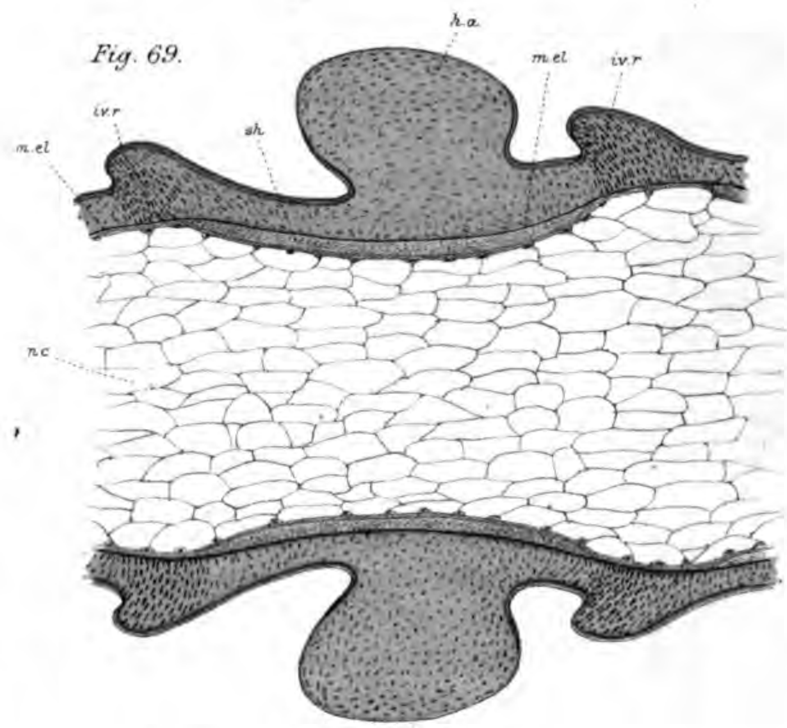

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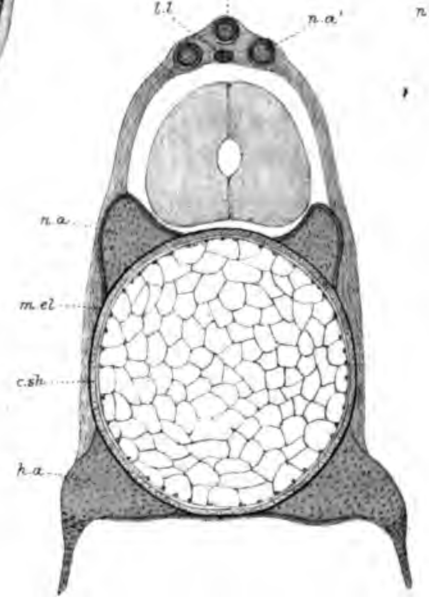

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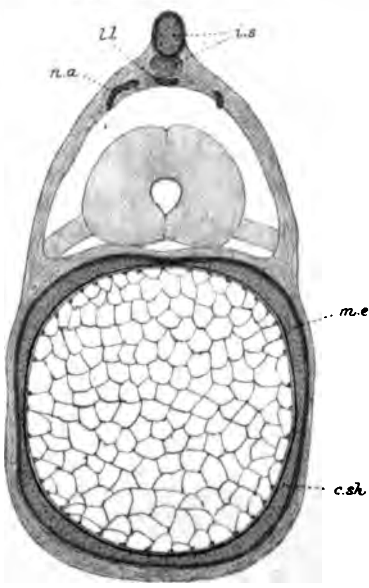

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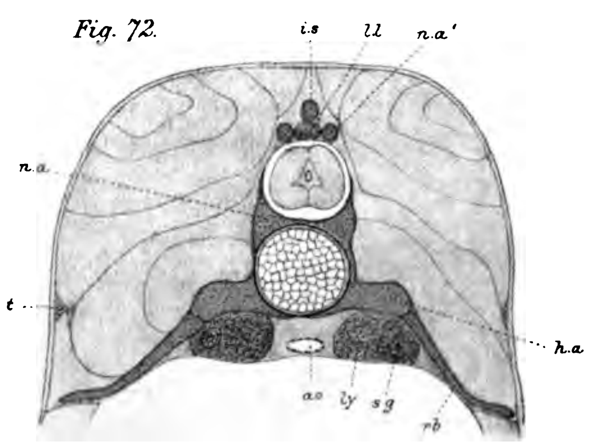

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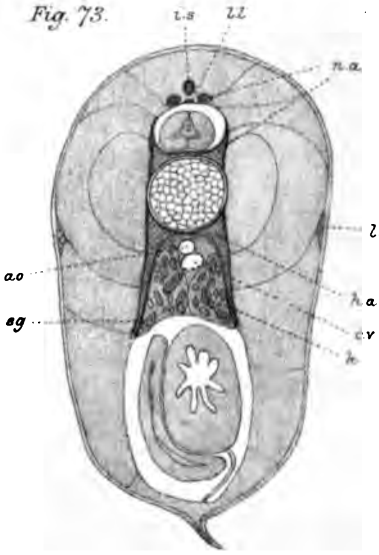

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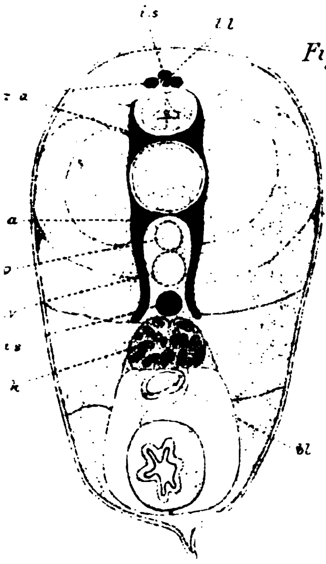

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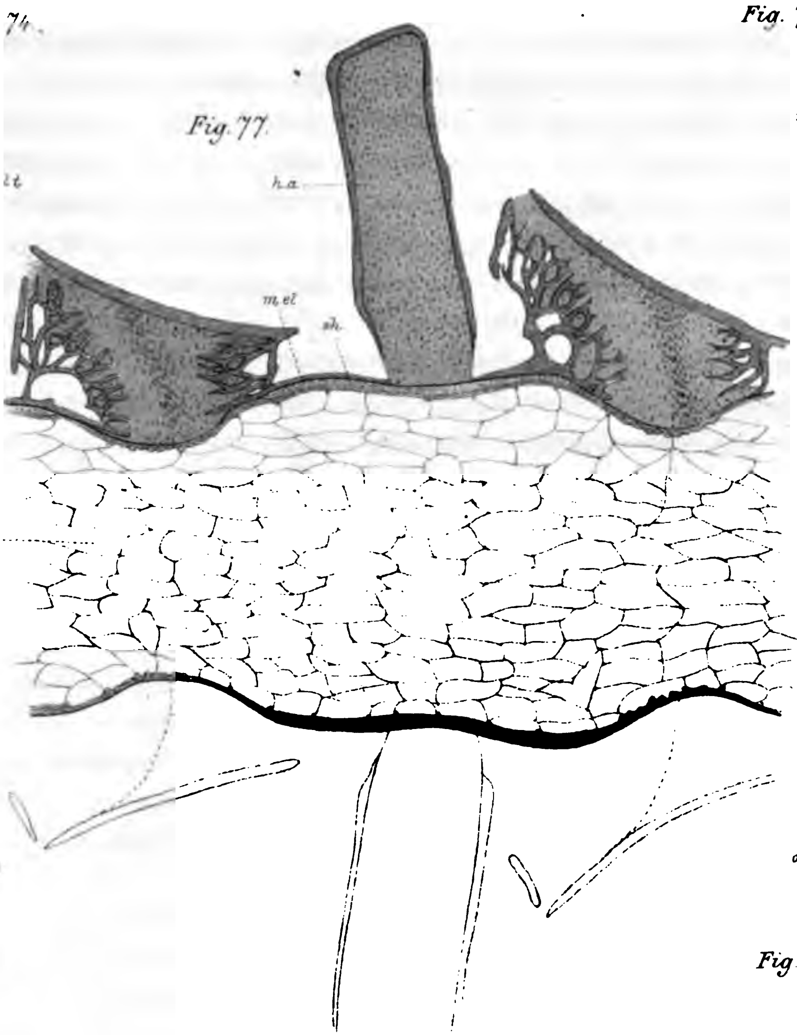

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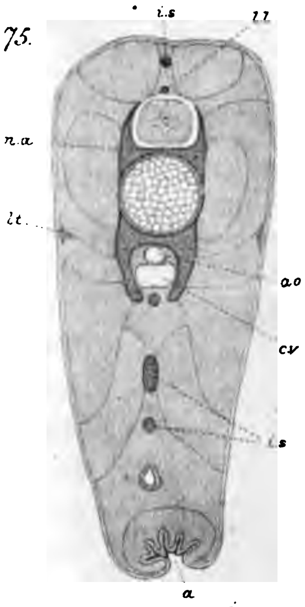

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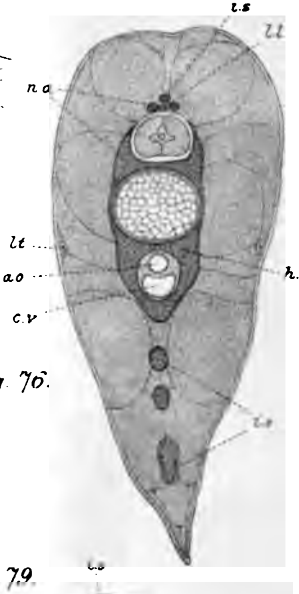

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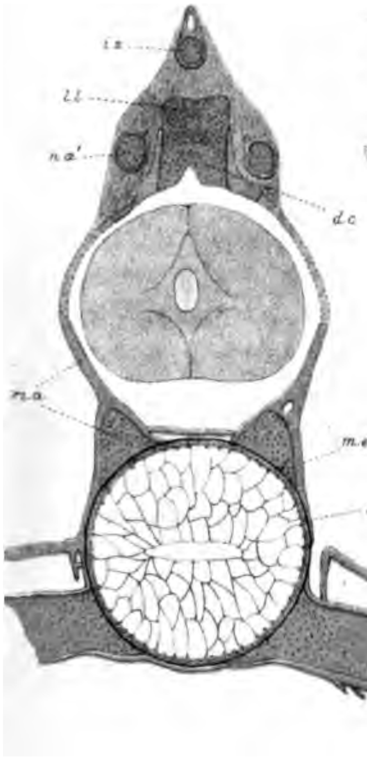

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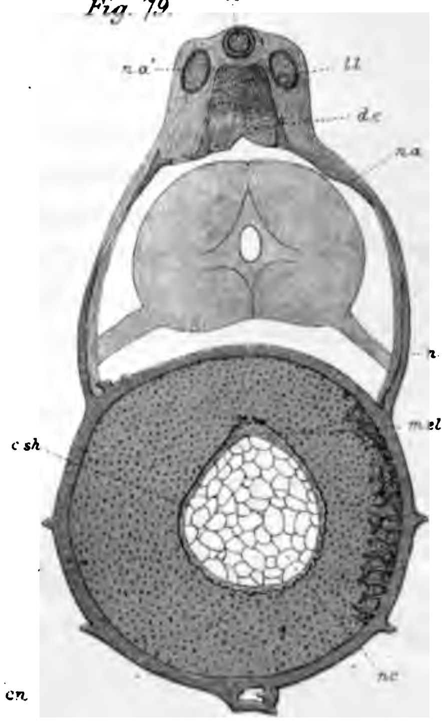

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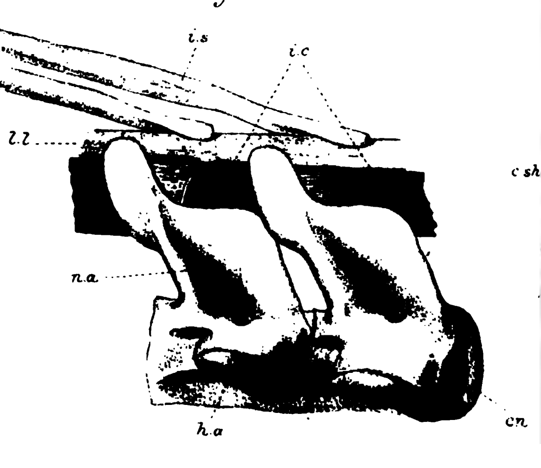

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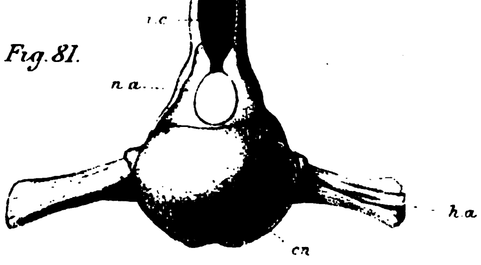

N.P. & M.P.P. del. ad nat.

F. Huth, Lith^r Edin^r

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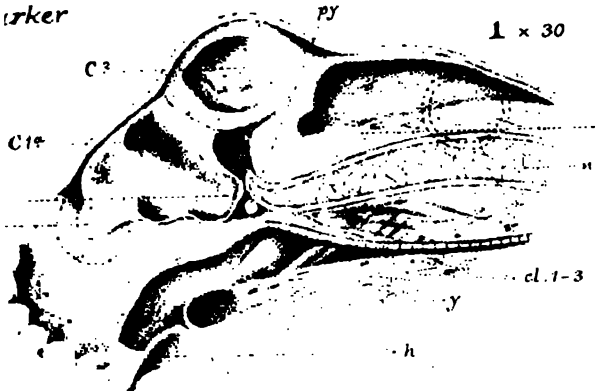
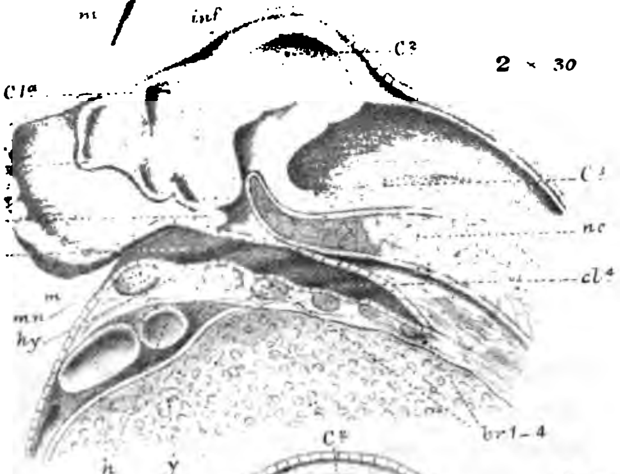
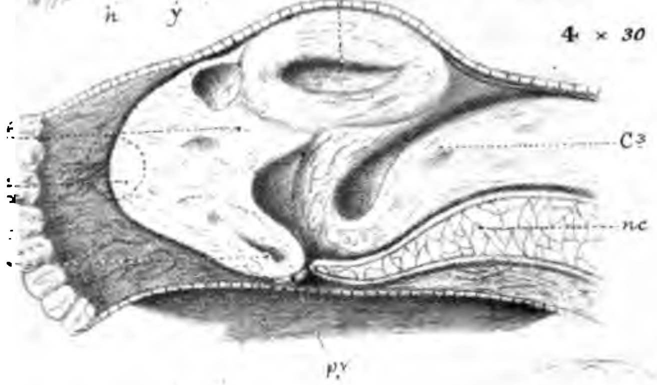


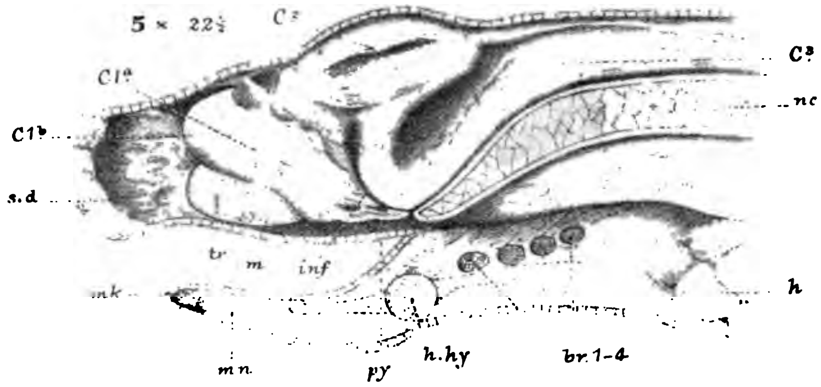
2 × 30



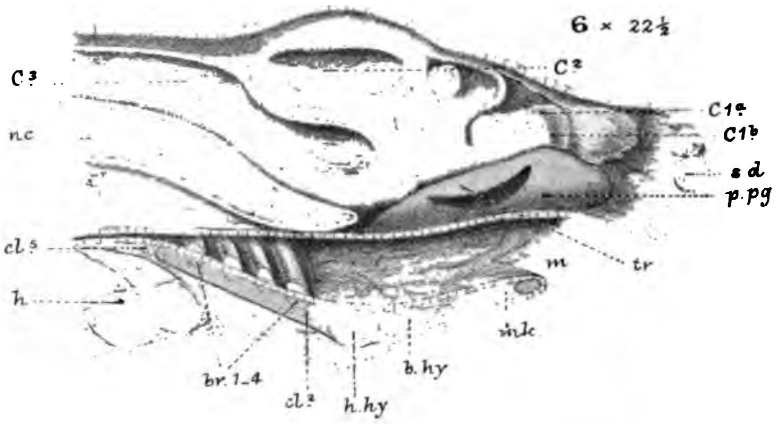
4 × 30



5 × 22½



6 × 22½



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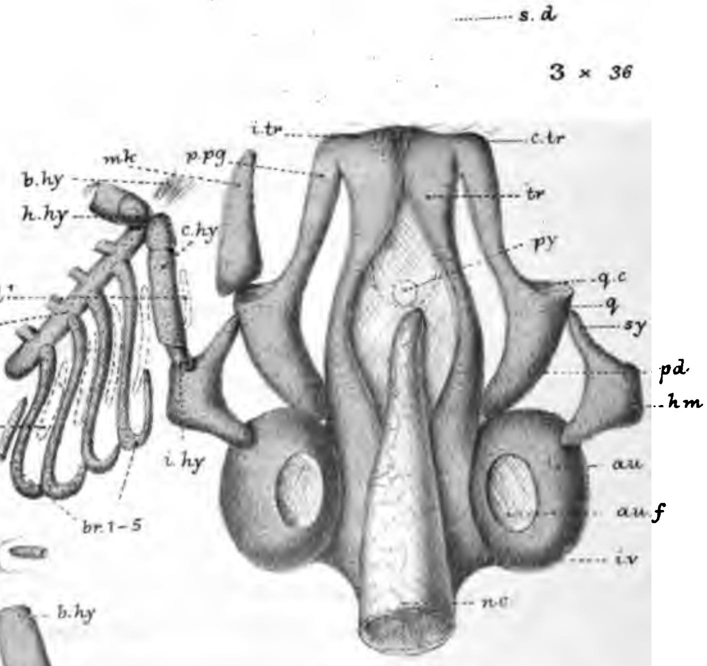

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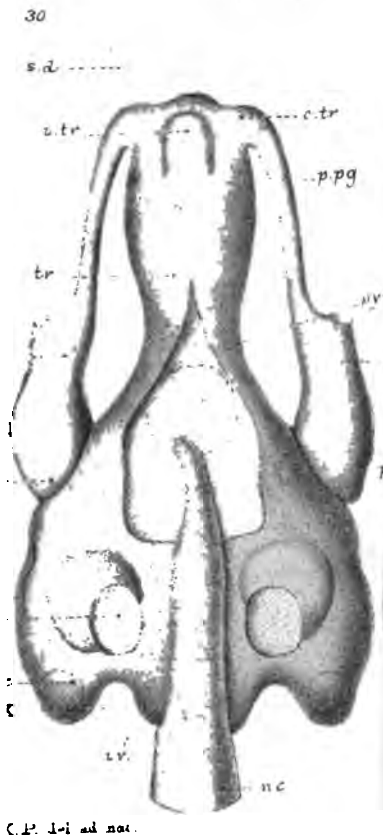
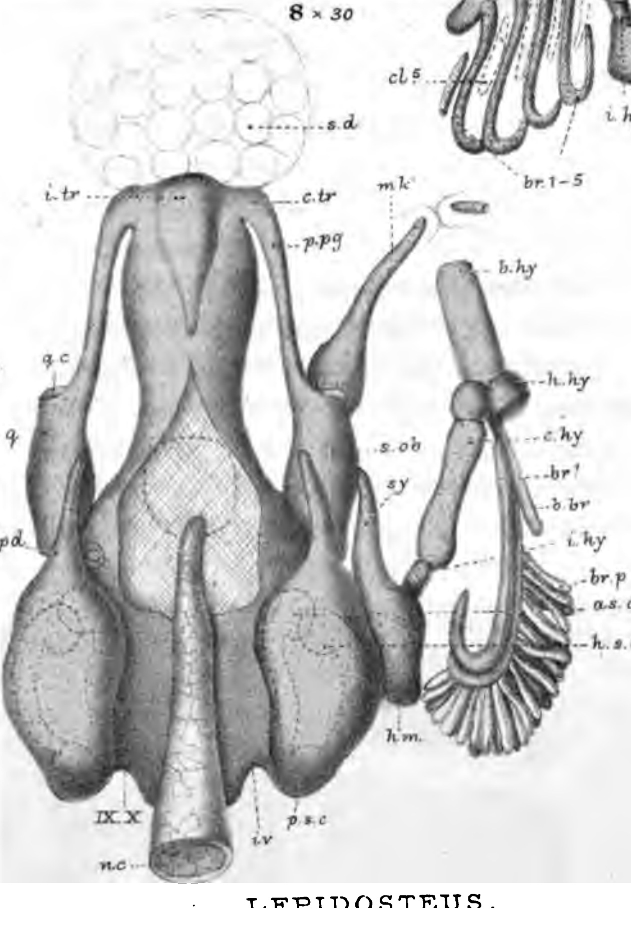
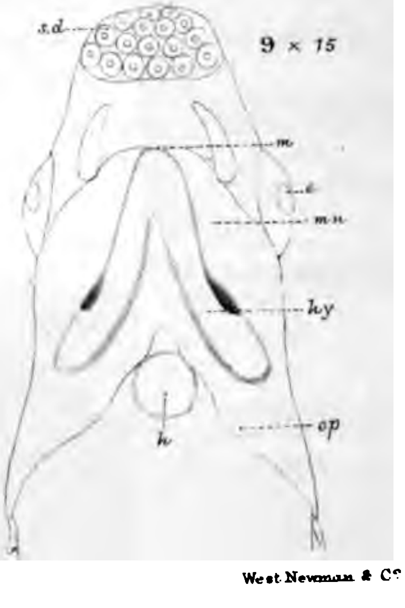


8 × 30



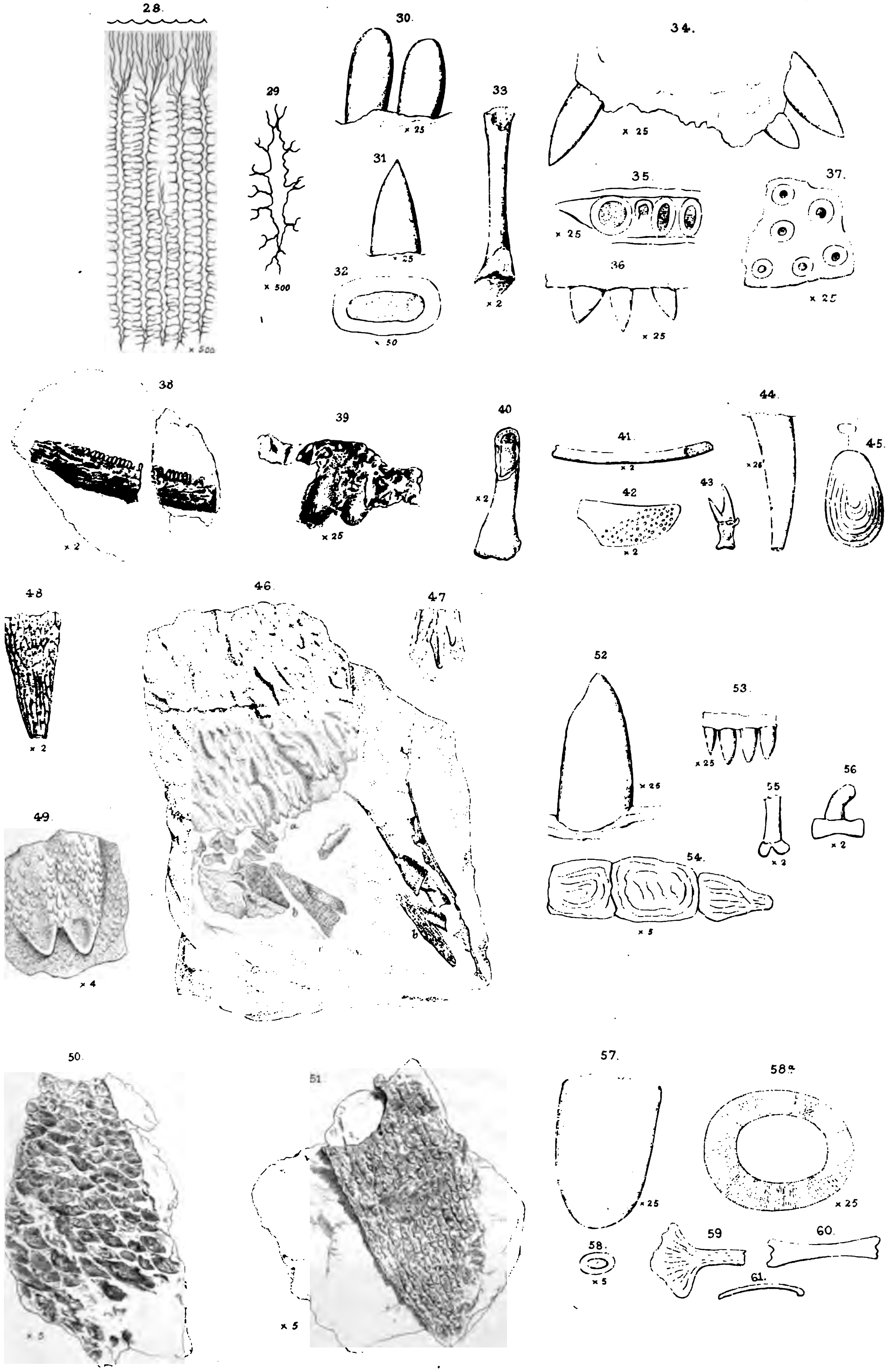
9 × 15



C.P. del ad nat.

LEPIDOSTEUS.

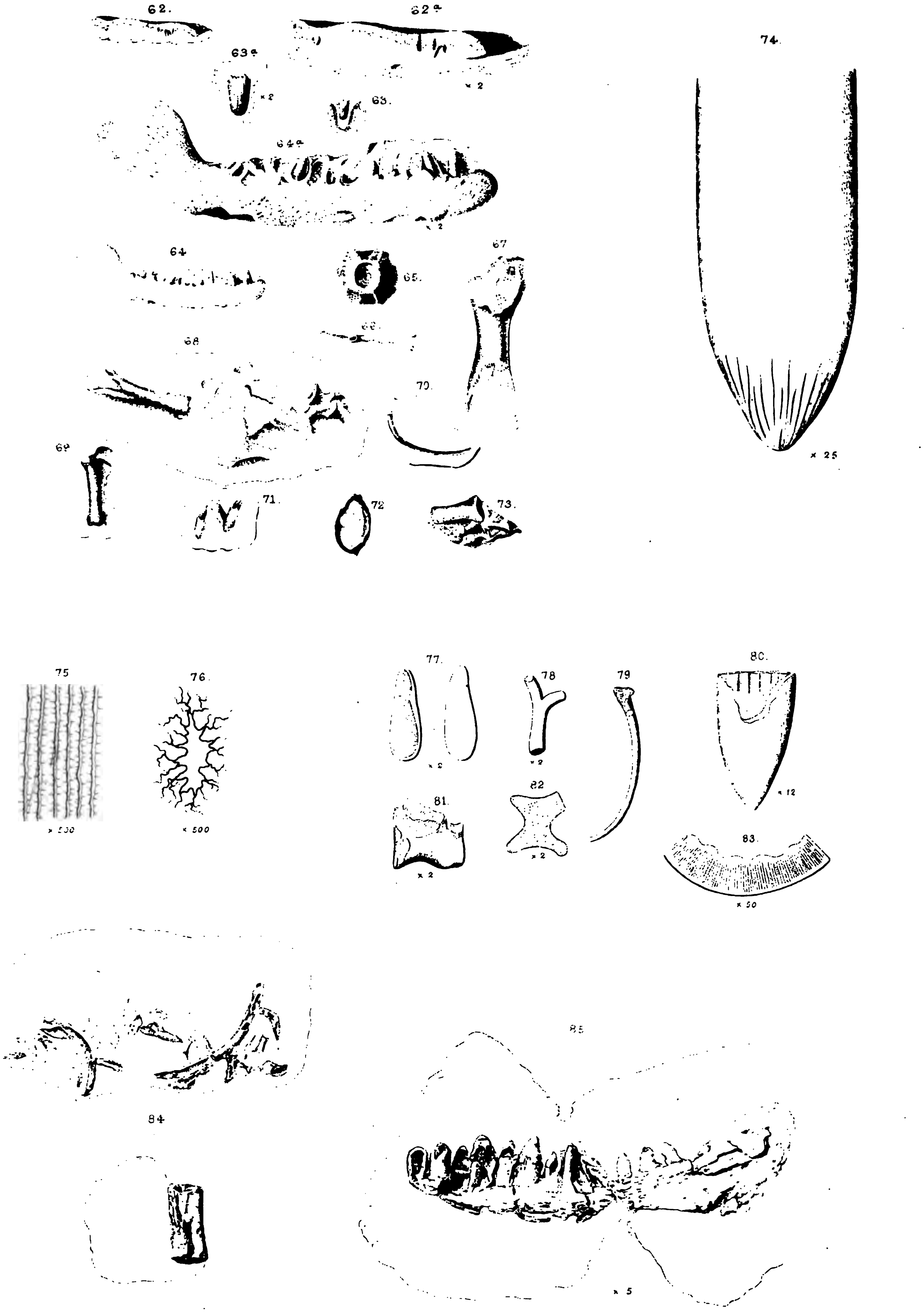
West Newman & Co imp.



West Newman & Cº sc.

SMILERPETON (28 to 45) DENDRERPETON (46 to 51) SPARODUS (52 to 56) AMBLYODON (57 to 61)

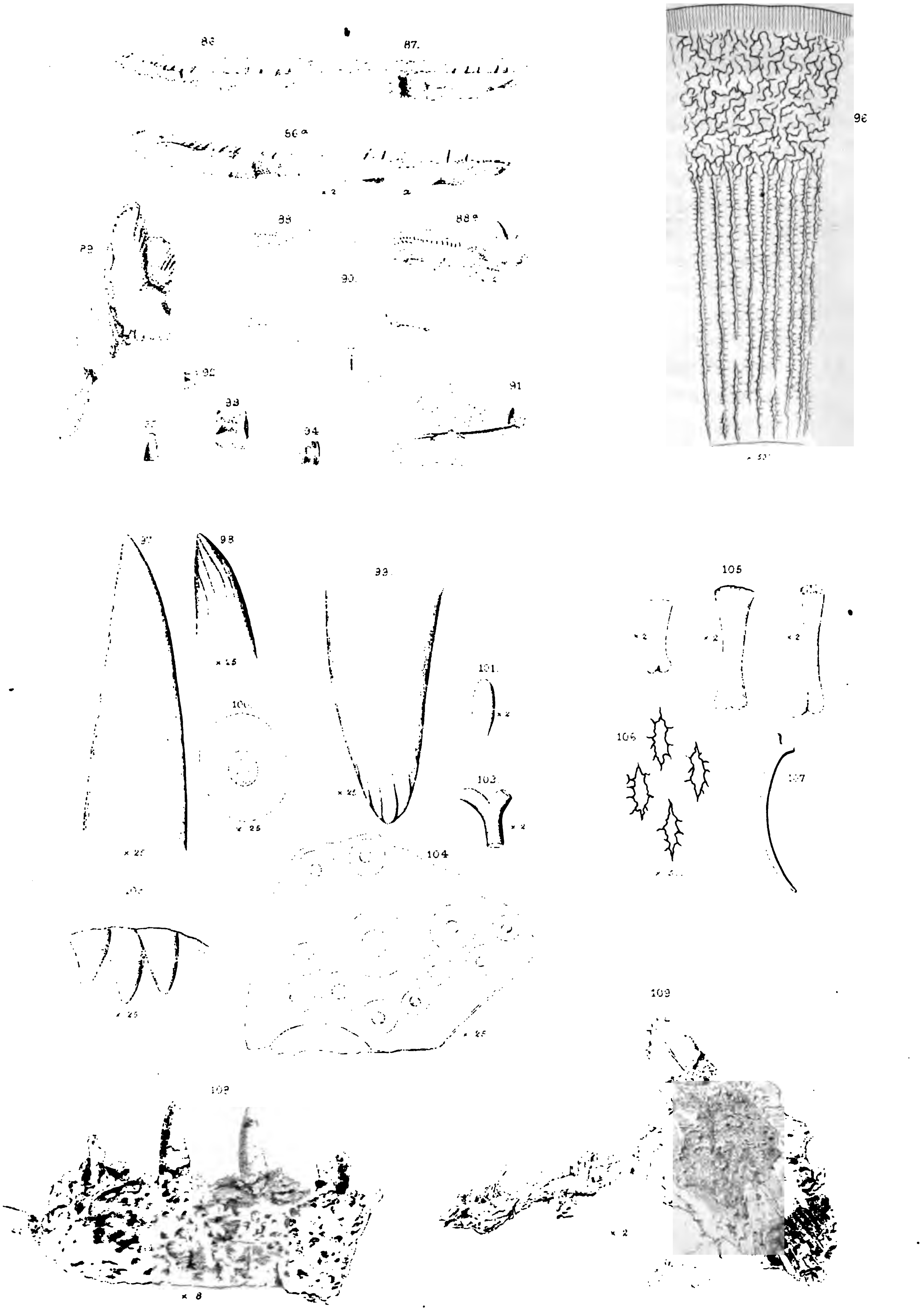
Dawson. *Phil. Trans.* 1882. *Plate* 41.



W. H. Wesley lith.

HYLERPETON DAWSONI.

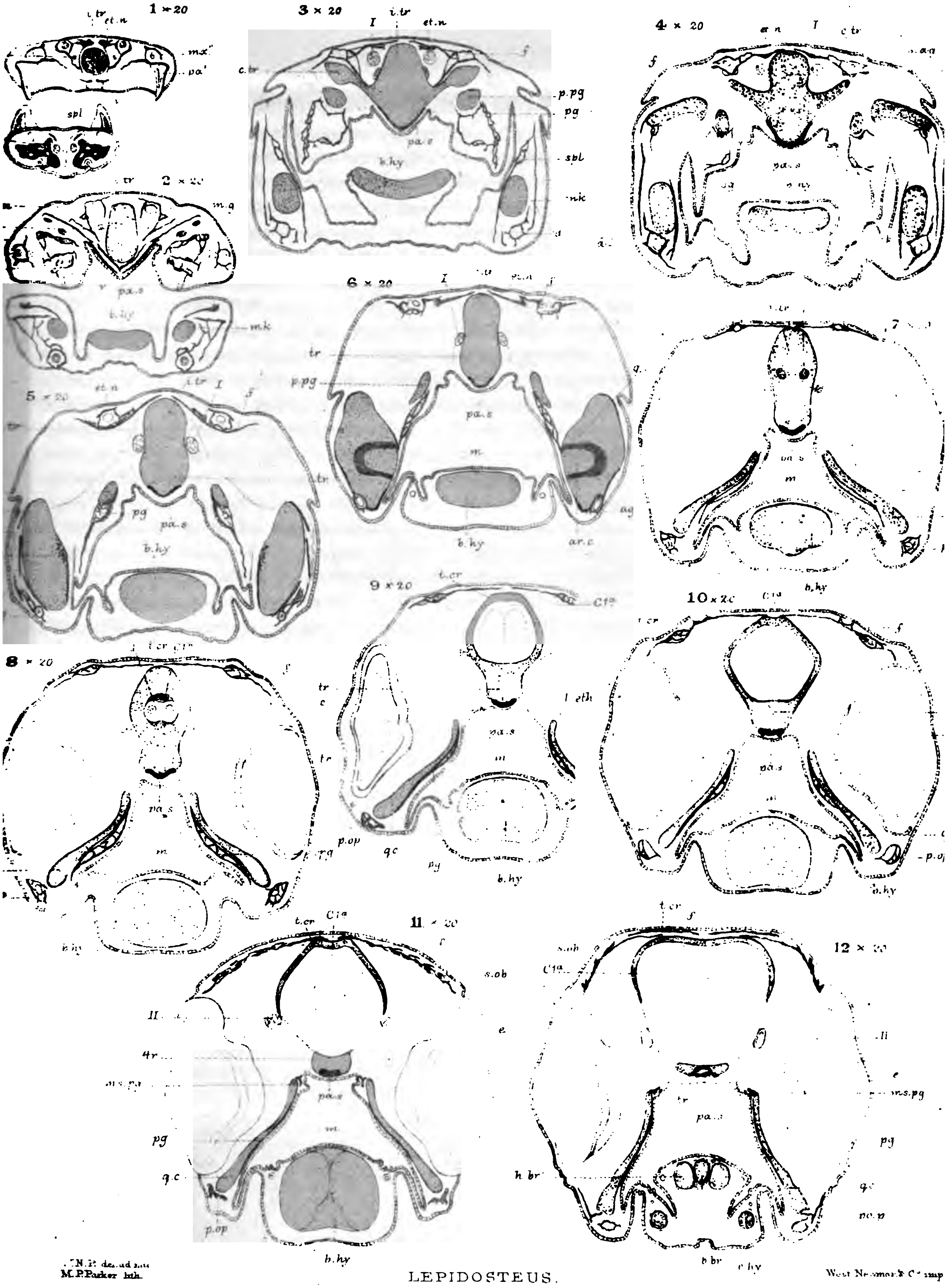
West Newman & Co. imp.



H. Wesley lith.

HYLERPETON LONGIDENTATUM.

West Newman & Co imp.

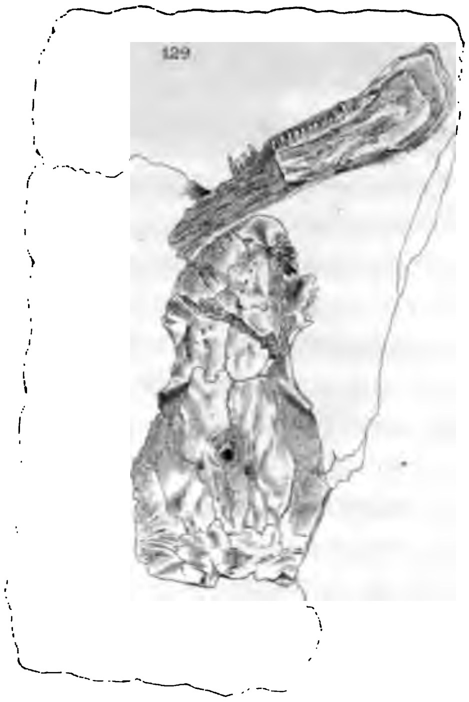


W.K.P. del. ad nat.
M.P. Parker lith.

LEPIDOSTEUS.

West Newman & Co. imp.

129.



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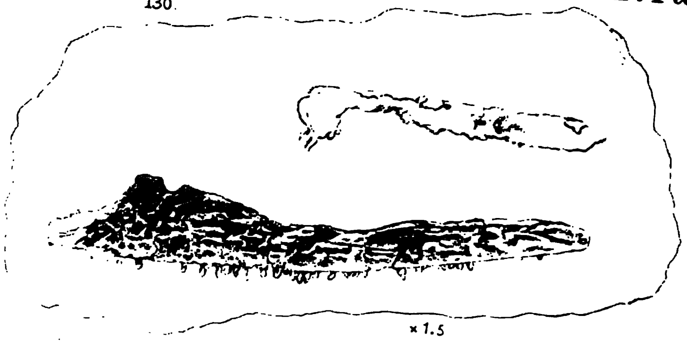
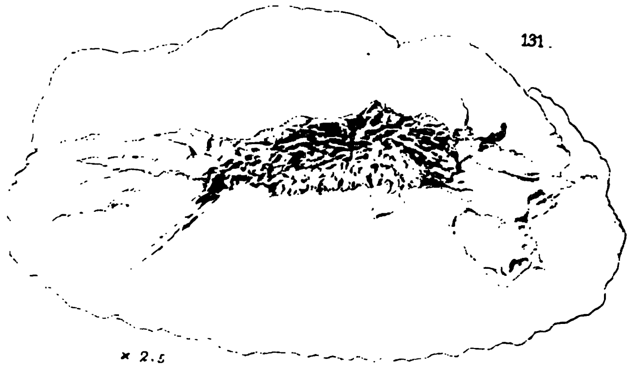


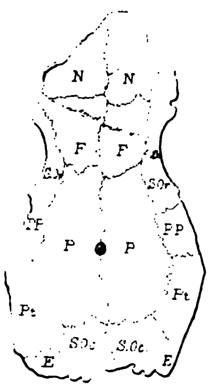
× 1.5

131.



× 2.5

132



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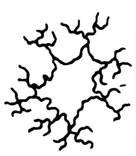
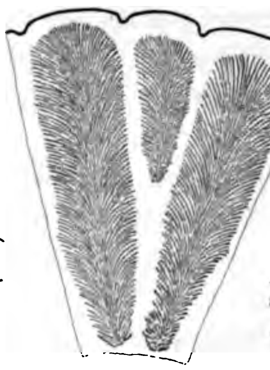


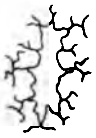
× 500

133.



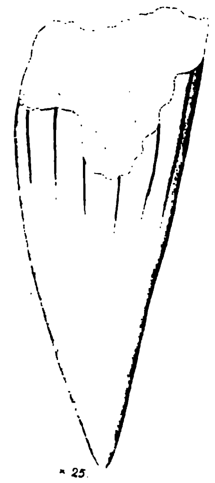
× 250

134



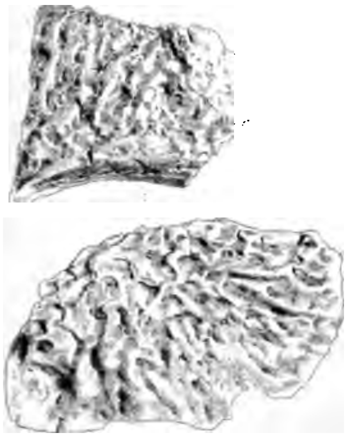
× 500

135



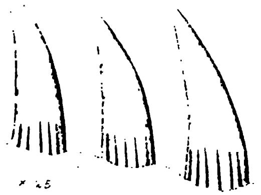
× 25

136



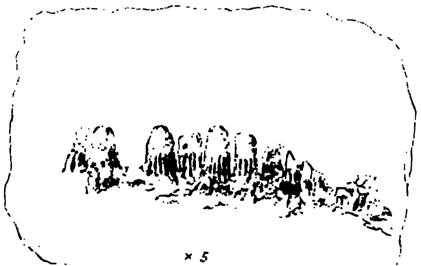
× 5

138.



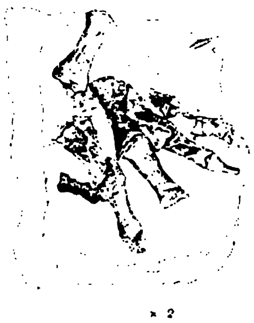
× 25

137



× 5

139.



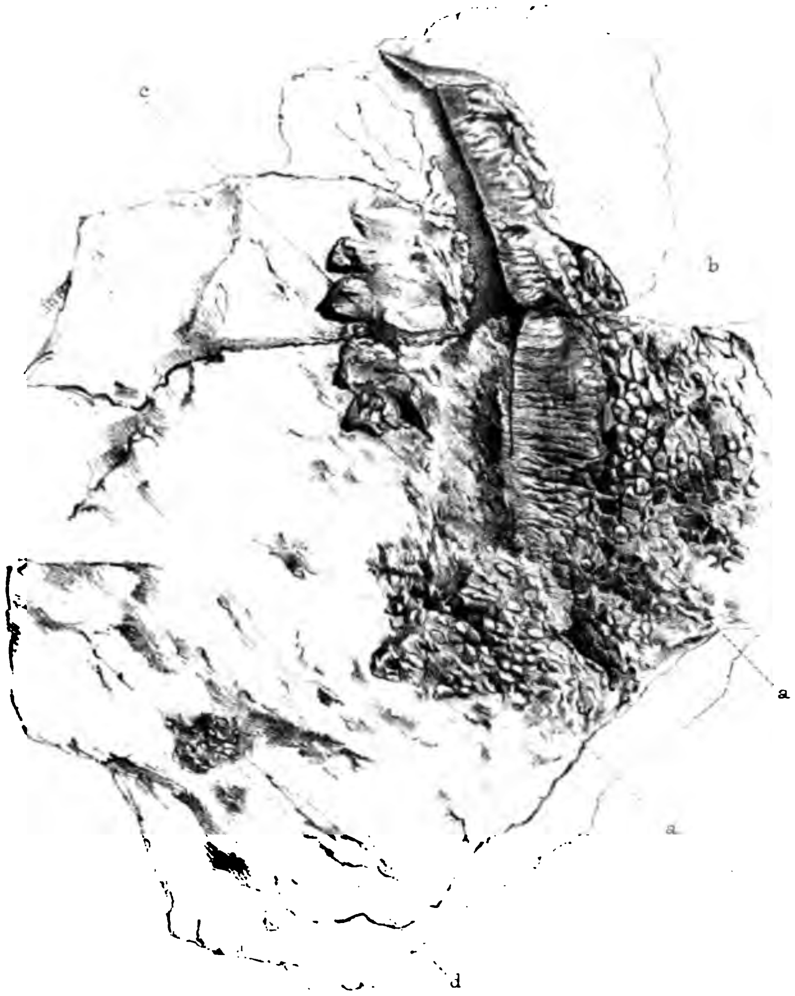
× 2

ley lith.

DENDRERPETON ACADIANUM, D. OWENI, (FIGS 131, 138, 139.)

West Newman & Co imp.

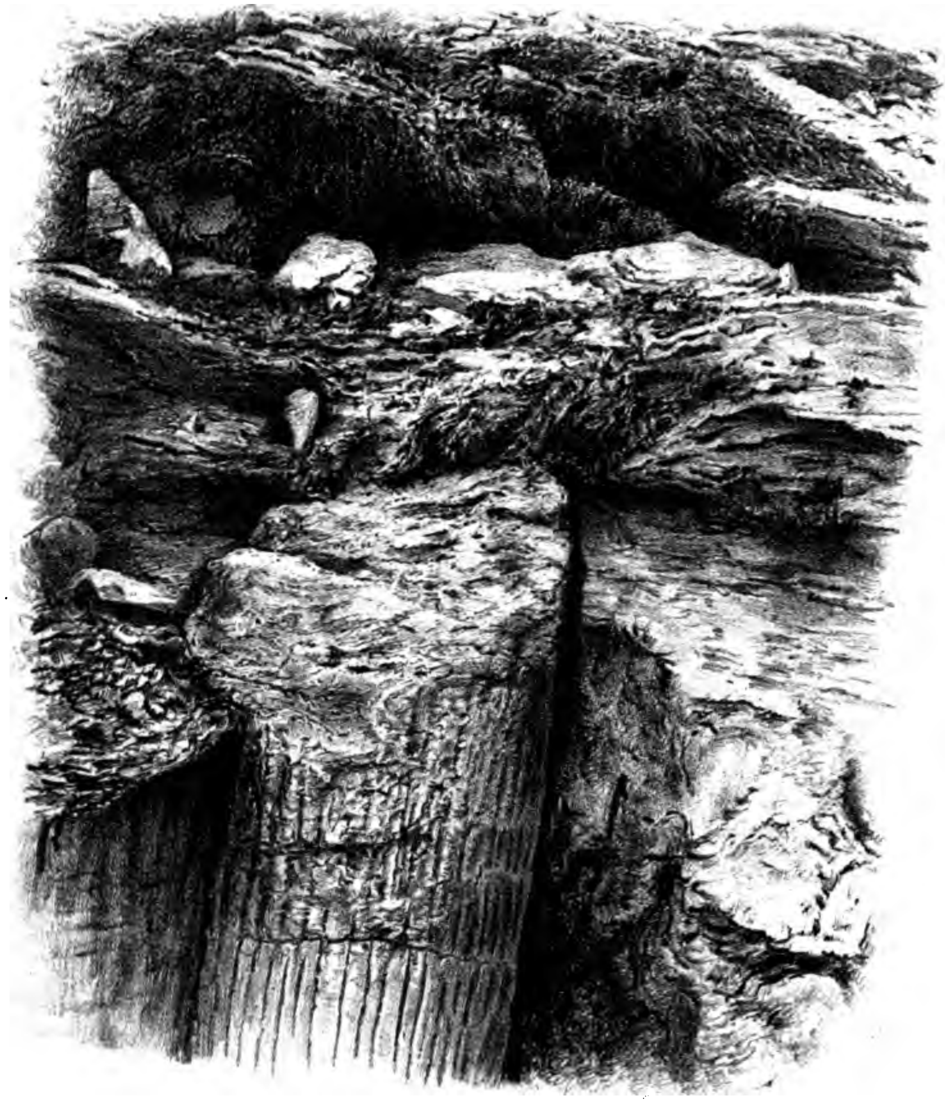
Fig. 140.



HORNY ARMOUR OF BACK OF HYLONOMUS.
× 5
(a) *Imbricated Scales*; (b) *Horny Plates*; (c) *Horny Spines or Tubercles*; (d) *Small imbricated Scales*.

From a Photograph by Inglis, Montreal.

Fig. 141.

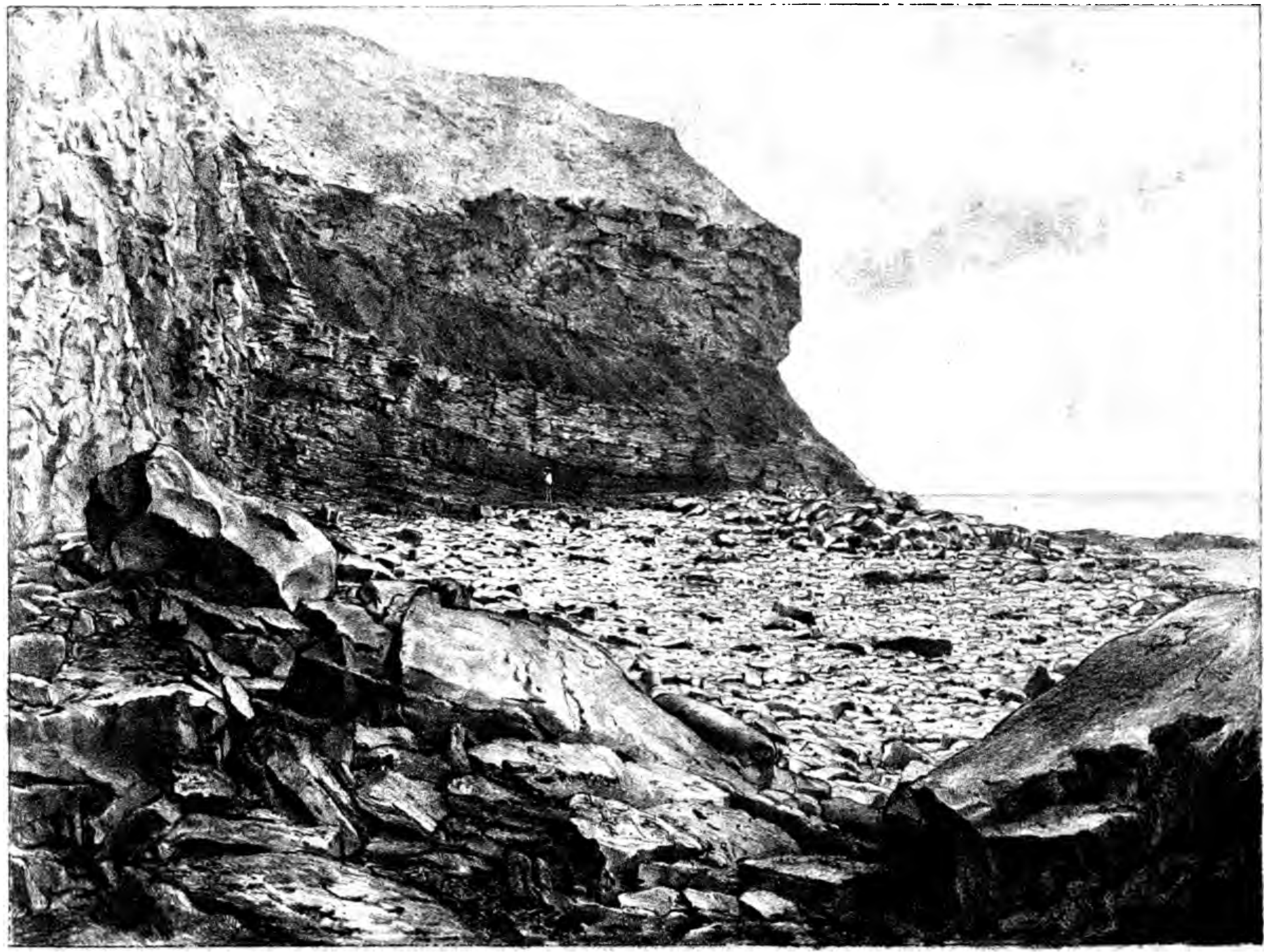


TREE No. 13.
Upper part, in situ, in the reef, after it had been exposed by blasting.

From a Photograph by Weston

W.H. Wesley lith.

West Newman & Co imp.

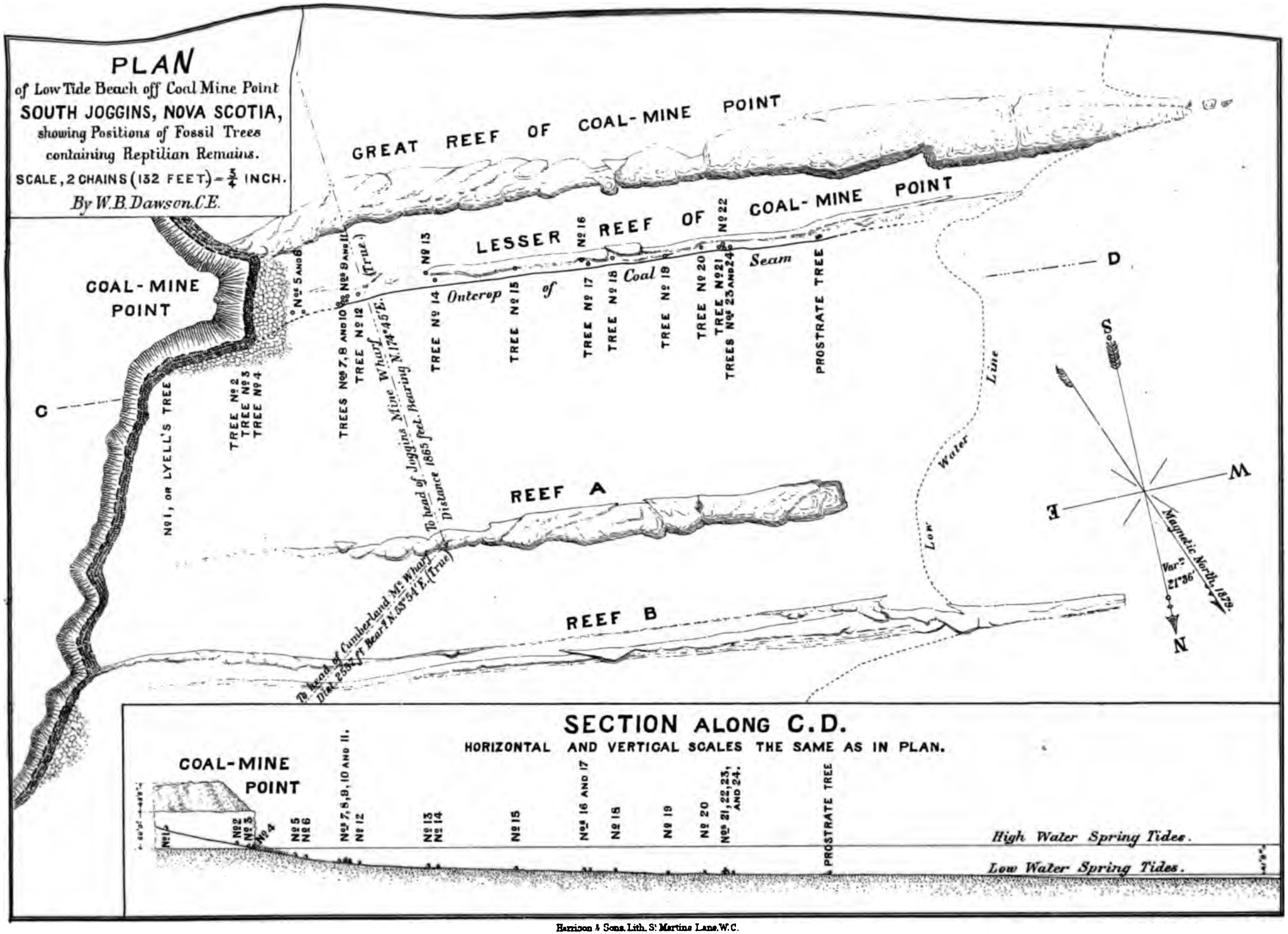


W. H Wesley lith.

West Newman & C° imp.

COAL MINE POINT, S. JOGGINS, 1879.

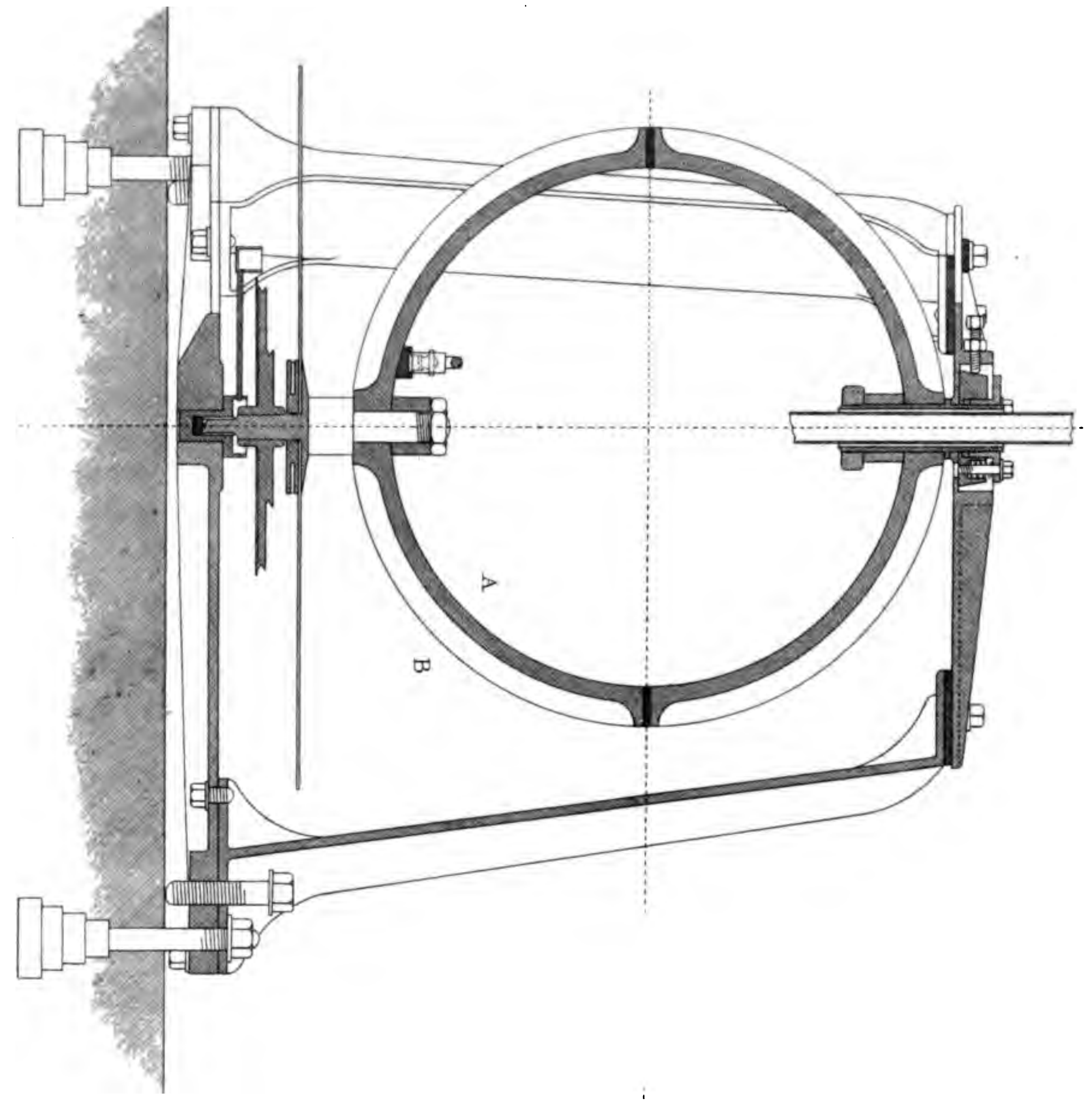
The head of the human figure is immediately below the Outcrop of the Six-inch Coal, on which the fossiliferous trees stand.

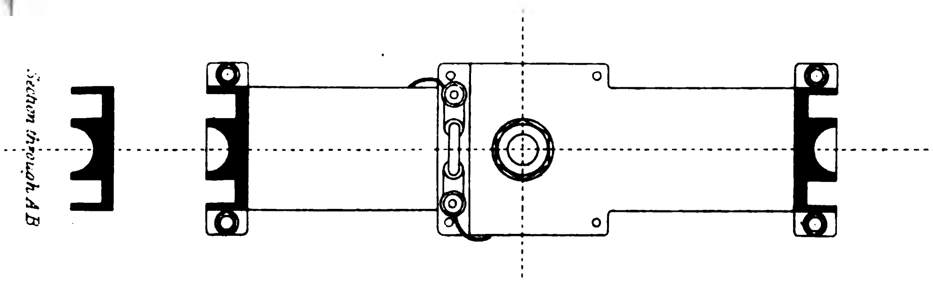


PLAN

of Low Tide Beach off Coal Mine Point
SOUTH JOGGINS, NOVA SCOTIA,
showing Positions of Fossil Trees
containing Reptilian Remains.
SCALE, 2 CHAINS (132 FEET) = 3/4 INCH.
By W. B. Dawson, C.E.

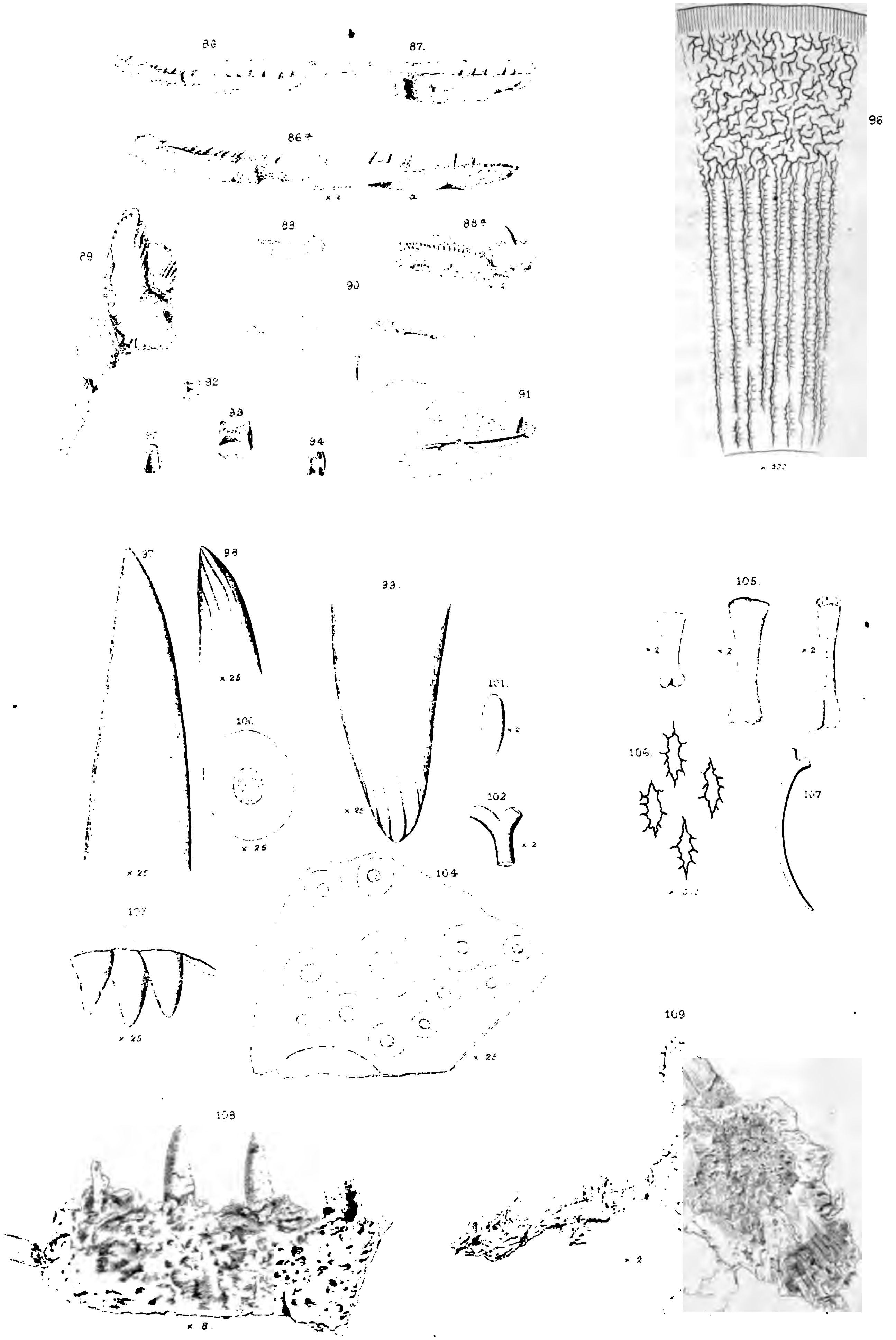
Harrison & Sons, Lith. St Martins Lane, W.C.





Scale one sixth.

Harrison & Sons Lith. St Martins Lane W.C.



J.H. Westley lith.

HYLERPETON LONGIDENTATUM.

West Newman & Co imp.